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Temperature density, temperature conductivity and conductivity-density  
relationships for marine-derived saline lake waters

John A.E. Gibson, John M. Ferris and Harry R. Burton



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# TEMPERATURE-DENSITY, TEMPERATURE-CONDUCTIVITY AND CONDUCTIVITY-DENSITY RELATIONSHIPS FOR MARINE-DERIVED SALINE LAKE WATERS

by

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## ABSTRACT

Equations, determined using water samples from marine-derived saline lakes in the Vestfold Hills, Antarctica, relating  $\sigma_T$  [(density-1000) kg/m<sup>3</sup> at temperature T] at temperatures between -15°C and 40°C to  $\sigma_{20}$ , electrical conductivity at temperatures from -15°C to 20°C to conductivity at 0°C, and  $\sigma_{20}$  to electrical conductivity at 0°C are given. These relationships allow *in situ*  $\sigma_T$  to be calculated from *in situ* electrical conductivity and temperature data.



## 1. INTRODUCTION

Limnologists studying *in situ* density profiles of stratified lakes can obtain data by two basic methods. Firstly, samples can be collected and the density, or another parameter related to the density, such as electrical conductivity or total dissolved salt content, can be determined in the laboratory. Secondly, *in situ* profiles of electrical conductivity (hereafter termed simply conductivity) can be obtained using a conductivity-temperature-depth logger or similar equipment. Both approaches have problems, however, in that density and conductivity are temperature dependant and it is therefore difficult to recover actual *in situ* densities from the data.

The variations of density and conductivity with temperature depend on the ratios of the ionic species present in the water and therefore differ from lake to lake. The water of the saline lakes of the Vestfold Hills, Antarctica have, generally speaking, similar ionic ratios. They were formed from pockets of seawater trapped as the surrounding land rose during the last 8000 years (Burton 1981a,b). Thus, it is expected that general relationships describing the temperature dependence of density and conductivity should hold reasonably well for all the saline lakes of the region.

Little information is available regarding the variations of density and electrical conductivity of hypersaline water with temperature, especially at low temperatures close to the freezing point of the solutions. This note reports the results of two sets of experiments that investigated, the relationships, firstly, of water densities determined at 20°C to densities at temperatures from -15°C to 40°C and, secondly, electrical conductivities at 0°C to conductivities at temperatures between -15°C and 20°C. The empirical relationships derived allow simple conversion of experimental data to more meaningful *in situ* densities.

## 2. EXPERIMENTAL METHODS

Water samples were obtained from various lakes in the Vestfold Hills. Some samples were diluted with distilled, deionised water in order to obtain a better range of water densities. The provenience of the water samples is listed in Tables 1 and 3.

The density of the lake water samples was determined over a range of temperatures between -15°C and 40°C (maintained using a MGW Lauda PMS water bath) with an Anton-Paar DMA 55 density meter accurate to five decimal places. The density meter measured absolute density at the experimental temperature (accuracy  $\pm 0.1^\circ\text{C}$ ).

Electrical resistivity of water samples was measured with a Phillips PW 9054 resistance bridge. The resistance readings, accurate to approximately  $\pm 1\%$ , were converted to conductivities using a cell constant determined by calibration with 1.00 molar potassium chloride. Water temperature, which ranged from -15°C to 20°C, was measured with a Jenway Water Analyser model number PWA 2, which was accurate to  $\pm 0.1^\circ\text{C}$ . Water samples were divided into two portions, one of which was cooled in a freezer. One of the portions was placed in a Dewar flask and the other added in small increments. Conductivity and temperature readings were taken after each addition. Care was taken to ensure that evaporation of samples was minimised by keeping them in sealed containers for as much time as possible. This procedure was repeated to give up to fifty data points for each sample. The density of the water samples at 20°C was measured with an Anton-Paar DMA 55 density meter as above.

*In situ* conductivity and temperature profiles were obtained using a Platypus Conductivity-Temperature-Depth recorder (CTD) (Platypus Engineering, Loyetee, Tasmania), which was deployed through a hole cut in the ice cover with a Jiffy ice drill. The CTD was lowered at 10 cm intervals and left at each depth for 30 seconds, which was found to be long enough for thermal equilibrium to be obtained.

### 3. RESULTS

#### 3.1 TEMPERATURE-DENSITY RELATIONSHIP

Density measurements of thirteen water samples from nine lakes of the Vestfold Hills, ranging in salinity from near fresh to over 200‰, were undertaken at range of temperatures from -15°C to 40°C. Density measurements were converted to  $\sigma_T$ , which was defined at temperature T as:

$$\sigma_T = (\text{Density} - 1000) \text{ kg/m}^3 \quad (1)$$

Plots of the experimental data are presented in Appendix I. For each sample the data formed a smooth and continuous curve, to which could be accurately fitted a quadratic equation of the form:

$$\sigma_T = A + BT + CT^2 \quad (2)$$

in which T is the temperature. The constants A, B and C, listed in Table 1, were found to form regular quadratic curves when plotted against experimentally determined  $\sigma_{20}$  (Figure 1). The reference temperature was chosen to be 20°C on the grounds of convenience in that it is readily attainable in the laboratory and because seawater densities at 20°C are commonly used in oceanography (Pickard 1979). The equations of the curves for A, B and C were as follows:

$$A = 1.9671 + 1.0621\sigma_{20} - 1.3392 \times 10^{-4}\sigma_{20}^2 \quad (3)$$

$$B = 5.6582 \times 10^{-3} - 3.8617 \times 10^{-3}\sigma_{20} + 8.5080 \times 10^{-6}\sigma_{20}^2 \quad (4)$$

$$C = -5.1773 \times 10^{-3} + 3.8604 \times 10^{-5}\sigma_{20} - 9.2424 \times 10^{-8}\sigma_{20}^2 \quad (5)$$

The regressions accounted for 100.00% of the variation sum of the squares for A, 99.95% for B and 99.36% for C. An estimate of the error was made by comparing  $\sigma_T$  calculated using equations (2)-(5) with  $\sigma_T$  calculated from the quadratic regressions (Table 1) derived from density measurements for individual water samples. Calculated densities were best near 20°C, but the maximum deviation was less than only 0.1 kg/m<sup>3</sup> between -15°C and 30°C, taking into account the approximate freezing temperatures of the less saline lakes.

These equations allow the calculation of *in situ*  $\sigma_T$  from  $\sigma_{20}$  and the *in situ* temperature. Figure 2 shows examples of the relationships applied to actual lake profiles. Measurements of  $\sigma_{20}$  of water samples were made in the laboratory and  $\sigma_T$  calculated using the water temperature recorded when the samples were collected. In both instances, *in situ*  $\sigma_T$  shows a steady increase with depth, as expected for these permanently stratified lakes.

Equations allowing similar calculations have been published (Fofonoff and Millard 1983). These equations, however, were determined for seawater, and therefore have a much lower applicable density range and higher minimum temperature. Results obtained using both sets of equations over the range of densities used in this study are compared in Table 2. The calculated  $\sigma_T$  are consistent at salinities up to that of seawater, but the published equations differ considerably at higher salinities and low temperatures.



Table 1.  $\sigma_{20}$  and quadratic coefficients of equation (2) for the water samples studied.

No.	Sample Source	$\sigma_{20}^1$	A	B	C
1	Abraxas Lake 3 m	5.29	7.5547	$-1.2065 \times 10^{-2}$	$-5.0369 \times 10^{-3}$
2	Abraxas Lake 15 m	12.61	15.346	$-4.1673 \times 10^{-2}$	$-4.7157 \times 10^{-3}$
3	Ekho Lake	53.19	58.128	$-1.7926 \times 10^{-1}$	$-3.3138 \times 10^{-3}$
4	Rookery Lake	82.68	88.875	$-2.5941 \times 10^{-1}$	$-2.4535 \times 10^{-3}$
5	Oblong Lake 4 m	128.99	136.74	$-3.4759 \times 10^{-1}$	$-1.7956 \times 10^{-3}$
6	Jabs Lake 5 m	130.23	138.00	$-3.5050 \times 10^{-1}$	$-1.7680 \times 10^{-3}$
7	Oval Lake 15 m	148.27	156.46	$-3.7448 \times 10^{-1}$	$-1.6958 \times 10^{-3}$
8	Lebed' Lake 1 m	148.99	157.13	$-3.7937 \times 10^{-1}$	$-1.4873 \times 10^{-3}$
9	Jabs Lake 21 m	149.06	157.36	$-3.8213 \times 10^{-1}$	$-1.4771 \times 10^{-3}$
10	Oblong Lake 12 m	151.62	160.00	$-3.8498 \times 10^{-1}$	$-1.5131 \times 10^{-3}$
11	Club Lake 25 m	172.33	181.07	$-4.1245 \times 10^{-1}$	$-1.0698 \times 10^{-3}$
12	Club Lake 1 m	173.34	182.06	$-4.0852 \times 10^{-1}$	$-1.2066 \times 10^{-3}$
13	Deep Lake 30 m	176.68	185.44	$-4.1093 \times 10^{-1}$	$-1.2657 \times 10^{-3}$

1. Determined experimentally

Table 2. Comparison of  $\sigma_T$  calculated using the temperature-density relationships described in this paper and similar equations derived for seawater (Fofonoff and Millard 1983).

$\sigma_{20}$	Temperature °C	$\sigma_T$ (UNESCO)	$\sigma_T$ (this work)	$\Delta \sigma_T$
5.29	0	7.42	7.58	0.16
	10	6.98	6.94	-0.04
24.76 <sup>1</sup>	0	28.11	28.19	0.08
	5	27.68	27.66	-0.02
82.68	10	26.95	26.91	-0.04
	-5	90.19	90.08	-0.11
	0	88.95	88.87	-0.08
130.23	5	87.58	87.52	-0.06
	10	86.08	86.05	-0.03
	-10	142.36	141.37	-0.99
	-5	140.36	139.74	-0.62
	0	138.38	138.01	-0.37
151.62	5	136.41	136.21	-0.20
	10	134.42	134.31	-0.11
	-15	167.63	165.36	-2.27
	-10	165.13	163.62	-1.51
	-5	162.76	161.81	-0.95
176.68	0	160.50	159.92	-0.58
	5	158.30	157.97	-0.33
	10	156.12	155.94	-0.18
	-15	194.63	191.32	-3.31
	-10	191.65	189.42	-2.23
	-5	188.91	187.46	-1.45
	0	186.34	185.44	-0.90
	5	183.90	183.35	-0.55
	10	181.51	181.20	-0.31

1. Seawater: salinity 35‰

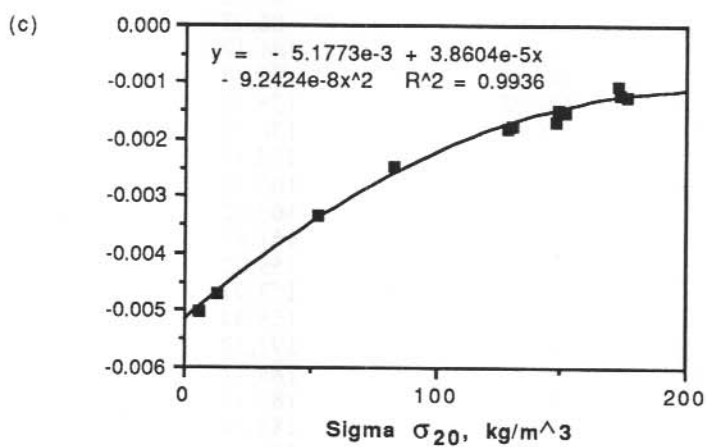
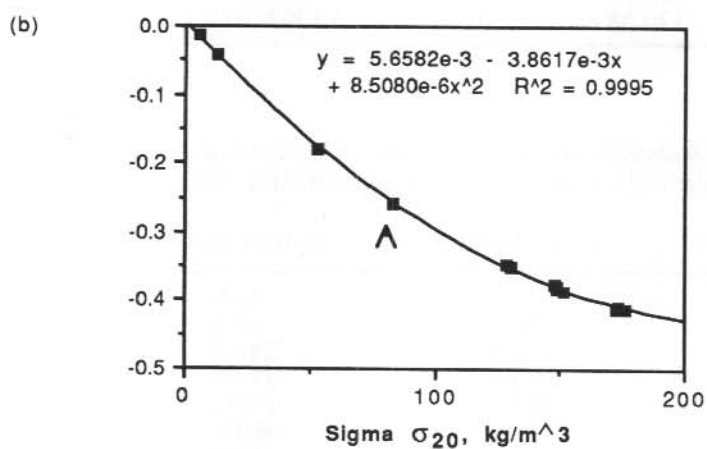
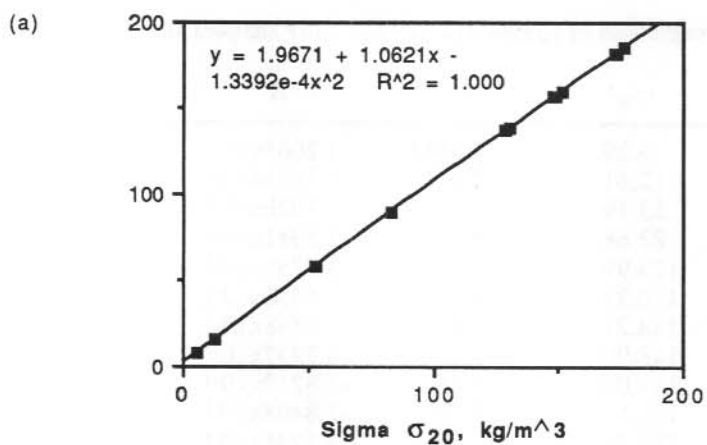


Figure 1. Plot of (a) A, (b) B and (c) C against measured  $\sigma_{20}$  for the water samples used in the study.

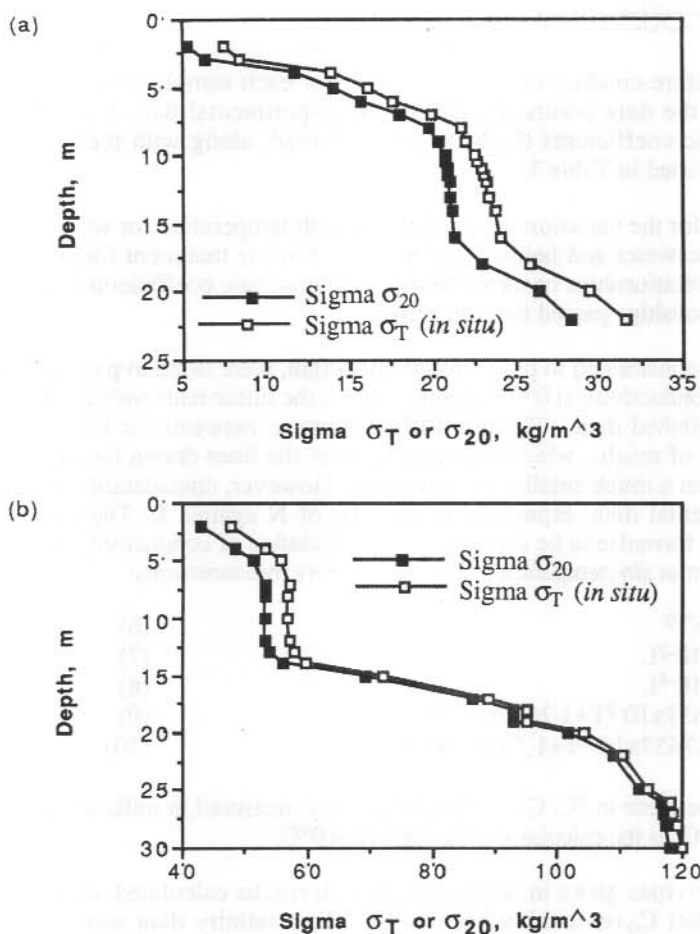


Figure 2. Profiles of  $\sigma_{20}$  and  $\sigma_T$  for (a) Ace Lake and (b) Ekho Lake.

### 3.2 TEMPERATURE-CONDUCTIVITY RELATIONSHIP

The experimental temperature-conductivity data obtained for each sample were plotted and a quadratic curve fitted to the data points. Graphs of the experimental data are presented in Appendix 2. The quadratic coefficients (L, M and N) obtained, along with the origin of the samples and their  $\sigma_{20}$ , are listed in Table 3.

Data have been published for the variation of conductivity with temperature for water of salinity approximately 1.5 times seawater and below (Smith 1974). Similar treatment (below) of these data suggested near linear relationships occurred between the quadratic coefficients L and M, and L and N, and that the relationships passed through zero.

Straight lines, which were constrained to pass through the origin, were fitted to plots of N and M against L (effectively the conductivity at 0°C). Figure 3 shows the linear relationships obtained for the experimental and published data. There is little difference between the lines of best fit calculated for the two sets of results, which is encouraging as the lines drawn for the published data were extrapolated from a much smaller salinity range. However, considerably more scatter occurred for the experimental data, especially in the plot of N against L. The experimental relationships allow general formulae to be derived for the calculation of conductivity at a standard temperature, 0°C (=L), from *in situ* temperature and conductivity measurements:

$$C_T = L + MT + NT^2 \quad (6)$$

$$M = 2.7657 \times 10^{-2} L \quad (7)$$

$$N = 1.2616 \times 10^{-4} L \quad (8)$$

$$C_T = L(1 + 2.7657 \times 10^{-2} T + 1.2616 \times 10^{-4} T^2) \quad (9)$$

$$C_0 = C_T / (1 + 2.7657 \times 10^{-2} T + 1.2616 \times 10^{-4} T^2) \quad (10)$$

where T is the *in situ* temperature in °C,  $C_T$  is the conductivity measured in millisiemens (mS) at the *in situ* temperature and  $C_0$  is the calculated conductivity at 0°C.

Table 4 compares conductivities given in Smith (1974) with results calculated using equations 6-10. The equations predict  $C_0$  reasonably accurately. High salinity data were used in the derivation of the relationships, and therefore accuracy at low salinity might be expected to be sacrificed in favour of better results at high salinity. For water samples with salinities below circa 50‰, it may be better to use the equations derived below from the more accurate data supplied in Smith (1974).

### 3.3 CONDUCTIVITY- $\sigma_{20}$ RELATIONSHIP

The  $\sigma_{20}$  of the samples studied in Section 3.2 were measured (Table 3). A plot of  $\sigma_{20}$  against  $C_0$  for the samples revealed that the two parameters were related by a smooth curve (Figure 5). A cubic function, constrained to pass through the origin, was fitted to the data, viz.:

$$\sigma_{20} = 7.0531 \times 10^{-1} C_0 + 7.4144 \times 10^{-5} C_0^3 \quad (11)$$

This relationship, along with those given above, allows the calculation of *in situ* water density from conductivity and temperature data obtained in lake profiles (a worked example is given in Section 3.5). Figure 6 gives two examples of *in situ* conductivity data treated by the equations given in Sections 3.1 and 3.2 and equation 11 to give *in situ*  $\sigma_T$  profiles.

Table 3. Measured  $\sigma_{20}$  and quadratic coefficients of equation (6) for the water samples studied.

No.	Sample Source	$\sigma_{20}$	L	M	N
14	Ace Lake	5.00	8.210	0.2496	$6.035 \times 10^{-4}$
15	Clear Lake	7.96	11.08	0.3592	$6.016 \times 10^{-5}$
16	Abraxas Lake	11.97	15.39	0.4792	$7.429 \times 10^{-4}$
17	Taynaya Bay	25.18	28.02	0.7645	$3.601 \times 10^{-3}$
18	Burton Lake	27.63	30.06	0.8714	$3.677 \times 10^{-4}$
19	No. 23 diluted	50.42	52.17	1.480	$2.925 \times 10^{-3}$
20	No. 22 diluted	55.49	52.74	1.489	$5.164 \times 10^{-3}$
21	Deep Lake	77.52	71.54	2.041	$2.739 \times 10^{-3}$
22	No. 26 diluted	83.01	74.64	2.045	$4.200 \times 10^{-3}$
23	Oval Lake	93.93	82.44	2.018	$1.443 \times 10^{-2}$
24	Shield Lake	115.80	88.33	2.397	$7.231 \times 10^{-3}$
25	No. 28 diluted	118.04	92.46	2.529	$9.882 \times 10^{-3}$
26	Organic Lake	120.71	94.55	2.468	$2.151 \times 10^{-2}$
27	Organic Lake	121.82	94.93	2.619	$1.748 \times 10^{-2}$
28	No. 30 diluted	153.39	103.4	2.950	$8.773 \times 10^{-3}$
29	Deep Lake	166.94	106.0	3.047	$2.055 \times 10^{-2}$
30	Deep Lake	180.77	107.4	3.083	$1.418 \times 10^{-2}$

Table 4. Comparison of conductivities (millisiemens) calculated using the temperature-conductivity relationships described in this paper and the published data (Smith 1974).

$C_T$	Temperature °C	$C_0$ (Smith) <sup>1</sup>	$C_0$ (This work) <sup>2</sup>	$C_0$ (This work) <sup>3</sup>
15.63	20	9.34	9.29	9.75
25.97	15	17.46	17.49	17.99
33.14	10	25.24	25.24	25.70
33.46	5	29.06	29.02	29.31
53.96	20	32.85	32.79	33.65
42.90	15	29.06	29.05	29.72
23.01	10	17.46	17.47	17.85
10.82	5	9.34	9.32	9.48
80	-10		111.4	108.7
90	-5		105.1	104.1
100	5		86.94	87.61
110	10		84.19	85.33
80	-5		93.47	92.50
70	5		60.83	61.33
60	10		45.85	46.54
50	15		33.90	34.64
40	1		38.84	38.92
30	5		28.02	26.28
20	10		15.17	15.51

1. Taken from Smith (1974)

2. Calculated using the relationship derived from the data in Smith (1974)

3. Calculated using the relationship derived from the lake samples in this work

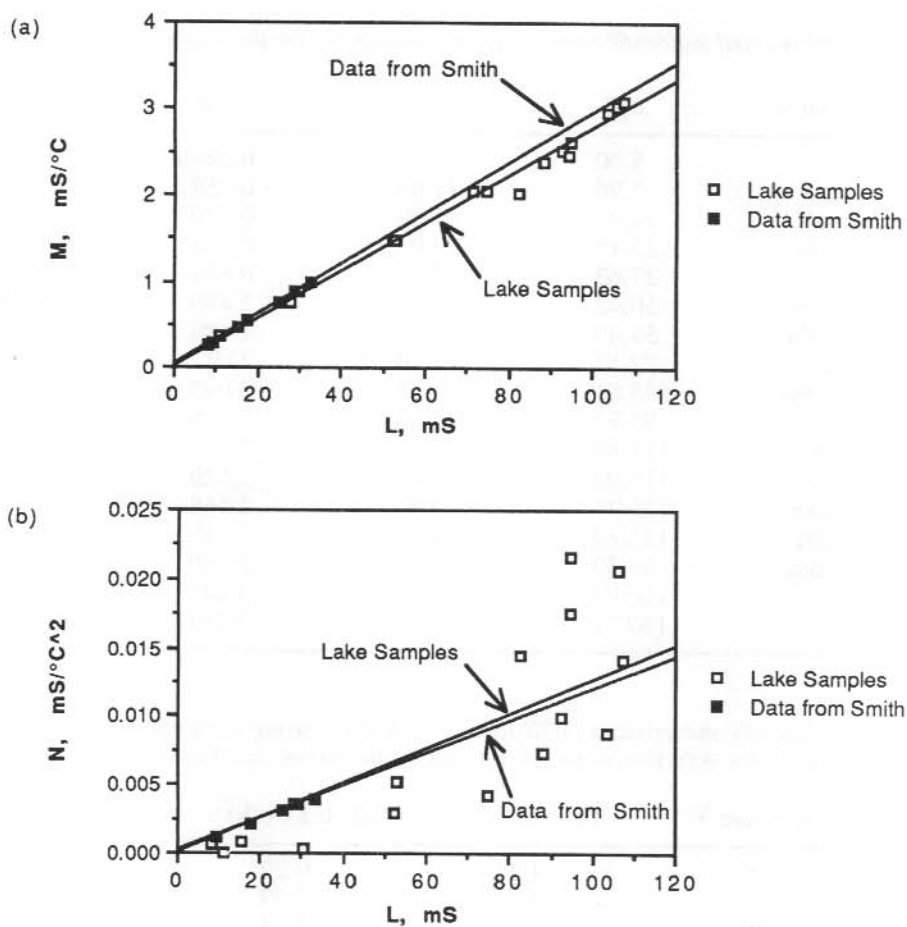


Figure 3. Plots of (a)  $M$  and (b)  $N$  against  $L$  for the lake samples and for the data given in Smith (1974).

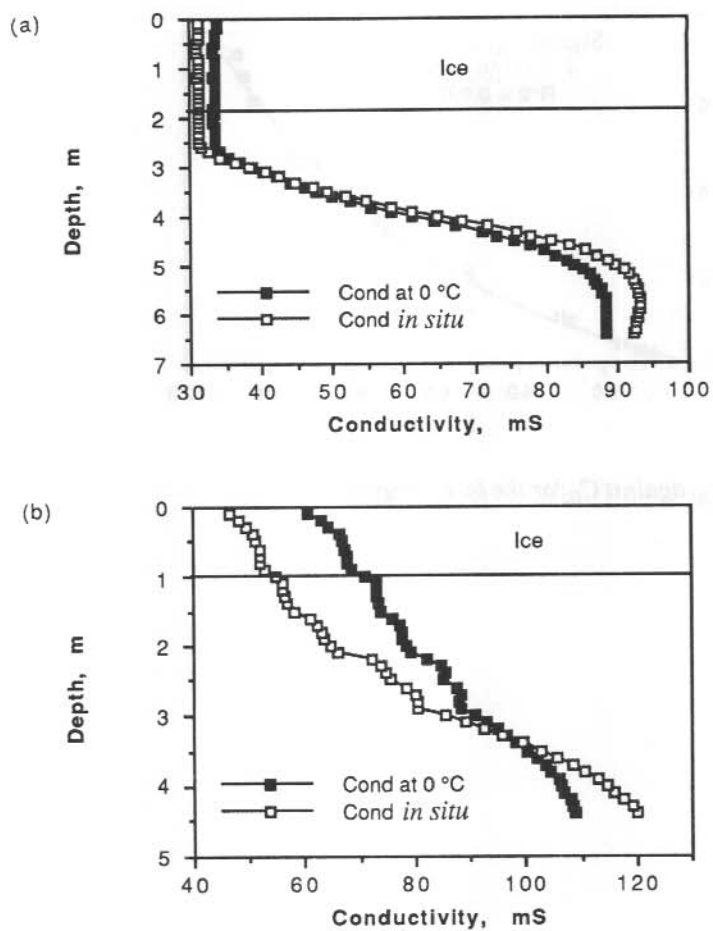


Figure 4. Conductivity profiles at 0 °C and in situ temperature for (a) Williams Lake and (b) Tassie Lake.

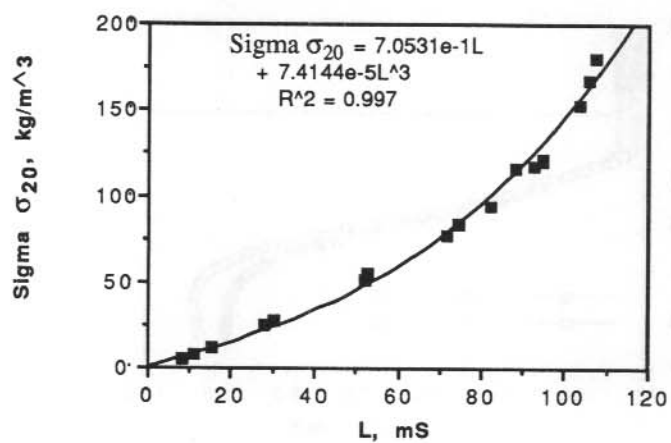


Figure 5. Plot of  $\sigma_{20}$  against  $C_0$  for the lake samples.



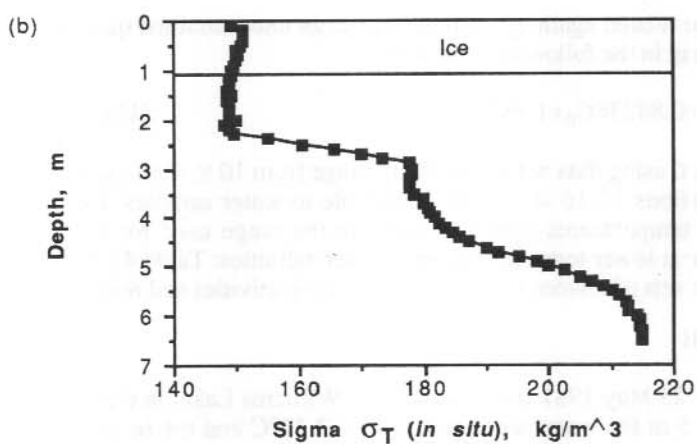
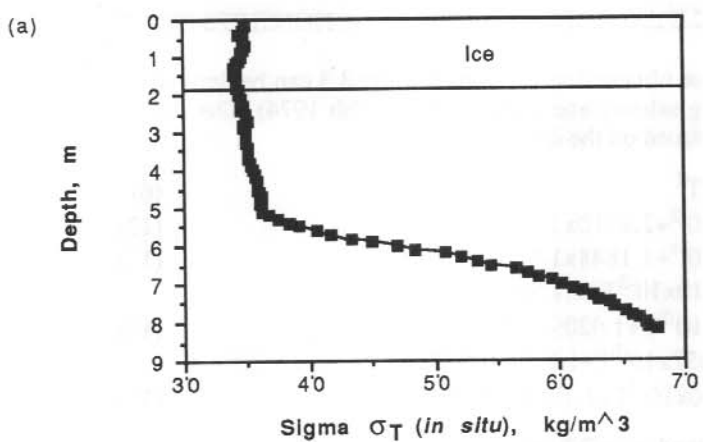


Figure 6. Calculated  $\sigma_T$  profiles for (a) Lake Fletcher and (b) Organic Lake.

### 3.4. CONDUCTIVITY-TEMPERATURE-DENSITY RELATIONSHIPS FOR SEAWATER

Similar relationships to those obtained in Sections 3.2 and 3.3 can be derived from conductivity data for seawater of varying salinity and temperature (Smith 1974). The equations obtained, in which no constraints were placed on the data, are as follows:

$$C_T = L + MT + NT^2 \quad (6)$$

$$M = 2.1457 \times 10^{-2} + 2.9210 \times 10^{-2} L \quad (12)$$

$$N = 1.0209 \times 10^{-4} + 1.1848 \times 10^{-4} L \quad (13)$$

$$C_T = L(1 + 2.9210 \times 10^{-2} T + 1.1848 \times 10^{-4} T^2) + (2.1457 \times 10^{-2} T + 1.0209 \times 10^{-4} T^2) \quad (14)$$

$$C_0 = (C_T - (2.1457 \times 10^{-2} T + 1.0209 \times 10^{-4} T^2)) / (1 + 2.9210 \times 10^{-2} T + 1.1848 \times 10^{-4} T^2) \quad (15)$$

where  $T$ ,  $C_T$  and  $C_0$  are defined as in 3.2 above.

$C_0$  and  $\sigma_{20}$  were found to be related again by a simple curve; an unconstrained quadratic function was fitted to the data resulting in the following equation:

$$\sigma_{20} = -2.6894 + 0.88236 C_0 + 1.9837 \times 10^{-3} C_0^2 \quad (16)$$

These functions were derived using data within a salinity range from 10 to 40‰ and temperatures between 0 and 30°C. Equations 12-16 should be applicable to water samples, especially from fjords, with salinities and temperatures within or close to the range used for the derivations. Larger variations will occur at lower temperatures and higher salinities: Table 4 compares results obtained employing the two sets of relations for a variety of conductivities and temperatures.

### 3.5. WORKED EXAMPLE

A CTD profile obtained on 23 May 1987 from meromictic Williams Lake, in the Vestfold Hills, revealed that at a depth of 5 m the water temperature was -3.13°C and the *in situ* conductivity 35.57 mS. From these data, the conductivity at 0°C can be calculated using equation 10:

$$C_0 = 35.57 / (1 + 2.7657 \times 10^{-2} \times (-3.13) + 1.2616 \times 10^{-4} \times (-3.13)^2) \\ = 38.89 \text{ mS}$$

Equation 11 can then be used to calculate the density of this water at 20°C:

$$\sigma_{20} = 7.0531 \times 10^{-1} \times (38.89) + 7.4144 \times 10^{-5} \times (38.89)^3 \\ = 31.79 \text{ kg/m}^3$$

From the calculated  $\sigma_{20}$ ,  $\sigma_T$  can be calculated using equations 2-5:

$$A = 1.9671 + 1.0621 \times (31.79) - 1.3392 \times 10^{-4} \times (31.79)^2 \\ = 35.60$$

$$B = 5.6582 \times 10^{-3} - 3.8617 \times 10^{-3} \times (31.79) + 8.5080 \times 10^{-6} \times (31.79)^2 \\ = -0.1085$$

$$C = -5.1773 \times 10^{-3} + 3.8604 \times 10^{-5} \times (31.79) - 9.2424 \times 10^{-8} \times (31.79)^2 \\ = -4.043 \times 10^{-3}$$

$$\begin{aligned}\sigma_T &= 35.60 - 0.1085x(-3.13) - 4.043 \times 10^{-3}x(-3.13)^2 \\ &= 35.90 \text{ kg/m}^3\end{aligned}$$

The water at this depth is calculated to have a  $\sigma_T$  of 35.90 kg/m<sup>3</sup>, and therefore an *in situ* density of 1035.90 kg/m<sup>3</sup>.

#### 4. DISCUSSION

Using the equations presented in this *ANARE Research Note*, it is possible to estimate *in situ* density from a temperature profile determined in the field and either a conductivity profile or a profile of sample densities measured in the laboratory. The calculated densities have an overall error estimated at  $\pm 1 \text{ kg/m}^3$ , which is broad compared with the accuracy expected of oceanographic data, but is sufficient to describe the much grosser stratification commonly encountered in the saline lakes of the Vestfold Hills. The relationships are empirical, and should not be used outside the range of the data from which they were derived viz.  $C_T$  from 8 to 170 mS,  $T$  from  $-15^\circ\text{C}$  to  $20^\circ\text{C}$  for field data from CTD, and  $\sigma_{20}$  from 5 to 177 kg/m<sup>3</sup> for measured density input. The estimate of  $\sigma_T$  will become progressively less reliable as  $T$  is decreased below  $-15^\circ\text{C}$  or increased above  $40^\circ\text{C}$ . The relationships should be used with caution when comparing lakes of ionic ratios very different to seawater. Comparing our density-temperature relationships (equations (2)-(5)) with the UNESCO formulation (Fofonoff and Millard 1983) demonstrates that the equation of state for seawater is not applicable at the high salinities and low temperatures commonly found in marine-derived saline lakes of the Vestfold Hills.

The form of the equations fitted to the data have no particular significance. Quadratic equations were chosen for the raw density and conductivity data, resulting in very good least squares fits ( $r^2 > 0.99$ ). When plotting the quadratic coefficients, the form of line which best described the data was chosen, a justified approach in this purely empirical study. UNESCO (Fofonoff and Millard 1983) presented their temperature-salinity-density formulation in a similar form, using two polynomial equations to predict the coefficients for use in a third equation. We could have imposed higher order polynomials on our data but we felt that this was not justified statistically, or in terms of the accuracy of the measurements. Furthermore, it would have made calculations more difficult.

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Appendix I: Data plots (i) density measurements

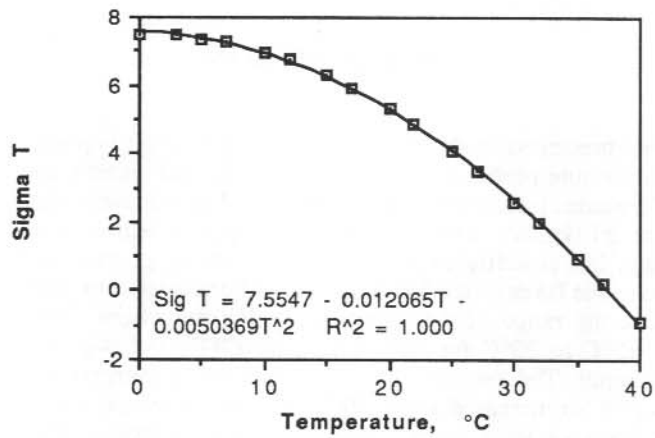


Figure 7. Effect of temperature on  $\sigma_T$ , Sample 1.

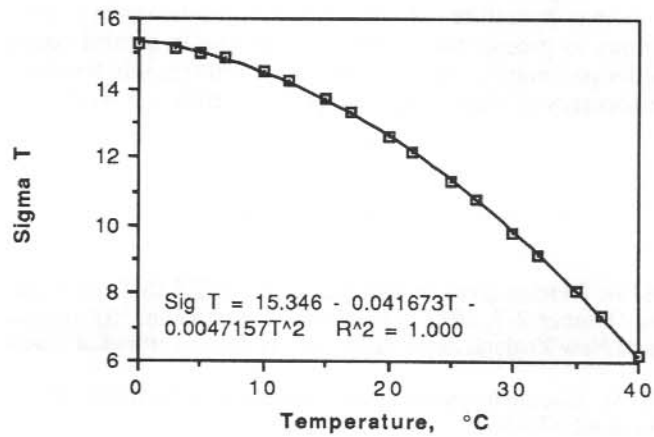


Figure 8. Effect of temperature on  $\sigma_T$ , Sample 2.

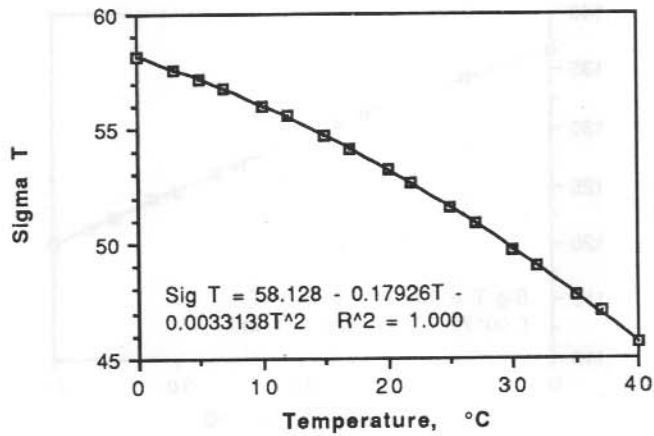


Figure 9. Effect of temperature on  $\sigma_T$ , Sample 3.

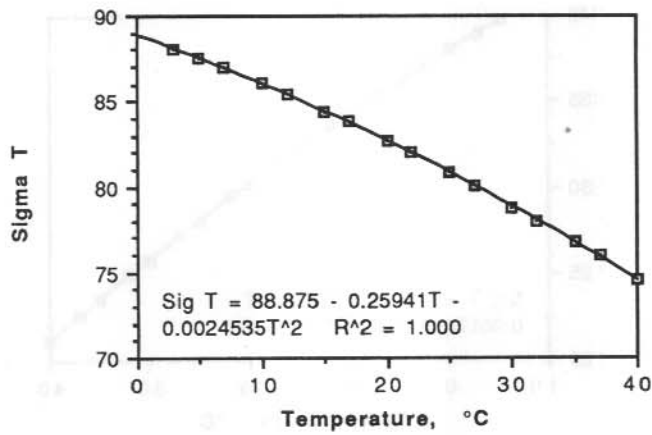


Figure 10. Effect of temperature on  $\sigma_T$ , Sample 4.

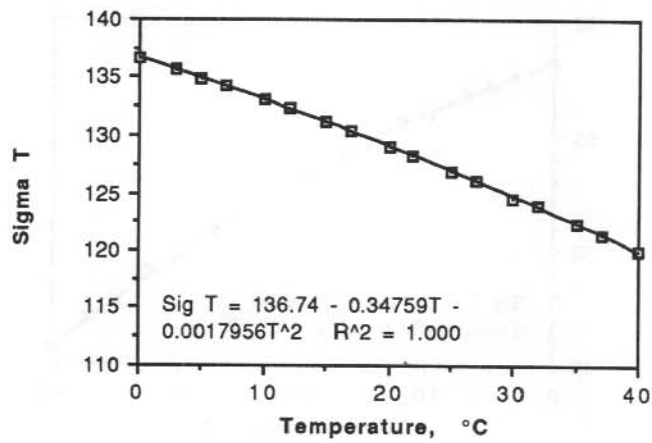


Figure 11. Effect of temperature on  $\sigma_T$ , Sample 5.

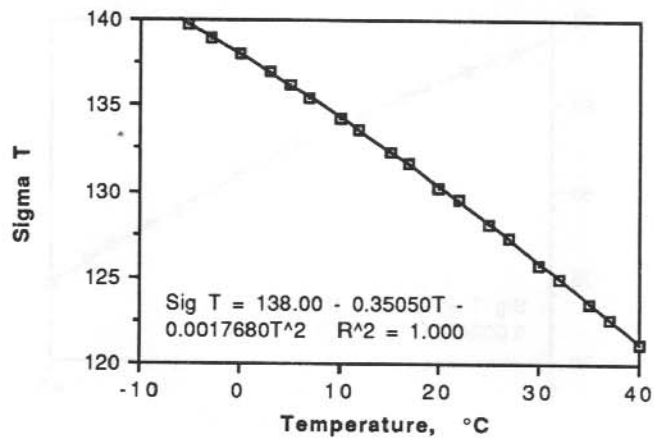


Figure 12. Effect of temperature on  $\sigma_T$ , Sample 6.

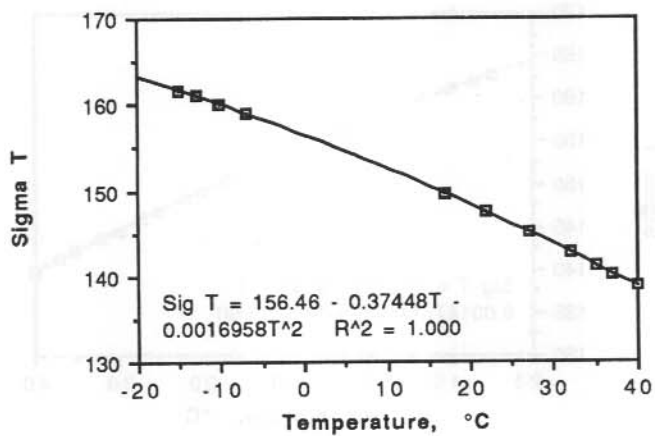


Figure 13. Effect of temperature on  $\sigma_T$ , Sample 7.

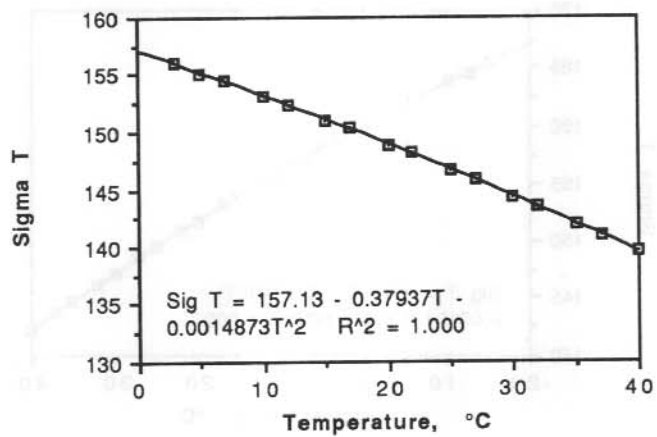


Figure 14. Effect of temperature on  $\sigma_T$ , Sample 8.

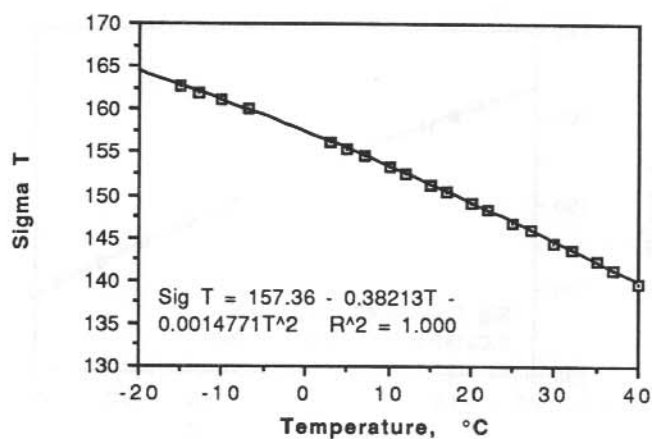


Figure 15. Effect of temperature on  $\sigma_T$ , Sample 9.

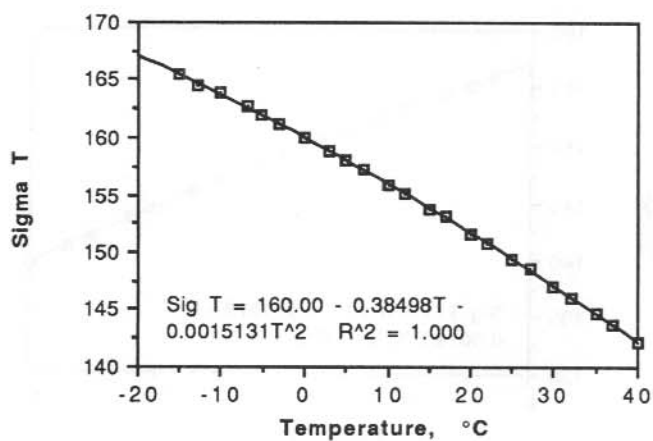


Figure 16. Effect of temperature on  $\sigma_T$ , Sample 10.



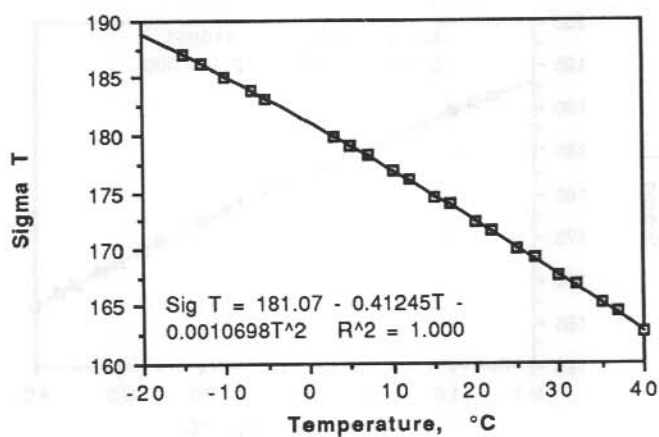


Figure 17. Effect of temperature on  $\sigma_T$ , Sample 11.

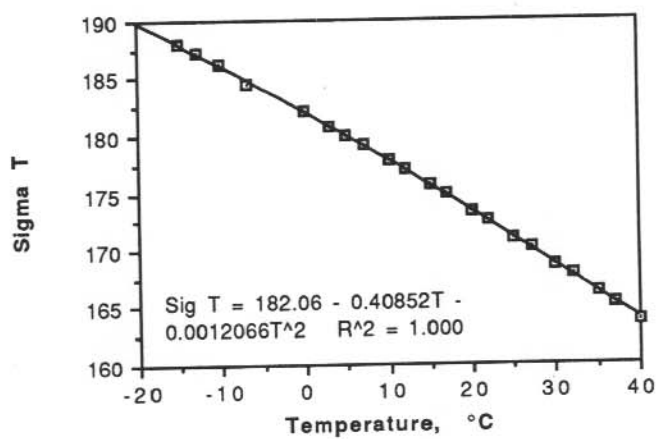


Figure 18. Effect of temperature on  $\sigma_T$ , Sample 12.

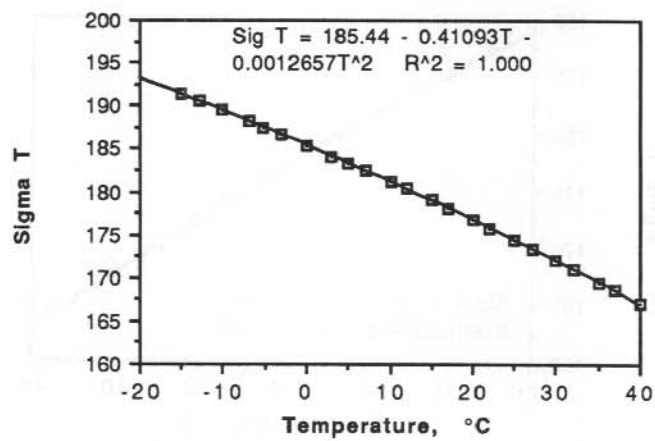


Figure 19. Effect of temperature on  $\sigma_T$ , Sample 13.

Appendix II: Data plots (ii) electrical conductivity measurements

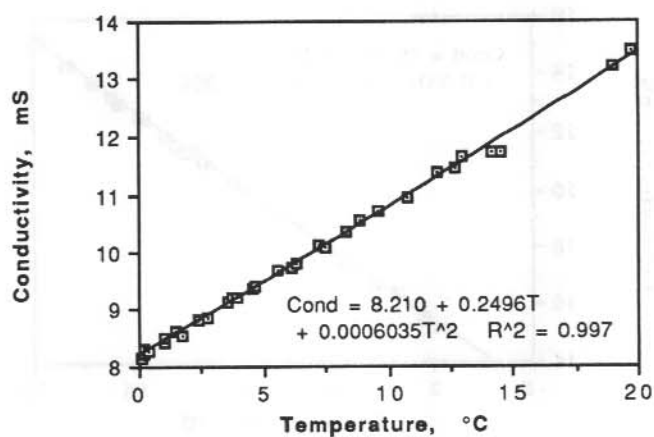


Figure 20. Effect of temperature on electrical conductivity, Sample 14.

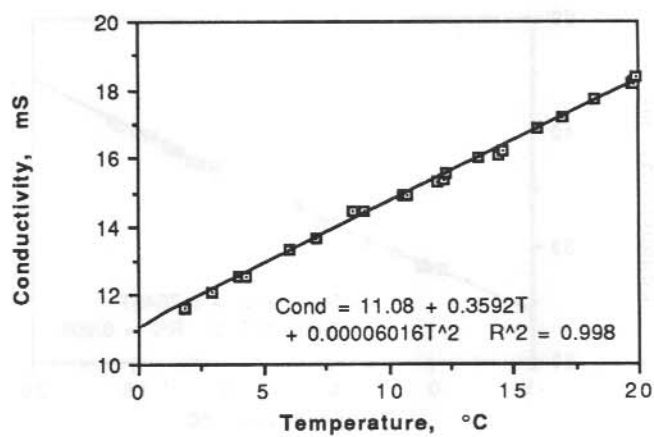


Figure 21. Effect of temperature on electrical conductivity, Sample 15.

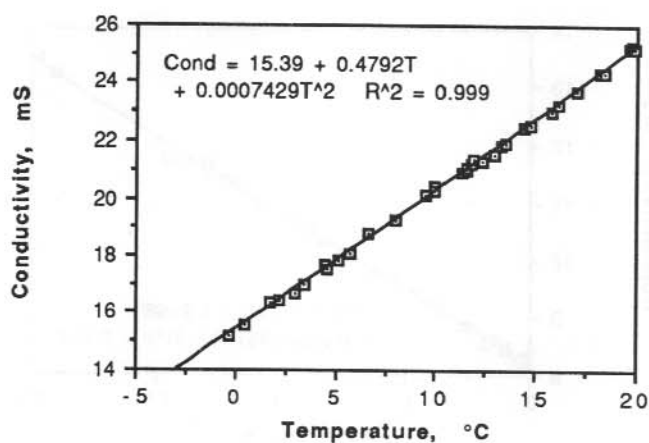


Figure 22. Effect of temperature on electrical conductivity, Sample 16.

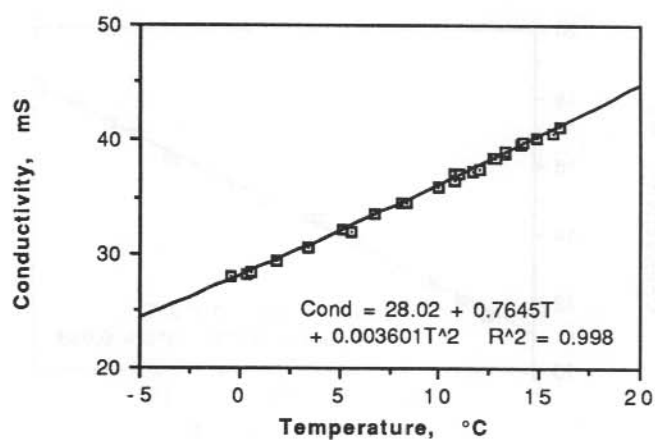


Figure 23. Effect of temperature on electrical conductivity, Sample 17.

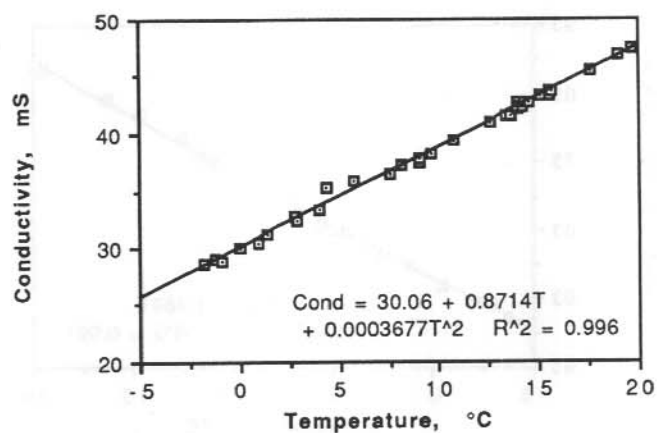


Figure 24. Effect of temperature on electrical conductivity, Sample 18.

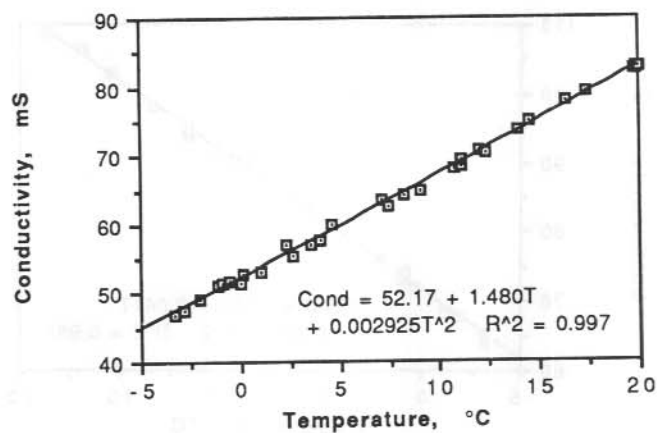


Figure 25. Effect of temperature on electrical conductivity, Sample 19.

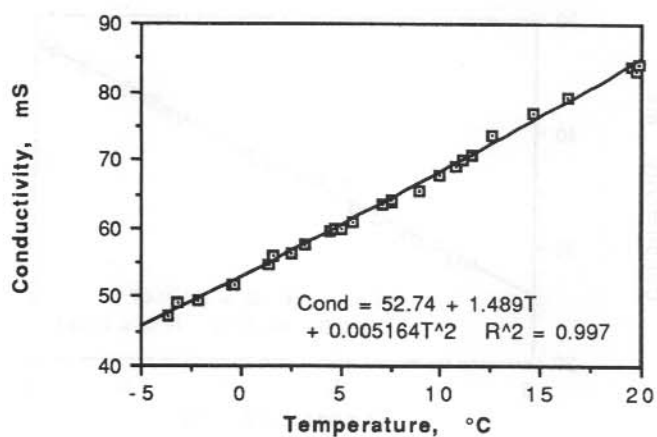


Figure 26. Effect of temperature on electrical conductivity, Sample 20.

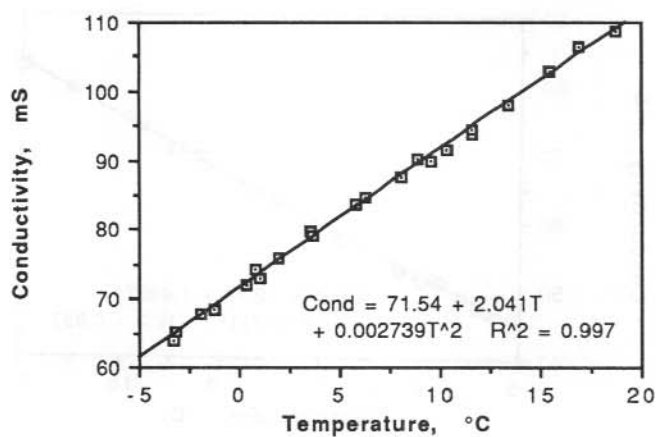


Figure 27. Effect of temperature on electrical conductivity, Sample 21.

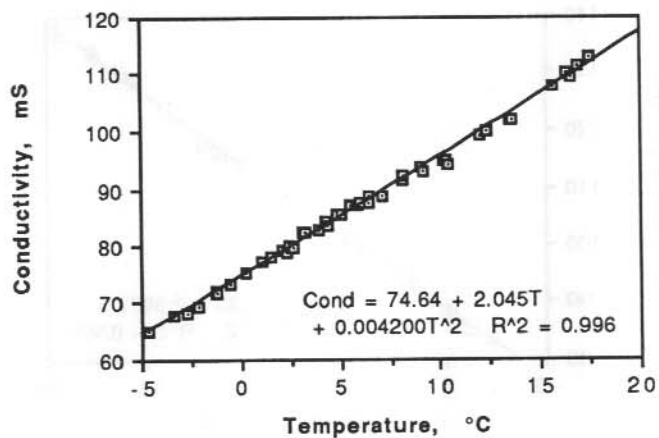


Figure 28. Effect of temperature on electrical conductivity, Sample 22.

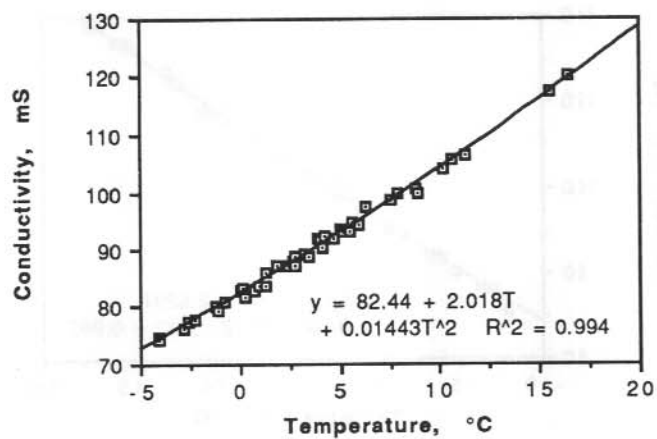


Figure 29. Effect of temperature on electrical conductivity, Sample 23.

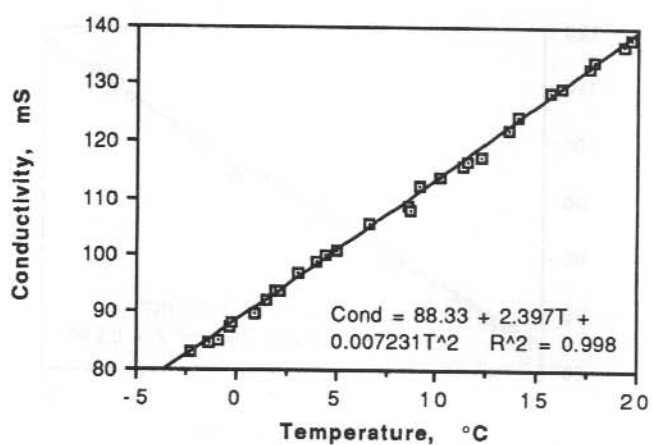


Figure 30. Effect of temperature on electrical conductivity, Sample 24.

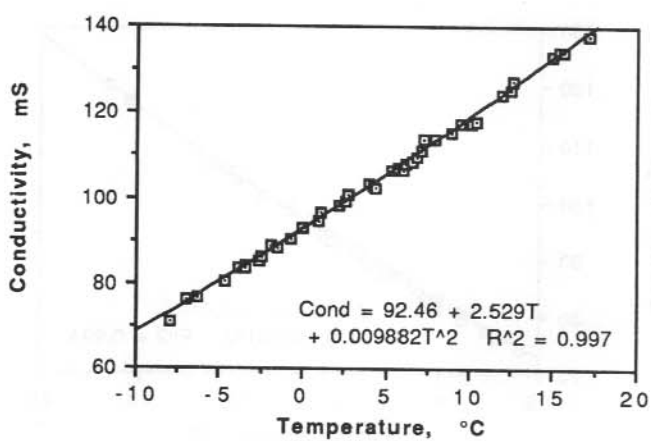


Figure 31. Effect of temperature on electrical conductivity, Sample 25.



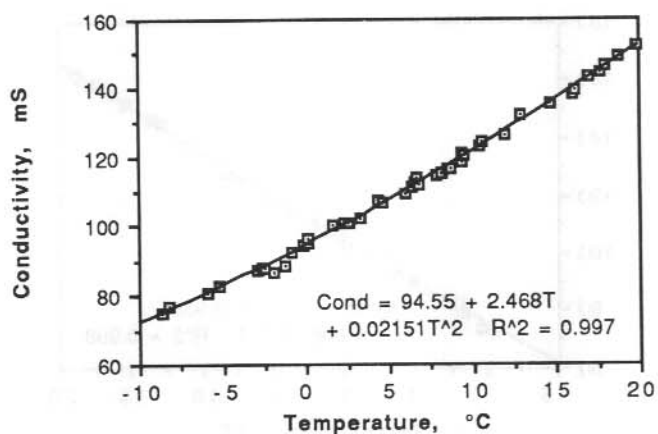


Figure 32. Effect of temperature on electrical conductivity, Sample 26.

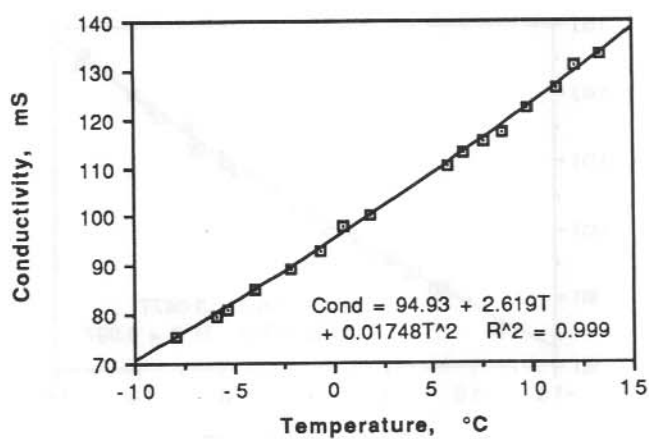


Figure 33. Effect of temperature on electrical conductivity, Sample 27.

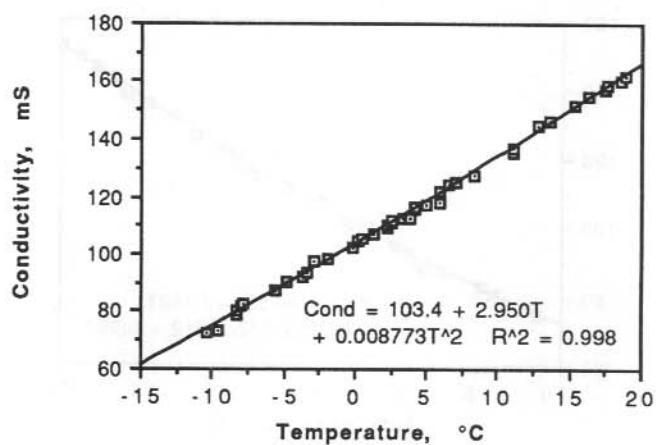


Figure 34. Effect of temperature on electrical conductivity, Sample 28.

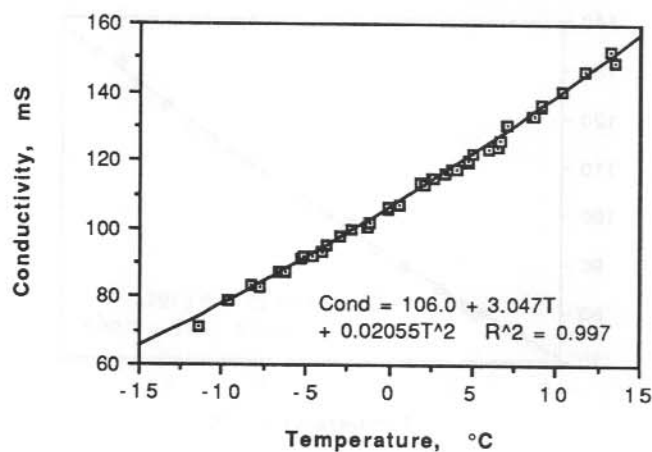


Figure 35. Effect of temperature on electrical conductivity, Sample 29.

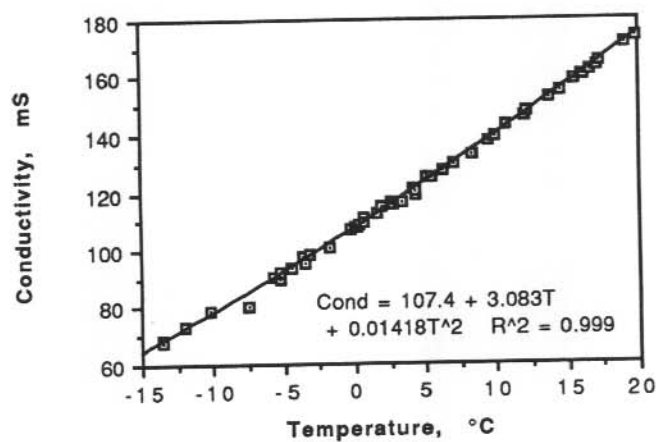


Figure 36. Effect of temperature on electrical conductivity, Sample 30.



Figure 1: Linear relationship between Time (min) and Concentration (mg/L)