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# STUDIES OF THE SIDEREAL DAILY VARIATION OF COSMIC RAY INTENSITY

## WITH PARTICULAR REFERENCE TO OBSERVATIONS AT 40 M.W.E. UNDERGROUND

by

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# PREFACE

For many years it has been known that, wherever we look in the sky, the intensity of high energy charged particles reaching us is, within errors of measurement, remarkably constant. This aspect of cosmic radiation is of fundamental interest and testifies to a chaotic stirring of the directions of particle motions in the galaxy. As the accuracy of measurements has improved, the more precisely has it seemed that the intensity is isotropic. Only at present levels of accuracy, when we are able to say that the intensity is constant in all directions to better than 99·9%, does there seem to be unequivocal evidence for a small but significant non-uniformity, or anisotropy.

It is the purpose of this report to describe various lines of investigation which have led in the first place to the conclusion that the charged particle intensity does vary with galactic direction of viewing. Following from this, a descriptive model for the anisotropy is proposed and the parameters are investigated. The material is essentially that of a doctoral thesis (University of Tasmania) completed in March 1967. Significant developments since that time are discussed in the concluding Section.

The report is concerned in the main with experiments carried out at the University's underground observatory, there being decisive factors in favour of an initial approach through observations with large detectors underground. Some of the findings concerning the anisotropy, such as the two-way (bi-directional) nature of the intensity maxima, have been widely published and have received substantial corroboration from experiments in other countries. However, the principal evidence as to a galactic origin for the bi-directional effect has come from an analysis of a latitude survey, leading to the first estimate of the declination. This is largely unpublished work as is other relevant material, including estimations of the energy range of the anisotropy, its energy dependence, the evidence of seasonal and long-term changes and that of a "streaming", uni-directional component. In the development of the model it was necessary to know something of most of these aspects of the new phenomenon.

The case for a galactic origin, as against a solar origin, for the two-way anisotropy rests largely on the estimate of the direction, particularly the declination, and of the energy range. However, no sensitive practical indicators of the latter have so far emerged from the analyses of underground data. On consideration of this, the investigation of the energy range has been taken up as a major objective of high energy experiments being planned by the Antarctic Division for Mawson. The experiments will make use of the fact that, at polar latitudes, the terrestrial magnetic field can be employed to maximum advantage as a magnetic spectrometer for determining energies of incident protons, up to quite high energies. Already the first results from a prototype high zenith angle telescope at sea level at Mawson have indicated a high threshold energy, compatible with a galactic rather than a solar origin for the bi-directional effect. The general methods of analysis set out in this report provide a basis for interpretation of the Antarctic data.

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## ABSTRACT

An investigation of the apparent sidereal daily variation of meson intensity at depths of approximately 40 m.w.e. underground is described. Continuous records, over the years 1958–1965, from vertical telescopes at Hobart, provide the main source of information, in conjunction with data from vertical telescopes at Budapest and from special experiments at Hobart with telescopes inclined 70° north, 30° north and 45° south of zenith. For comparison with the underground data, the available observations of an apparent sidereal daily variation at sea level are examined. Developments since 1965 are summarized and discussed at the end of the report.

It is concluded from analyses based on the anti-sidereal technique, from discriminatory experiments and from the coherence of the evidence for a genuine sidereal component, that spurious contributions, if any, constitute only a small part of the main effect, which is attributed to a small two-way sidereal anisotropy.

The observations are shown to be compatible with an empirical model which specifies the anisotropy in interplanetary space outside the earth's field region as consisting of two parts:

(1) A bi-directional component with equal intensity maxima in opposite directions;

(2) A smaller uni-directional component.

The co-ordinates of the axis of the two-way anisotropy are estimated to be RA = 0640 ± 0115, Dec. = − 38 ± 7° in the southward direction and, correspondingly, RA = 1840 ± 0115, Dec. = + 38 ± 7° in the northward direction. These estimates are shown to be insensitive to values of the index of the variation spectrum, $\gamma$, in the tested range 0 to —2, and to the primary threshold rigidity, $R_c$, from 50 GV to at least 200 GV. It is also shown that the observations are incompatible with bi-directional intensity maxima in the direction normal to the plane of the ecliptic (Dec. = ± 66° 30′).

Although most of the evidence does not depend on assumptions as to the nature of the interplanetary magnetic field, its role is considered both in relation to the

cut-off rigidity $R_c$, which it presumably imposes, and to the pronounced seasonal changes that are thought to have occurred in the sidereal daily variation at sunspot maximum.

On the available evidence it is suggested that an excess flux of primaries with small pitch angles, propagating in both directions along the galactic magnetic field, may be responsible for the two-way anisotropy, but that there could be a different origin for the uni-directional component.

## 1. INTRODUCTION

### 1.1. INTRODUCTORY REMARKS

There is now considerable evidence that the charged primary cosmic radiation to which detectors at the earth respond is essentially isotropic in free space, at least in the seven decades of rigidity from $10^{10}$ v to $10^{17}$ v. In fact, many careful investigations over some thirty years have failed to produce convincing evidence for even a small excess of charged particle flux from any fixed galactic direction. More particularly, if the amplitude of the observed anisotropy is specified as

$$A = \frac{I_{max} - I_{min}}{I_{max} + I_{min}},$$

there would appear to be zero anisotropy, with a margin of uncertainty of amplitude varying from less than 0·1% at the lowest primary rigidities to about 3% at the highest rigidities. Nevertheless, small anisotropies have been predicted, of special interest being those that should be associated with the propagation of cosmic rays through the galaxy in the presence of large-scale magnetic fields.

In the first part of this report, up to the end of Section 5, evidence is given that there is indeed a small sidereal anisotropy, such as would be expected to be associated specifically with a relatively uniform local galactic magnetic field (see Section 2 below). The evidence derives from daily variations of intensity, obtained in the main from meson telescopes located at depths of approximately 40 m.w.e. underground, where it is believed most of the response would be due to primary protons in the rigidity range $5 \times 10^{10}$ v $- 10^{12}$ v. The observations differ in a number of respects from those obtained at ground level. Of particular note is the fact that the amplitude of the solar daily variation is much smaller, and that the characteristics of the apparent sidereal effect reproduce themselves remarkably well from year to year. The initial problem is to determine to what extent the observed sidereal effect is due to a sidereal anisotropy, extracting, if necessary, spurious contributions that may have been generated by systematic modulation of solar components of the daily variation. If, as a result, there is significant evidence for a genuine sidereal component, it should be possible to ascertain the general character of the anisotropy, provided that observations can be obtained from several well-separated latitudes of viewing, preferably spanning both hemispheres.

The remaining sections lead to estimates of the mean direction of the anisotropy in free space beyond the earth's field region, using assumed values for the index of the variation spectrum and for the expected cut-off rigidity due to the interplanetary magnetic field. While the time of maximum of the observed sidereal effect, after removal of any spurious sidereal components, gives some indication of the right ascension, the declination is not approximated by the latitude of maximum amplitude that might be obtained from a latitude survey, being in fact, independent of it. The determination of declination has to be made in three stages. Using observations from a number of detectors, scanning different strips of latitude, transformations are effected from the observed sidereal daily variations to the corresponding daily variations in free space, using the technique of the asymptotic cone of response. Independently of these calculations, a model that is appropriate to the type of anisotropy is formulated, yielding a free-space sidereal daily variation and an average intensity term. The amplitude of the harmonics of the daily variation term depend in part on the latitude of observation and on the declination of the anisotropy. Using the expressions for the harmonics, a value of the declination can be so determined as to give a best fit agreement between the free-space amplitudes of the model and the amplitudes derived from the observations.

It will be shown that the estimated directions of the intensity maxima in free-space, when taken together with the type of two-way anisotropy that is indicated by the free-space harmonics, suggest that the primary cosmic radiation may be propagating along a rather uniform local galactic magnetic field.

### 1.2. ISOTROPY AND ANISOTROPY

It appears that the overall high degree of isotropy of cosmic radiation must be due to the randomizing of the directions of motion of the charged primaries as they travel from their region or regions of origin to the solar system, rectilinear propagation being disallowed in the light of the evidence that large-scale magnetic fields of $10^{-5}$ to $10^{-6}$ gauss permeate the interstellar medium in the galactic disc. Optical and radio measurements indicate that the galactic magnetic field in the vicinity of the solar system lies approximately in the direction (to within 20°) of the local spiral arm, towards $l^{II} = 70°$ or 250° [optical polarization of starlight in dust clouds (e.g., Hiltner 1951; Hall 1958); Zeeman measurements relating to dense neutral hydrogen clouds (e.g., Davies, Slater *and* Wild 1964); linear polarization of radio emission from nearby "synchroton" regions (e.g., Mathewson *and* Milne 1965; Mathewson, Broton *and* Cole 1966); Faraday rotation in HII regions, of polarized emission from radio sources (e.g., Morris *and* Berge 1964; Gardner *and* Davies 1966)] and that it is a well-ordered field on the average, with typical local deviations of direction of about 0·1 radians. It is thought that the mean distance to the "synchroton" regions is $\sim$ 150 parsecs, while radio and optical data in general refer to regions within 3 kiloparsecs of the sun. That interstellar magnetic fields probably extend throughout the galactic disc had been suggested earlier from the requirements for storage of cosmic radiation in the galaxy (Fermi 1949, 1954) and from the need to explain the dynamical stability of the spiral arms, the kinetic pressure of the diffused matter being insufficient to counterbalance the gravitational pressure exerted on it (Chandrasekhar *and* Fermi 1953).

Although the most recent measurements of linear polarization of synchroton emission and of Faraday rotation give further evidence of a uniform magnetic field in the local spiral arm, it has been argued that the presence of random as well as ordered fields would be compatible with the type of uniformity of direction that has been observed as an average of contributions distributed along the arm (Ginzburg *and* Syrovatskii 1964). It appears that the scale of a disordered segment of field would be so large in relation to the gyroradius of the average particle, however, that in the absence of small scale irregularities the motions of particles would be regarded as being tied to magnetic field lines in the highly conducting interstellar medium.

At the low end of the range of primary energies, the scattering and trapping of particles in the interplanetary medium would be expected to cause galactic anisotropies to be smeared out and it will be of importance to enquire here as to what the upper limiting energy for such an effect would be. At higher energies it may be inferred from the considerations given above that the charged primaries in the vicinity of the solar system spiral along a relatively uniform magnetic field, yet exhibit a high degree of randomization of pitch angles, a situation which must prevail over distances of the order of a parsec if it applies over the range of particle rigidities up to $10^{17}$ v.

By what process isotropy is brought about at these higher energies is not clear, yet it is evidently a characteristic of cosmic radiation on the galactic scale. Conjectured mechanisms range from the propagation of small-scale irregularities along the spiral arm field, or reflections and diffusion along a uniform field containing randomly distributed constrictions, to the mixing of cosmic rays in the weak random fields of the galactic halo in association with a free interchange of particles between the halo and the galactic disc. The choice of a suitable mechanism is governed by a number of interdependent factors. Notably, if, as is widely believed, all but the most energetic cosmic rays originate in the galactic disc, they must be allowed to diffuse out of the plane of the galaxy in a time of the order of $10^6$ years (Parker 1965) and yet not exhibit pronounced anisotropy due to streaming. Consistent with this requirement, the cause of isotropy seems to depend on the distributions of sources, on the configuration of magnetic fields in the disc and the halo and on the dynamical properties of the medium (including cosmic radiation). In this last respect, it has been pointed out (e.g., Parker 1965) that on the galactic scale cosmic radiation is a hot collisionless gas whose pressure ($\sim 10^{-12}$ dynes/cm$^2$) is comparable to that of the magnetic field and considerably greater than that of interstellar hydrogen ($\sim 10^{-13}$ dynes/cm$^2$), and that it must therefore contribute very largely to the dynamical stability of the medium. By the same token, cosmic ray pressure, as well as motions of the interstellar gas, can produce local deformations in the field which may lead to dynamical instability, disordering of the field and the mixing of particle trajectories. It has also been suggested that plasma instabilities may occur in the cosmic ray gas, associated with excess pressure normal or parallel to the magnetic field, and that these could assist in the achievement of isotropy.

It is evident that the observed cosmic ray isotropy does not in itself greatly restrict conjecture as to the configuration of magnetic fields in the galaxy and the distribution of sources. On the other hand, it is known that rather definite information may be given by small anisotropies, notably those associated with particular configurations of magnetic fields. For example, in the model for propagation of cosmic rays put forward by Ginzburg *and* Syrovatskii (1964), who favour disordered fields in the galactic arms, cosmic rays diffuse outwards from the central regions and a small excess of intensity should be observed in the direction of the galactic centre. On the other hand, if the magnetic fields are essentially uniform, an intensity maximum along the axis of the local field due to net streaming would be expected, if, for instance, diffusion out of the galaxy along the arms were important or if particles were diffusing away from a nearby source. Of particular interest would be the intensity maxima connected with the reflection from regions of strong field and the leakage, through these regions, of particles whose motions were essentially adiabatic (although they may be subject to some degree of scattering by small-scale irregularities). In these situations the distribution of pitch angles would not be entirely random and a balance of factors would decide whether intensity maxima occurred along the axis of the field or at right angles to it. A possibility of great importance here is the two-way anisotropy that may occur if, as the result of the trapping between constrictions in the magnetic field, there are excess fluxes of particles with steep helices (large pitches) in both directions along the axis of the field. Leverett Davis (1954) has given a treatment of this possibility in connection with statistical acceleration processes whereby particles are reflected from moving inhomogeneities or in regions of time-varying field. The two-way anisotropy produces a terrestrial sidereal daily variation consisting essentially of a diurnal component and a semi-diurnal component whose characteristics exhibit a well-defined dependence on the latitude of observation.

It should be mentioned that an effect additional to simple streaming (see below) occurs if cosmic radiation which is isotropic in a region of strong field moves adiabatically into a region of weaker field. It becomes anisotropic in the moving frame of reference, there being an excess flux of particles with steep helices in the direction of motion. It has been noted that this is likely to give rise to first and second harmonics in the sidereal daily variation (L. Davis, private communication). Thus it would appear that a two-way anisotropy may be connected with streaming of previously isotropic radiation from both directions into a region of weaker field in which the observer is located.

In simple streaming (Compton *and* Getting 1935) which originates outside the solar system, the latter moves with uniform velocity relative to some frame of reference in which the cosmic radiation is isotropic—it may be the frame of reference of neighbouring stars, that of the extra-galactic medium or that which moves with the velocity of diffusion of cosmic radiation. Ignoring local modifying influences, the resulting terrestrial effect that is of interest here is a sidereal diurnal variation, corresponding to enhanced intensity in the upstream direction of viewing and reduced intensity in the opposite direction, the amplitude being proportional to the velocity of streaming. This is partly due to the apparent steepening of helices when viewing upstream and the apparent flattening of helices when viewing downstream. It seems likely that a net effect would be observed, noting particularly that in the event of streaming motions along the spiral arm there may be contributions from both directions. Thus the observed intensity anisotropy may be an unreliable indicator of individual streaming velocities.

It is evident that the two-way anisotropy is the one which specifically indicates that particles are being propagated along a relatively well-ordered magnetic field and that any indications of associated streaming along the field would relate to a balance of contributions from opposite directions.

Only a small fraction of the primary protons thought to be responsible for the mesons observed at a depth of 40 m.w.e. would have gyroradii in excess of hundreds of astronomical units (a.u.) in magnetic fields of $10^{-5}$ to $10^{-6}$ gauss. Therefore, if a type of anisotropy is found that is indicative of a relatively uniform galactic field, it may also enable the direction of the axis of the field in the immediate vicinity of the solar system to be estimated, provided that distortions of the effect that would occur within the solar system are not too serious or can be taken adequately into account.

An anisotropy of a different character should also be mentioned. It has been suggested that mesons produced by high-energy (BEV) $\gamma$-radiation may have been responsible for the large but variable intensity spike observed in the approximate direction of W-ORIONIS by Sekido *et alia*, using very narrow angle alt-azimuth detectors, horizontally directed (e.g., Murahama 1960; Sekido *et alia* 1963).

With their high counting rates, the wide-angle meson detectors employed at moderate depths underground are designed specifically to detect very small anisotropies that give rise to a world-wide pattern of response. While there may be an extremely inefficient response to point sources of $\gamma$-radiation, the observations to be described here give no indication of it. The possibility would have to be considered more seriously, however, if a genuine sidereal component was observed that was not accompanied by the characteristic effects due to the deflections of primary charged particles in the terrestrial magnetic field.

Table 1.1 lists possible sources of anisotropy in the charged primary cosmic

TABLE 1.I.

PREDICTED SOURCES OF SIDEREAL ANISOTROPY IN THE CHARGED COSMIC RADIATION AT MODERATE PRIMARY ENERGIES. GALACTIC CO-ORDINATES REFER TO THE NEW SYSTEM.

| Source | Reference | First harmonic *Direction of maximum* | | | | Second harmonic |
|---|---|---|---|---|---|---|
| | | Celestial | | Galactic | | |
| | | RA (hr) | Dec (deg) | Lat. (deg) | Long. (deg) | |
| Diffusion outwards from central regions of Galaxy | Ginzburg *and* Syrovatskii (1964) | 1800 | −30 | 0 | 0 | Absent |
| Streaming due to motion of solar system relative to neighbouring stars | Hogg (1949) Parker (1964) | 1800 | +30 | 25 | 55 | Absent |
| Streaming due to motion of solar system relative to extra-galactic medium | Compton *and* Getting (1935) | 2100 | +50 | 0 | 90 | Absent |
| Propagation along Spiral Arm field | Davis (1954) | | | | | |
| (a) Simple streaming | | 0830<br>2030 | −35<br>+35 | 0<br>0 | 250<br>70 | Absent |
| (b) Excess of particles with steep helices | | 0830<br>2030 | −35<br>+35 | 0<br>0 | 250<br>70 | in phase |
| (c) Excess of particles with flat helices | | 0830<br>2030 | +35<br>−35 | 30<br>−30 | 190<br>10 | 6 hours out phase |
| (d) Inhomogeneities of cosmic ray energy density normal to direction of field | | 1230<br>0030 | +30<br>−30 | +90<br>−90 | | Absent |

radiation that have been suggested in the literature, together with the estimated directions of maximum intensity.

### 1.3. THE DETECTION OF A SIDEREAL ANISOTROPY

The methods that are employed to identify a sidereal anisotropy are conditioned to some extent by the type of anisotropy that is expected to be observed. This is reflected in the present investigation by a change in approach to the problem when it was realized that the observed sidereal effect was evidently conforming, not with a single direction of maximum intensity in free space as would be associated with simple streaming, but with intensity maxima in opposite directions such as might derive from trapping in regions of magnetic field that were of large scale by comparison with the average gyroradius.

At first, the investigations to be described here were concerned with observations at Hobart of the apparent sidereal effect in the vertical dirction underground. The main difficulty of interpretation of the effect is caused by the presence of a solar daily variation of cosmic ray intensity. Some of the observed meson flux responds to solar modulation of the primaries and it has become clear in recent years that this accounts in part for the solar daily variation (dv), there being additional contributions of local (e.g., atmospheric) origin. At depths of about 40 m.w.e. underground the amplitudes of the observed solar and apparent sidereal dvs are roughly equal, about 0·1% or less, while at ground level the solar dv is usually considerably the greater of the two, with the typical amplitudes of about 0·3%. Changes in the solar daily variation within a year, particularly seasonal modulation, may produce a residual spurious sidereal effect when the daily variation is arranged in sidereal time and averaged over a complete year. However, the smaller the average amplitude of the solar dv, the smaller this effect is likely to be. Thus it is important to note that, unless the solar dv at moderate depths underground comprises rather large contributions that are out of phase, spurious sidereal effects should be a factor of about 3 smaller than they are at ground level for the same percentage modulation.

Close attention is given to the problem of seasonal modulation of the solar daily variation and the well-known anti-sidereal technique for calculating the spurious sidereal effects due to seasonal variation of amplitude of a solar diurnal variation (Farley *and* Storey 1954) is extended to include seasonal and secular changes of various kinds. The application of the anti-sidereal technique to secular and seasonal modulation of a *sidereal* diurnal variation is developed and it is found that semi-annual changes in particular could give rise to large anti-sidereal effects. It is shown that the distinctive seasonal changes observed in 1958 could be explained rather better on the basis of the existence of a sidereal component exhibiting a semi-annual variation than by attributing seasonal modulation to a solar component.

Seasonal modulation is difficult to interpret when the daily variation contains solar and sidereal components of comparable magnitude and it will be shown that, in seeking an explanation, it is desirable to examine trends in the seasonal effect over a number of years.

Very significant evidence for the existence of a genuine sidereal component is obtained if, as might happen when solar activity decreases, the amplitude of the solar component becomes so small that the sidereal component dominates the daily variation and the resultant time of maximum becomes orientated in sidereal time. The phenomenon is referred to here as a phase anomaly. The appearance of such an effect in the daily variation underground during 1961 is examined closely. It is important that the sidereal time of maximum given by the phase anomaly should be compatible with the phase of the annual apparent sidereal effect observed over a number of years, and it is also desirable to have evidence that the occurrence of the anomaly synchronizes with secular trends of amplitude of the individual solar and sidereal components.

The indications from the underground telescopes at Hobart for a persistent sidereal component with an averaged time of maximum between 0500 and 0700 local sidereal time (LST) contrasted with observations from the northern hemisphere that had accumulated over many years from detectors at ground level. Although the observed times of maximum varied considerably, both from year to year and from place to place, there was a tendency, particularly from the ionization chambers, for the annual sidereal diurnal maximum in the northern hemisphere to occur somewhere between 1800 and 2100 local sidereal time (LST). Relatively little evidence was available from the southern hemisphere over the same period, certainly not enough to substantiate the indications from the ionization chamber at Christchurch (New Zealand) for a much earlier time of maximum at southern latitudes of viewing.

The evidence for a phase difference of approximately 12 hours between the apparent sidereal diurnal maxima in the two hemispheres was to be substantiated by the results from the high counting rate detectors underground at Budapest and London. Two phenomena are notably associated with a 12-hour phase difference of this type:

(1) A spurious sidereal diurnal variation, produced by seasonal modulation of a solar component of the daily variation and appearing as a residual when the data are arranged in sidereal time and averaged over a complete year. The reversal of phase between hemispheres follows from the fact that the time of maximum of the spurious effect is tied to the time of the year when maximum seasonal change occurs.

(2) A two-way sidereal anisotropy. When it became clear that the essential problem of detection was to distinguish between possibilities (1) and (2) above, experimental studies were commenced that would give more specific evidence of the nature of the sidereal effect, as distinct from existing evidence for a genuine sidereal component in the vertical direction.

A simple two-way anisotropy lends itself quite well to identification. It produces a sidereal daily variation comprising two harmonics. If the anisotropy is due to an excess of primary particles with steep helices, the two harmonics are in phase, whereas in the case of an excess of particles with flat helices the harmonics are six hours out of phase (Table 1.1). In both cases the harmonics exhibit a distinctive latitude dependence of amplitude. However, it should be noted that these characteristics relate to free-space conditions and, as mentioned in Section 1.1, it is necessary to employ the technique of the asymptotic cone of response to arrive at the corresponding characteristics that should be observed with underground detectors. The cone of acceptance technique has already been developed by McCracken and others (McCracken, Rao *and* Shea 1962; Rao, McCracken *and* Venkatesan 1963) for the interpretation of the solar daily variation of neutron intensity at ground level, and the treatment is extended here in the somewhat different application to data from the meson telescopes underground.

In calculating the response to a sidereal anisotropy, an empirical method is used in an attempt to take some account of the influence of the interplanetary magnetic field. The magnetic field should impose a primary threshold rigidity for observation of the anisotropy, which is assumed for the present to originate beyond the interplanetary field and independently of it. To a first approximation this threshold rigidity is equated with the upper limiting rigidity of response to the solar anisotropy. There is now considerable evidence to show that the extra-terrestrial part of the observed annual solar diurnal variation is due to charged primaries which are constrained by the spiral interplanetary field to stream past the earth with azimuthal velocity of rigid rotation with the sun (Rao, McCracken *and* Venkatesan 1963; Parker 1964). A method is described of determining the upper limiting rigidity of the solar anisotropy by comparison of observations from meson telescopes underground and high latitude neutron monitors, and average values for some individual years are estimated.

To obtain free-space first and second harmonics of a sidereal daily variation as functions of asymptotic latitude of viewing, a latitude survey is required. This was undertaken during the years 1961 and 1962, with underground telescopes scanning three different strips of asymptotic latitude near the plane of the meridian at Hobart. The results were combined with the data from vertical telescopes located at approximately the same depth underground at Budapest. The latitude dependence of amplitude of the estimated free-space harmonics gives important evidence for the existence of a two-way anisotropy and provides a means of determining the declination (arbitrarily specified looking southward). The resulting estimated direction (RA and Dec.) of the axis of the anisotropy should be appropriate to the observed phase relation between the two harmonics. That is to say, if the harmonics are in phase (steep helices) the direction might be expected to approximate that of the local galactic field, whereas if the harmonics are six hours out of phase (flat helices) the analysis should give a direction normal to the field (Table 1.1).

A further experiment was designed to distinguish specifically between the 12-hour phase difference in the first harmonic that would be connected with a two-way anisotropy and the 12-hour phase difference associated with a spurious sidereal effect produced by seasonal modulation of a solar component. Towards the end of 1963 a narrow angle telescope was mounted at a sufficiently steep inclination to the north of zenith underground to enable it to scan asymptotically entirely within the northern hemisphere. If the apparent sidereal maximum observed in the vertical direction differed from the maximum observed in the north-pointing direction by 12 hours, a genuine two-way anisotropy would be indicated. On the other hand, if the maxima were in phase, this, combined with the existing evidence for a 12-hour difference in the two hemispheres, would conform with a spurious sidereal effect. A significant result for the first two complete years of observation is presented.

In summary, although at first the object of the investigation was to determine whether or not there was a genuine sidereal component in the daily variation observed underground, it will be shown that this has led to evidence for intensity maxima in both directions along an axis running parallel with the estimated direction of the local galactic magnetic field. While second-order features (e.g., year-to-year changes, seasonal variations, rigidity dependence of amplitude) give indications of a more complex structure, the evidence to be presented here is centred on the fulfilment of the following observational requirements for a sidereal component produced by a simple two-way anisotropy:

1. There should be evidence from the anti-sidereal analysis of annual averages that there is a sidereal component of the daily variation as distinct from spurious effects produced by seasonal modulation of a solar component. It is desirable that the identifying features—time of maximum, nature of harmonics (see 4, below)—should be observed to maintain approximate constancy from year to year.

2. A phase anomaly should be compatible with secular trends of amplitude of the solar and sidereal components and should give a sidereal time of maximum that conforms with the characteristic annual average.

3. The 12-hour phase difference between the sidereal diurnal maxima observed in the northern and southern hemispheres should be demonstrated to be consistent with a two-way anisotropy and not with a spurious sidereal effect.

4. The sidereal component should comprise first and second harmonics that should be approximately in phase or six hours out of phase, depending on the type of two-way anisotropy that is being observed. The corresponding free-space harmonics should exhibit the appropriate latitude dependence of amplitude.

5. The direction of the anisotropy estimated from the free-space harmonics should be appropriate to the type of anisotropy indicated by 4. above.

### 1.4. DEFINITIONS

To clarify some of the terms already used and to introduce the notion of sidereal time and its connection with solar time, the following definitions are appended.

A *sidereal day* is the time interval between successive transits of the observer's meridian by a fixed star. Because of the motion of the earth around the sun, the meridian transit of a star in solar time occurs approximately four minutes earlier on successive days, with the result that there are 366 transits in a year of 365 solar days. It follows that the unit of sidereal time is shorter than solar time by a factor of 365/366. Since it is necessary to designate some moment in the year when solar and sidereal time are the same, solar time and *local sidereal time* (LST) are synchronized at the September equinox (September 22 or 23). In other words, sidereal time is reckoned from the noon transit (upper meridian passage) of the First Point of Aries, which occurs at the March equinox. Since *right ascension* (RA) is measured eastward of the First Point of Aries, the sidereal time of meridian transit of any fixed point in the sky is also its right ascension.

As far as the present study is concerned, some of these definitions need not be applied with astronomical strictness. For example, although solar and sidereal times should be synchronized at the moment of the equinox, it is sufficient to synchronize them at any time of the day of the equinox, conveniently at GMT zero, since this does not involve more than two minutes of error at any one place. Again, mean solar time is used in calculating the solar daily variation, whereas strictly speaking it is orientated in apparent local solar time—this introduces maximum seasonal errors of timing approximately $\pm$ 18 minutes.

The maximum of a genuine sidereal daily variation of cosmic ray intensity must occur progressively earlier day by day in solar time, in such a manner that, on being averaged in solar time over a complete year, the daily variation cancels itself out—provided that, in sidereal time, it is of constant amplitude and phase. Similarly, the maximum of a genuine solar daily variation occurs progressively later in sidereal time on successive days and the daily variation cancels itself out when averaged in sidereal time over a complete year—again, provided that it is a constant daily variation in solar time. The annual averaging of daily variations with respect to solar and sidereal time is therefore a completely effective method of separating out solar and sidereal components of all degrees of complexity provided that the components are constant.

A daily variation that is arranged in solar time and averaged over a period of less than a year is referred to here as *the daily variation in solar time, the total daily variation* or *the daily variation*, and usually relates to a single day or a monthly average. It may contain both solar and sidereal components.

A *sidereal component* of a daily variation is defined as a contribution which is orientated in sidereal time, the maximum having characteristically a fixed right ascension. It should perhaps be added that *orientated* implies that it exists independently of other componets of the daily variation.

A *solar component* is defined as a contribution which is orientated in solar time, the identifying maximum having characteristically a fixed direction relative to the earth-sun line.

The annual average of a daily variation arranged in sidereal time is known as *the annual apparent sidereal daily variation*, since, as we shall see, it may not have been entirely produced by a genuine sidereal component.

The annual average of a daily variation arranged in solar time is known only as *the annual mean solar daily variation*. More strictly, it is the annual apparent solar dv, since there may be a spurious contribution due to modulation of a sidereal component. If, as is usually the case at ground level, the solar component of the daily variation is very much larger than any suspected sidereal component the contamination of the annual average solar dv would be insignificant. The situation is rather different when the two components are of comparable amplitude and there is moduation of the sidereal component.

## 1.5. THE INFLUENCE OF THE INTERPLANETARY MAGNETIC FIELD

Proceeding from the evidence that a genuine sidereal component of the daily variation exists, it is proposed to obtain a description of the sidereal anisotropy as it manifests itself immediately beyond the earth's field region, to the extent that the observations will allow. To do this, it will be necessary to assume that the observed effect is due to charged primaries, to assume a spectrum of variation and to estimate a cut-off for observation imposed by the interplanetary magnetic field. The resulting free-space characteristics, as they will be called for the present, should in themselves give some indication that the sidereal effect is indeed due to charged primaries. Naturally, it is desirable to demonstrate this at an early stage, since it would constitute further evidence that the effect was not spurious and that it was not due to neutral radiation to which there might be a response at the earth. Anisotropies of the latter kind appear to be suggested in very different ways, by the experiments of Sekido *et alia* (1963) and of Cowan *et alia* (1965).

A quantitative extrapolation to the effective outer boundary of the interplanetary field will not be attempted in this report. It will be shown that this would concern in the main the calculation of asymptotic trajectories for primaries of rigidities greater than $\sim$ 100 GV which arrived at the magnetosphere from directions steeply inclined to the plane of the ecliptic. Of crucial importance here would be a knowledge of the manner in which the field strength changes as one proceeds outward from the plane of the ecliptic (where the value is $\sim 5 \times 10^{-5}$ gauss at the earth's orbit) to distances of the order of astronomical units. It would seem from the lack of observations in these regions that an adequate description of the field in three dimensions cannot be given yet. On the other hand, it is suggested that the underground observations in themselves may give new and significant information as to the properties of the field. We now briefly mention three relevant types of observation connected with the present investigation.

In the first place, a large decrease in amplitude of the solar daily variation underground has been observed to occur during the years following the recent solar maximum. It is believed that this constitutes evidence of a great reduction in the ability of the interplanetary field to trap primaries of high rigidity. Values of the maximum primary rigidity of response, in relation to the solar anisotropy, are estimated from comparisons of the solar daily variations observed above and below ground. These give some indication of the threshold rigidity for observation of a sidereal anisotropy.

The second type of observation that might prove to be of considerable relevance is seasonal modulation of a sidereal daily variation. The Right Ascension of the outward (or inward) direction of the interplanetary field as seen from the earth must rotate through 360° in the course of a year. It also appears, from the Imp–I measurements that during that time there may be as many as 50 roughly equally spaced reversals of the observed direction of the field (Wilcox *and* Ness 1965). Therefore it seems that, except for the imposition of a threshold rigidity for observation of a sidereal anisotropy and a considerable smoothing of amplitude, the influence of the field would be mainly seasonal. If, as will be proposed, strong seasonal modulation of the sidereal daily variation was observed underground at solar maximum, this may have been connected with a tendency for the primaries to mirror in the inter-planetary field as they neared the sun. A discussion of this possibility is given in Section 5.7.

Support for the view that the interplanetary field may not produce important directional changes other than those of a seasonal character will come from a consideration of annual average free-space characteristics. A corollary to the evidence for a sidereal anisotropy, derived from a presumed charged particle response integrated over all rigidities above a threshold of $\sim$ 100 GV, will be that there appears to have been no large distortion of the directions of maximum intensity between the source region and the magnetosphere.

It will be indicated, then, that distortions of the anisotropy produced by the interplanetary field may be considerably less, over annual averages, than the observed field strength at the earth's orbit might at first suggest.

## 2. MODULATION OF COMPONENTS AND THE SPURIOUS SIDEREAL EFFECT

### 2.1. MODULATION OF A SOLAR COMPONENT

Almost any type of modulation of a solar component of the daily variation is liable to give rise to a spurious sidereal effect when the data are arranged in sidereal time and averaged over a complete year. Included are irregular changes, uniform linear changes, cyclic changes with a period of one year (referred to as annual seasonal changes), and cyclic changes with a period of more than one year and/or an odd sub-multiple of a year. Notable exceptions are the cyclic changes with periods of one half-year or sub-multiples of this. The guiding rule which covers most eventualities is that changes during the year that cause a solar component to be different, at any time, from its value six months previously are liable to give rise to a residual annual apparent sidereal effect. In most cases the residuals would be expected to be very small, but more significant effects may be generated by the annual seasonal changes and by sizeable linear displacements of phase.

Generally speaking, if all seasonal changes are regarded as ultimately deriving from the annual variation of the earth's ecliptic longitude, it is convenient to classify them (a) as being connected with the obliquity of the ecliptic (e.g., annual variation of the declination of the sun or semi-annual variation of the modulus of the declination) or (b) as not belonging to (a) above (e.g., changes connected with the position of the earth relative to the solar equator, its heliocentric position, or with its distance from the sun, or with its position in the cavity swept out by the solar wind).

Seasonal modulation has been detected or predicted quantitatively in two solar components of the c/r daily variation, and in both cases derives from the annual variation of the declination of the sun. In the first instance, there are the annual seasonal changes in the diurnal variation of atmospheric temperature that are of crucial importance in the study of the apparent sidereal daily variation of meson intensity at ground level. As Dorman (1957) has shown, the component of the daily variation due to atmospheric temperature is, properly speaking, the integrated effect of contributions from all levels of the atmosphere up to the production level for pi-mesons. It is difficult to predict theoretically the effect for a given locality, because it appears to vary markedly from place to place according to local conditions. In those places where it can be calculated, using radiosonde data, it seems that the true effect in the past has been seriously masked by radiation errors in the data (see Dorman 1965). Again, the nature of the contribution to the daily variation depends greatly on the energy-response of the detector. Consequently it seems not quite clear yet what the characteristics are. The time of maximum (of the induced cosmic ray diurnal variation) once thought to be about midnight has recently been estimated to be near 0600 local solar time (Quenby *and* Thambyah-pillai 1960). The amplitude varies considerably from place to place [e.g., see (a) the discussion by Kitamura and Kodama (1960) on the difference between Syowa Base and Mawson in relation to the negative temperature effect and (b) the conclusion reached by Bercovitch (1963) that the effect at Deep River was extremely small]. However, it is generally agreed that the diurnal variation is a maximum in summer. The daily variation of neutron intensity is, of course, not subject to temperature effects of this kind. It is hoped to show in later sections that what contribution there is from atmospheric temperature in the daily variation of meson intensity underground must be relatively small at Hobart.

The other type of seasonal modulation of a solar component concerns the diurnal variation due to a solar anisotropy. The anisotropy is assumed to have a fixed orientation with respect to the E-S line and to be constant in time. The seasonal effect is a frame of reference phenomenon and is essentially a semi-annual variation of amplitude of the diurnal variation of the primaries. Thus, although the magnitude is small, it is inherent in the daily variation from any detector which responds to the anisotropy. The transformation from the frame of reference of the anisotropy to the rotating terrestrial co-ordinate system is described in Paper 1 (Jacklyn 1963a) and will be referred to again in Section 6B.1. The procedure is relevant in that it demonstrates that the seasonal effect is essentially of semi-annual character, not annual as might be at first supposed, and therefore cannot be responsible for spurious annual sidereal components.

There are two very important indicators of a spurious sidereal effect produced by seasonal modulation of a solar component:

(1) an anti-sidereal effect (not necessarily matching in amplitude) which is described below, and

(2) a difference of 12 hours between the time of maximum of a spurious sidereal effect observed in the northern hemisphere and the time of maximum observed in the southern hemisphere, because of the six months' difference in the seasons. Now it so happens that there appears to be a genuine sidereal component of the daily variation that exhibits precisely these two characteristics. This constitutes the essence of the problem of interpretation of observations from the point of view of seasonal modulation. Consequently, as mentioned in the Introduction, it becomes necessary (a) to devise an experiment to distinguish specifically between the two possibilities and (b) to find supporting evidence for this particular type of sidereal component, independently of seasonal modulation.

## 2.2. MODULATION OF A SIDEREAL COMPONENT

Modulation of a sidereal component has not been considered by others, perhaps because there has been no compelling evidence for it and because it may have been thought that, if there is a genuine sidereal daily variation, it should remain constant over periods of time of the order of a solar cycle. However, to mention one situation that could arise, the interplanetary magnetic field may impose the effective primary cut-off rigidity, $R_c$, for observation of a sidereal anisotropy by the given detector, and $R_c$ may be greater than $R_l$ the lower limiting rigidity of the anisotropy. It would be reasonable to expect both seasonal and year-to-year changes in the cut-off to occur and it seems likely that the consequent seasonal changes in the observed sidereal effect would belong to category (a) described in Section 2.1, with a tendency to be semi-annual in character.

It will be necessary to consider modulation of a sidereal component very closely when examining the evidence for the existence of a sidereal anisotropy. It will be shown below that seasonal modulation of a sidereal component can give rise to a very significant anti-sidereal effect. If this possibility is not taken into account, the presence of a significant annual component in anti-sidereal time comes to be regarded as unambiguous evidence that at least part of the observed sidereal daily variation is spurious. It is stressed that this view of the role of the anti-sidereal effect is incorrect.

### 2.3. THE ANTI-SIDEREAL TECHNIQUE

A theoretical treatment of this topic is given in Paper 2 (Jacklyn 1962). Only the essential ideas are discussed here.

*(a) Seasonal and secular modulation of a solar component*

The anti-sidereal technique was originally devised by Farley *and* Storey (1954) as a means of determining the spurious annual sidereal component produced by an annual variation of amplitude of a solar diurnal variation. We use their case to illustrate a method that applies in much the same manner to other types of modulation.

The diurnal variation is regarded as a carrier wave whose frequency is 365 cycles per year. The annual variation of amplitude becomes amplitude modulation at a frequency of 1 cycle per year. It follows from the theory of modulation of radio waves that, upon averaging over complete cycles of modulation, matching sideband components will appear at frequencies of 366 cycles per year and 364 cycles per year. The former is the sidereal frequency, there being 366 sidereal diurnal cycles in one year. Thus, by arranging the modulated solar diurnal variation in sidereal time, by the day or group of days, and averaging over a complete year, the appropriate sideband components will appear as an apparent sidereal diurnal variation. However, it has none of the characteristics of a genuine sidereal component.

The matching sideband at 364 cycles per year is known as *the anti-sidereal component*. It is extracted by arranging the data, for individual days, in units of time according to day lengths that are approximately four minutes longer than a solar day and averaging over a complete year. Note that anti-sidereal time, like sidereal time, is synchronized with solar time at the September equinox.

In the case we are considering, the anti-sidereal component is very simply related to its counterpart, the spurious sidereal diurnal variation. Denoting the carrier wave by $A \sin \phi$, the modulated solar dv is given by

$$i_d = A \{1 + k \cos (\theta - \theta_o)\} \sin \phi \qquad (2.1)$$

where $k$ is the maximum fractional change of amplitude, $\theta$ is time of year in angular measure, and $\theta_o$ is the phase constant of the annual variation. It is given by

$$\theta_o = \frac{2\pi T_o}{12}$$

where $T_o$ is time of the year in months measured from the September equinox. Following the usage in Paper 2, where the distinction was found to be convenient in some sections, $i_d$ is replaced by $I_d$ to denote the annual average and $\phi$ is replaced by $\Phi$, although the two symbols are synonomous.

The annual mean diurnal variations in solar, sidereal and anti-sidereal time are

Solar: $$I_d = A \sin \Phi \quad (2.2)$$

Sidereal: $$I_s = \frac{Ak}{2} \sin (\Phi' - \theta_o) \quad (2.3)$$

Anti-sidereal: $$I_{as} = \frac{Ak}{2} \sin (\Phi'' + \theta_o) \quad (2.4)$$

where $\Phi$, $\Phi'$ and $\Phi''$ represent solar, sidereal and anti-sidereal time respectively.

We note that the sideband terms $I_s$ and $I_{as}$ have equal amplitudes and that, within the respective time frameworks, the times of maximum are displaced symmetrically about the time of maximum of the carrier wave. In practice the annual averaging of the data in solar and anti-sidereal time would give $I_d$ and $I_{as}$, whereas in sidereal time we would have $I_s$ plus, perhaps, a genuine sidereal component. The equations show that $I_s$ can be deduced from $I_{as}$ and $I_d$ very simply. This allows $I_s$ to be subtracted from the annual mean apparent sidereal d.v. leaving the genuine part as the residual. This is usually done vectorially on a first harmonic dial. Knowledge of $I_d$ and $I_{as}$ also allows us to determine $k$ and $\theta_o$, the constants of seasonal modulation.

In general, annual seasonal changes and secular changes (e.g., uniform displacement of phase) in a solar diurnal variation give rise to sidereal and anti-sidereal sideband components. Semi-annual changes do not. Various cases of interest are examined in Paper 2. The figures reproduced here show how the annual characteristics are related, in respect of

(1) annual linear displacement of phase (Figures 2.1, 2.2 and 2.3); and
(2) annual sinusoidal variation of phase (Figure 2.4).

A treatment of combined annual variation of amplitude and phase is given in Paper 4 (Jacklyn 1963b) and will be considered in Section 5.

*(b) Seasonal and secular modulation of a sidereal component*

In the context of modulation of a sidereal component the carrier wave is at the sidereal frequency, 366 cycles per year. We are particularly concerned here with modulations which produce sidebands at the anti-sidereal frequency, 364 cycles per year. Simple annual variations cannot do this, but an obvious possibility is a semi-annual variation of amplitude, the modulating frequency being 2 cycles/year. This produces sidebands at 368 cycles/year and 364 cycles/year. It is perhaps the most important case and is dealt with on page 18 of paper 2. In brief, if the carrier is denoted by $A \sin \psi'$, the modulated sidereal d.v. is given by

$$i_s = A \{1 + k \cos (2\theta - \theta_o)\} \sin \psi' \quad (2.5)$$

where $k$, $\theta$ and $\theta_o$ have the appropriate meanings. The annual mean diurnal variations in sidereal and anti-sidereal time are

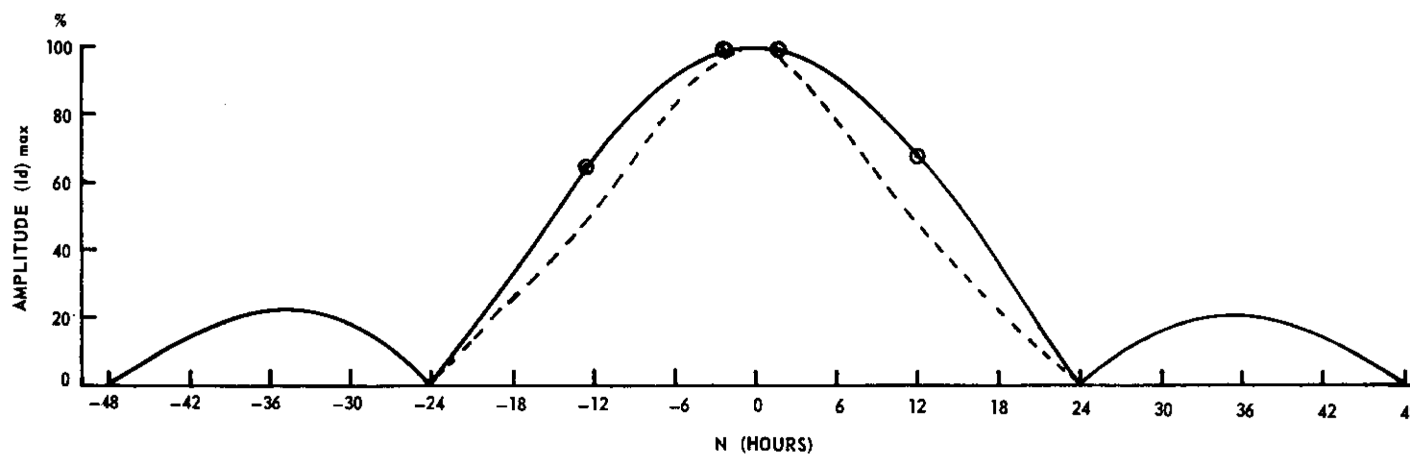



FIG. 2.1. The annual mean amplitude of a solar daily variation versus total annual displacement (N) of phase.
Full lines : Theoretical Relationship
Dashed lines : Relationship obtained from the "underground" model
Circled points: Values from a sinusoidal model.

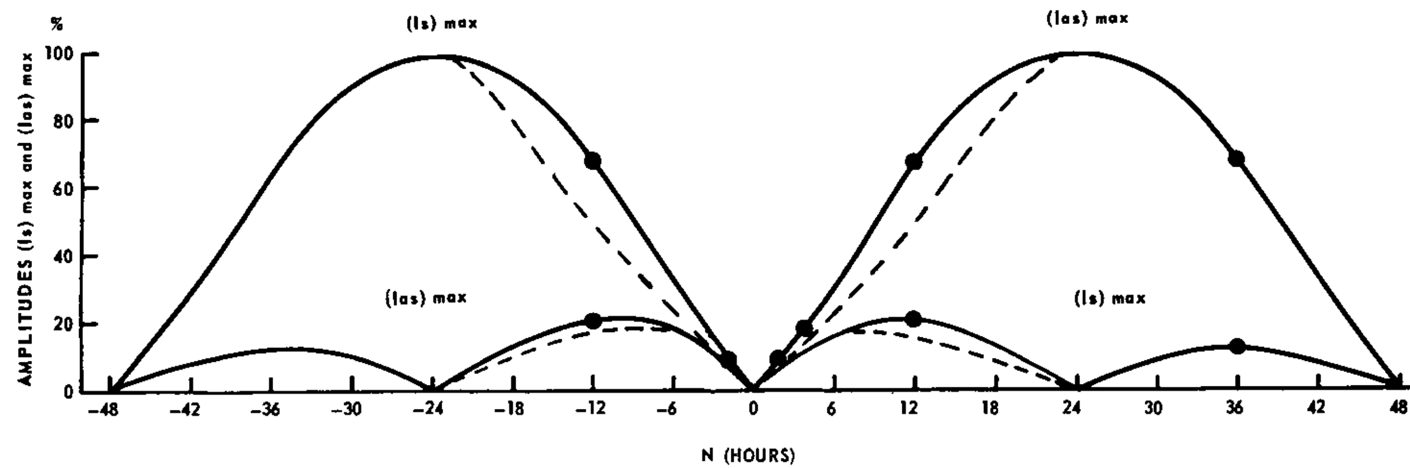



FIG. 2.2. The amplitudes $(I_s)_{max}$ and $(I_{as})_{max}$, versus total annual displacement $N$ of phase of a solar diurnal variation. $(I_s)_{max}$ is the amplitude of the annual mean d.v. in sidereal time and $(I_{as})_{max}$ is the corresponding amplitude in anti-sidereal time.
Full lines : Theoretical Relationship
Dashed lines : Relationship obtained from the "underground" model
Circled points: Values from a sinusoidal model.

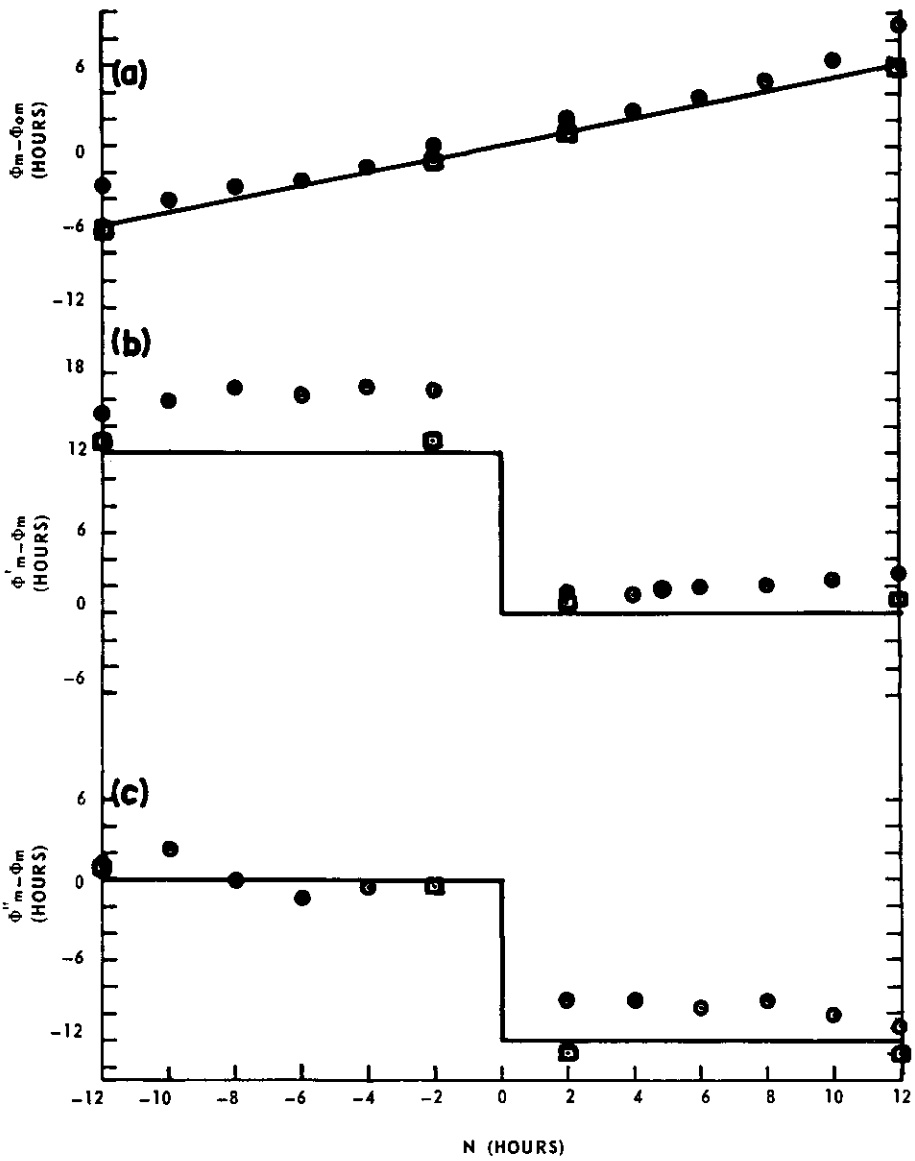



FIG. 2.3. The relationships between the annual mean solar ($\Phi_m$), sidereal ($\Phi'_m$) and anti-sidereal ($\Phi''_m$) diurnal maxima for a solar daily variation with linear displacement of phase, versus $N$, the total annual phase displacement. $\Phi_{om}$ is the time of maximum of the unmodulated d.v.

Full lines : Theoretical relationships
Squared points: Values from a sinusoidal model
Circled points : Values from the "underground" model.

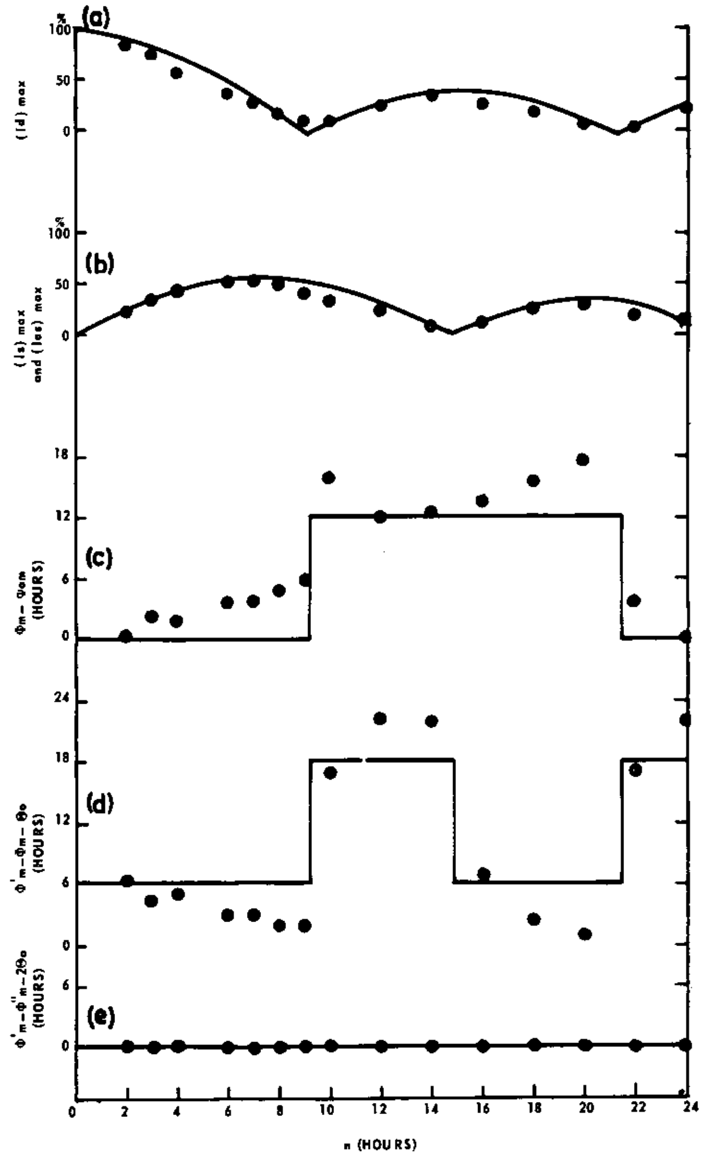



FIG. 2.4. The amplitude and phase relationships for annual mean solar ($I_d, \Phi_m$), sidereal ($I_s, \Phi'_m$) and anti-sidereal ($I_{as}, \Phi''_m$) diurnal variations produced by a solar d.v. with an annual variation of phase, versus $n$, the maximum phase displacement. $\Phi_{om}$ is the time of maximum of the unmodulated d.v. and $\Theta_o$ as is the phase constant determined by the time of year when the maximum displacement of phase occurs.

Full lines : Theory

Circled points: Values from the "underground" model.

Sidereal: $I_s = A \sin \psi'$ (2.6)

Anti-sidereal: $I_{as} = \frac{Ak}{2} \sin (\psi' + \theta_o)$ (2.7)

A relatively large anti-sidereal effect may be generated in this manner. It is not at all easily distinguished from an effect produced by annual variation of amplitude of a solar d.v. if the total daily variation contains both solar and sidereal components. In either situation, if the solar d.v. is the greater of the two components, the end-points of the monthly mean diurnal vectors trace out an ellipse in an anti-clockwise manner. The critical point of difference is that a spurious annual sidereal component is generated in the case of modulation of the solar component.

Other types of modulation of a sidereal component are considered in Paper 2, as for example,

(a) linear displacement of phase of a sidereal diurnal variation (Figures 2.5 and 2.6),

(b) annual variation of phase (Figure 2.7), and

(c) semi-annual variation of phase (Figure 2.8).

Combined semi-annual variation of amplitude and phase of a sidereal diurnal variation is treated in Paper 4 (Jacklyn 1963b) and will be referred to in Section 5.

*(c) Application of the technique*

In order to use the anti-sidereal technique in a given situation it is necessary to decide as to the component or components being modulated and the nature of the modulation. This seems to be the chief source of error in the use of the technique. The tendency is to assess modulation of the daily variation on the basis of physical mechanisms that are thought likely to apply, the result being that the anti-sidereal effect is generally attributed to seasonal changes in the diurnal variation of atmospheric temperature. In Paper 4 an attempt is made to elucidate a pronounced seasonal effect in the daily variation underground by comparing the observations with the predictions of several widely differing hypotheses of seasonal modulation. The hypothesis of best fit is then provisionally chosen as the correct one.

Strictly speaking, the anti-sidereal technique in its present form applies only to first harmonic components of the daily variation. It is conceivable that serious departures from the prediction of the theory might occur if the component being modulated consisted of more than a first harmonic. In Paper 2 an attempt was made to investigate this problem. The mathematical predictions from the integration of modulated sine waves were compared with the results from two types of models (see page 5 of the paper). In one of the models the carrier was a sine wave and it was processed in the same way as the cosmic ray data. The purpose was to detect any significant bias or smoothing that might result from records that were analysed in the form of monthly groups of bi-hourly averages. Comparison with the mathematics revealed no significant discrepancies. The basis of the other model was a carrier constructed from the long-term average solar daily variation observed underground. It is referred to as the "underground" model in the paper and contains a considerable second harmonic. In most cases the results from the model

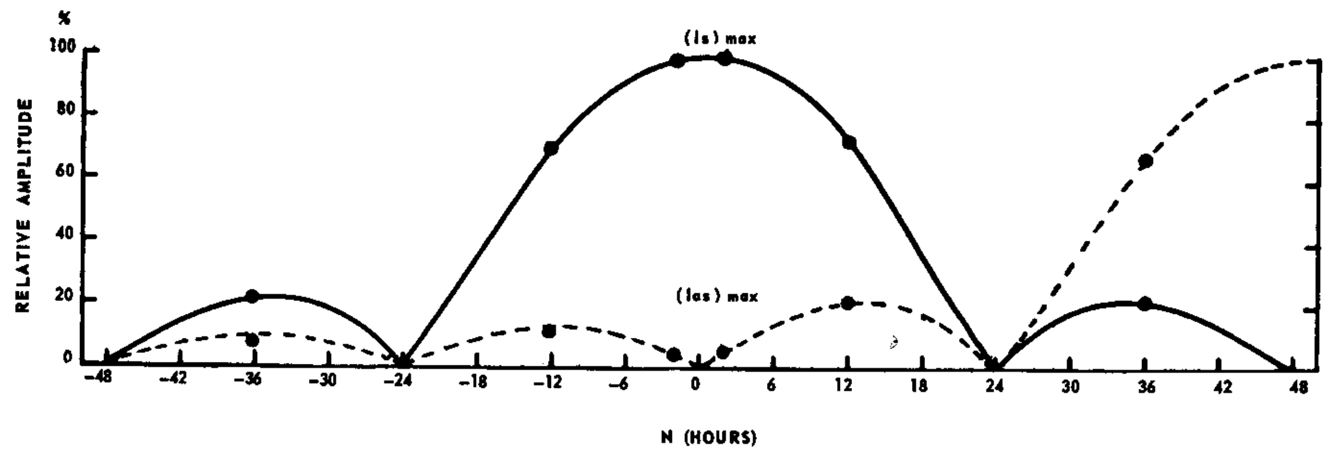



FIG. 2.5. The amplitudes of the annual mean sidereal ($I_s$) and anti-sidereal ($I_{as}$) diurnal variations associated with a linear displacement of phase of a sidereal diurnal variation, versus $N$, the total annual phase displacement. The amplitudes are measured relative to the amplitude of the unmodulated sidereal d.v.

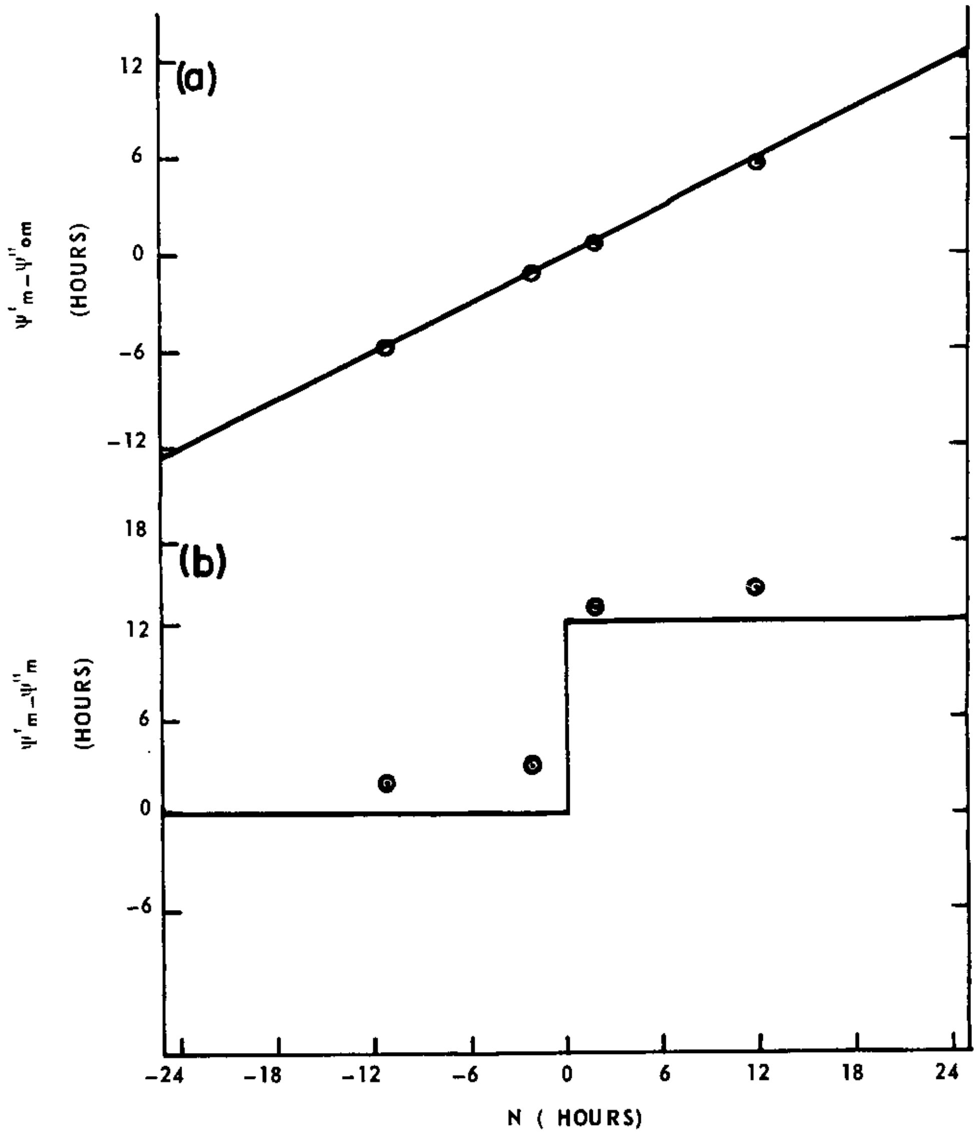



FIG. 2.6. The relationships between the annual mean sidereal ($\psi'_m$) and anti-sidereal ($\psi''_m$) diurnal maxima for a sidereal diurnal variation with linear displacement of phase, versus $N$, the total annual phase displacement. $\psi'_{om}$ is the time of maximum of the unmodulated d.v.
Full lines : Theoretical
Circled points: Values from the "underground" model.

were in good agreement with the mathematics (Figures 2.7 and 2.8). However, in the important case of a solar component with annual variation of amplitude the disagreement was significant. The case is considered in Paper 2 (see Section 5). This kind of difficulty may not be relevant to the atmospheric temperature effect, which appears to derive from a simple diurnal variation, but it would certainly

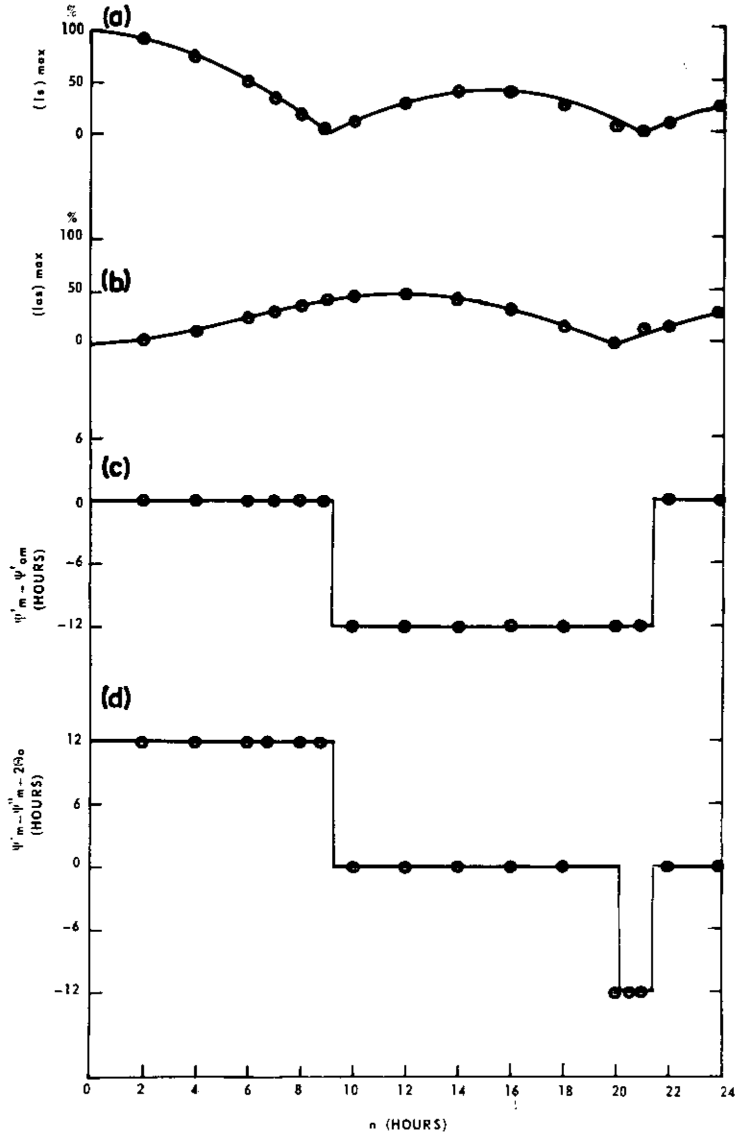



FIG. 2.7. The amplitude and phase relationships between the annual mean sidereal ($I_s$, $\psi'_m$) and anti-sidereal ($I_{as}$, $\psi''_m$) diurnal variations for a sidereal d.v. with an annual variation of phase, versus $n$, the maximum phase displacement. $\psi'_{om}$ is the time of maximum of the unmodulated d.v. and the phase constant $\Theta_o$ is determined by the time of the year when maximum displacement of phase occurs.

Full lines : Theory

Circled points: Values from the "underground" model.

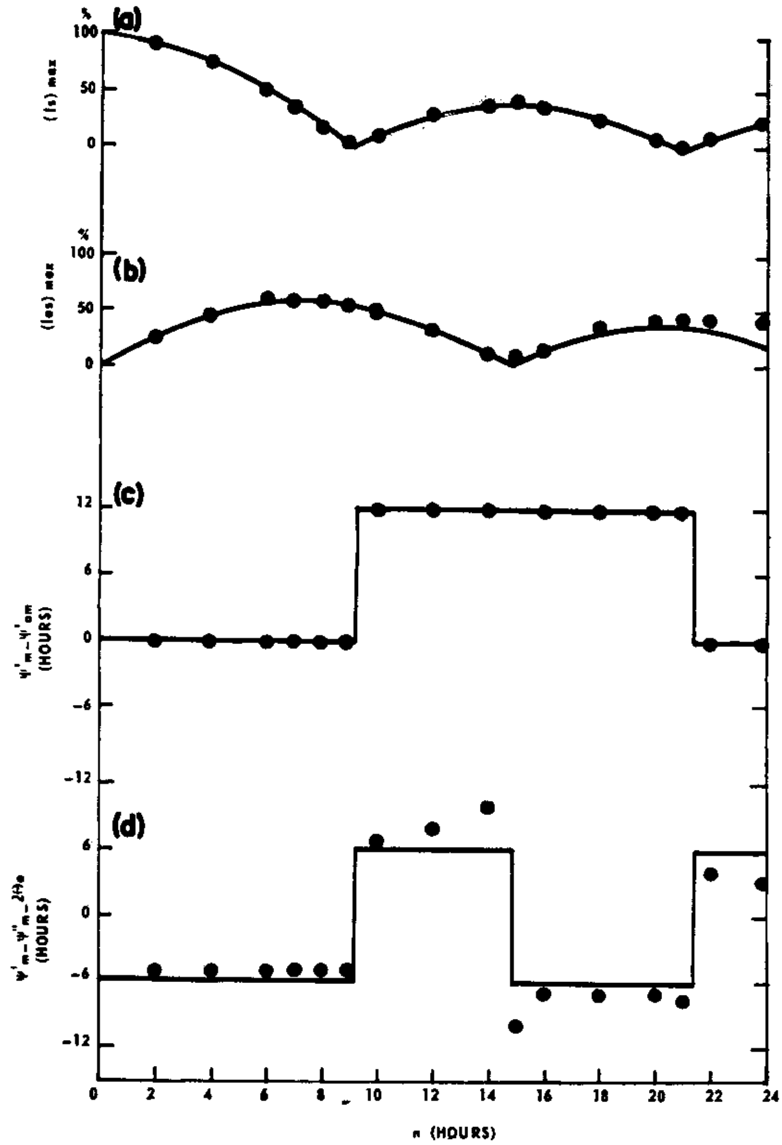



FIG. 2.8. The amplitude and phase relationships between the annual mean sidereal ($I_s$, $\psi'_m$) and anti-sidereal ($I_{as}$, $\psi''_m$) diurnal variations for a sidereal d.v. with semi-annual variation of phase, versus $n$, the maximum displacement of phase. $\psi'_{om}$ is the time of maximum of the unmodulated d.v. and the phase constant $\Theta_o$ is determined by the time of the year when maximum displacement of phase occurs.

Full lines : Theory

Circled points: Values from the "underground" model.

apply if significant annual variations were found in the daily variation produced by the solar anisotropy.

If seasonal modulation happens to be non-sinusoidal it should suffice to break it down into annual and semi-annual terms and treat them separately.

### 2.4. SECOND ORDER SIDEBANDS

Dorman (1965) has extended the sideband technique to include the second order sideband components produced by combined phase- and amplitude-modulation of the solar diurnal variation. These sideband components, at the frequencies $365 \pm 2$ cycles/year, appear in the expansion, by means of Bessel functions, of the expression for the modulated diurnal variation (equation (4.3) of Paper 4). A simpler example, the expansion of a phase-modulated diurnal variation, is given in Section 11 of Paper 2. Dorman shows that the seasonal phase and amplitude constants can be derived from the harmonic coefficients of the first and second order sidebands, obtained by suitably arranging observations and averaging over complete years.

In the application of this interesting method, it is evident that essentially the same difficulties that have just been mentioned in connection with the anti-sidereal technique must be considered. In addition, it should be noted that the higher order sidebands tend to be smaller, so that unless the carrier is of large amplitude there are liable to be difficulties in obtaining significant harmonic coefficients. Thus the technique may be better suited to the analysis of observations from ground level than from underground.

The over-riding problem, of course, is that of identifying phase- and amplitude-modulation of a solar component in the first place. Sideband components at one or more of the four frequencies of interest (363, 364, 366 and 367 cycles/year) can be generated by other types of seasonal modulation of a solar diurnal variation (e.g., a semi-annual variation of phase, treated in Paper 2, Section 11) and by annual and semi-annual variations of a sidereal component (e.g., see Paper 2, Sections 10 and 12).

## 3. REVIEW OF GROUND-LEVEL STUDIES AND COMPARISON OF RECENT GROUND-LEVEL AND UNDERGROUND OBSERVATIONS

### 3.1. REVIEW OF EXPERIMENTS AT GROUND-LEVEL

The history of experimental studies of the sidereal effect at ground level goes back in the main to the 1930s, although as early as 1923 D. Kolhorster *and* von Salis (1926) observed an apparent effect with an ionization chamber on the Jungfrau, the intensity maximum occurring at about 2000 local sidereal time (LST). Longer-term studies became possible with the accumulation of data from ionization chambers, notably from instruments on the Hafelekar, from 1932 to 1934 (W. Illing 1936) and from 1936 to 1937 (Demmelmair 1937); at Capetown, from 1933 to 1935 (Schonland, Delaritsky *and* Gaskell 1937); at Canberra, from 1936 to 1940 (Hogg 1949); and at Cheltenham (U.S.A.), Huancayo (Peru) and Christchurch (N.Z.), from 1938 onwards (Lange *and* Forbush 1948).

From the mid-1930s the theory of Compton *and* Getting (1935) appears to have greatly stimulated the studies at ground level. Following a suggestion of Lowry, these authors predicted a world-wide diurnal maximum of 0·1% within a factor of two, at about 2000 LST, in connection with the motion of the earth towards the constellation Cygnus, in the direction 47°N, 20 hours 40 minutes RA. The motion, with a velocity of 300 km/sec, was being imparted chiefly by the rotation of the galaxy and it was proposed that there should be a sidereal effect due to cosmic rays coming from outside the galaxy and not sharing in the rotation.

Their evidence for the existence of the predicted effect came from ionization chamber measurements on the Hafelekar (lat. 47°N, 2300 m), published by Hess *and* Steinmaurer (1933). Their Figure 2 showed that there was good agreement with the predictions.

In a later review of the evidence for the Compton-Getting effect, Wollan (1939) showed that, when the data from ionization chambers were averaged over several years (either those from the Hafelekar or from Capetown), the amplitudes of the apparent sidereal diurnal variations were not statistically significant. His Figure 8 gives a useful summary of results up to 1939. He also demonstrated the difficulty of detecting a sidereal component, superimposed on an amplitude-modulated solar component, by Thompson's method of fitting an ellipse to the end-points of monthly mean solar diurnal vectors on an harmonic dial (Thompson 1939). His method is also discussed in Paper 2, page 2.

The Hafelekar, Canberra and Capetown results when compared by Hogg attested to the year-to-year variability of the apparent sidereal effect at any one place. His figure 17 shows that it was not possible to detect over three or four years either a reproducible time of maximum or, for that matter, a 12-hour difference between the two hemispheres. Thus the evidence gave little support to the theory of cosmic ray streaming put forward by Compton *and* Getting, nor was the sidereal effect obviously fictititious, arising from seasonal modulation of a solar component.

In a well-known study of the sidereal effect at ground level, Elliott *and* Dolbear (1951) attempted to minimize effects due to seasonal changes of the solar component of the daily variation. They examined

(a) the difference of intensity between crossed Geiger counter telescopes, inclined 45° south and 45° north of zenith at Manchester in 1949;

(b) the ion chamber data from Huancayo for the nine complete years available, assuming that seasonal effects would be small at the low latitude concerned (12°S); and

(c) the combined ion chamber data from Christchurch (geog. lat. 35°S) and Cheltenham (38·7°N) for the six complete years available. They assumed that spurious sidereal components at the two places would cancel out in the combining process because of the 12-hour difference in phase expected between the two hemispheres.

The time of maximum of the annual apparent sidereal d.v.'s found by methods (a), (b) and (c) were in reasonably good agreement, being 0800, 0400 and 0600 LST respectively. However, the result was at variance with the predictions of Compton *and* Getting and with most of the earlier ion chamber measurements.

The authors were able to demonstrate the anti-clockwise cyclic arrangement of the bi-monthly vectors in respect of the Huancayo and Christchurch/Cheltenham data, indicative of a sidereal vector superimposed on a relatively constant solar vector. In the case of the Manchester N-S difference the same test was not successful, the statistical errors apparently masking any positive indication of this type either for or against a genuine sidereal maximum at 0800 LST. As it now appears, the two results which derived partly or wholly from the southern hemisphere were understandable and it was the essentially northern hemisphere result from Manchester that was not.

The variability of the combined Cheltenham/Christchurch result becomes evident when one examines data averaged over a different period, namely the eight complete years from 1947 to 1954. The time of maximum of the apparent sidereal effect from these data is approximately 2200 LST, differing by eight hours from the result obtained by Elliott *and* Dolbear. It would appear (Figure 3.1) that the maximum of the combined daily variation tends to coincide with the maximum at either Christchurch or Cheltenham, depending on which has the greater average amplitude at the time of observation.

Very significant and somewhat complex developments accompanied the minimum of solar activity in 1954. By now, G-M counter telescopes had become the prevailing instruments for recording the intensity of $\mu$-mesons and the first neutron monitors had been put into operation. Without enquiring into the many complexities of the observations, it seems that, broadly speaking, there were two major occurrences of interest:

(1) The time of maximum of the annual mean solar diurnal variation of meson intensity, having maintained a relatively constant value, approximately 1400 hours

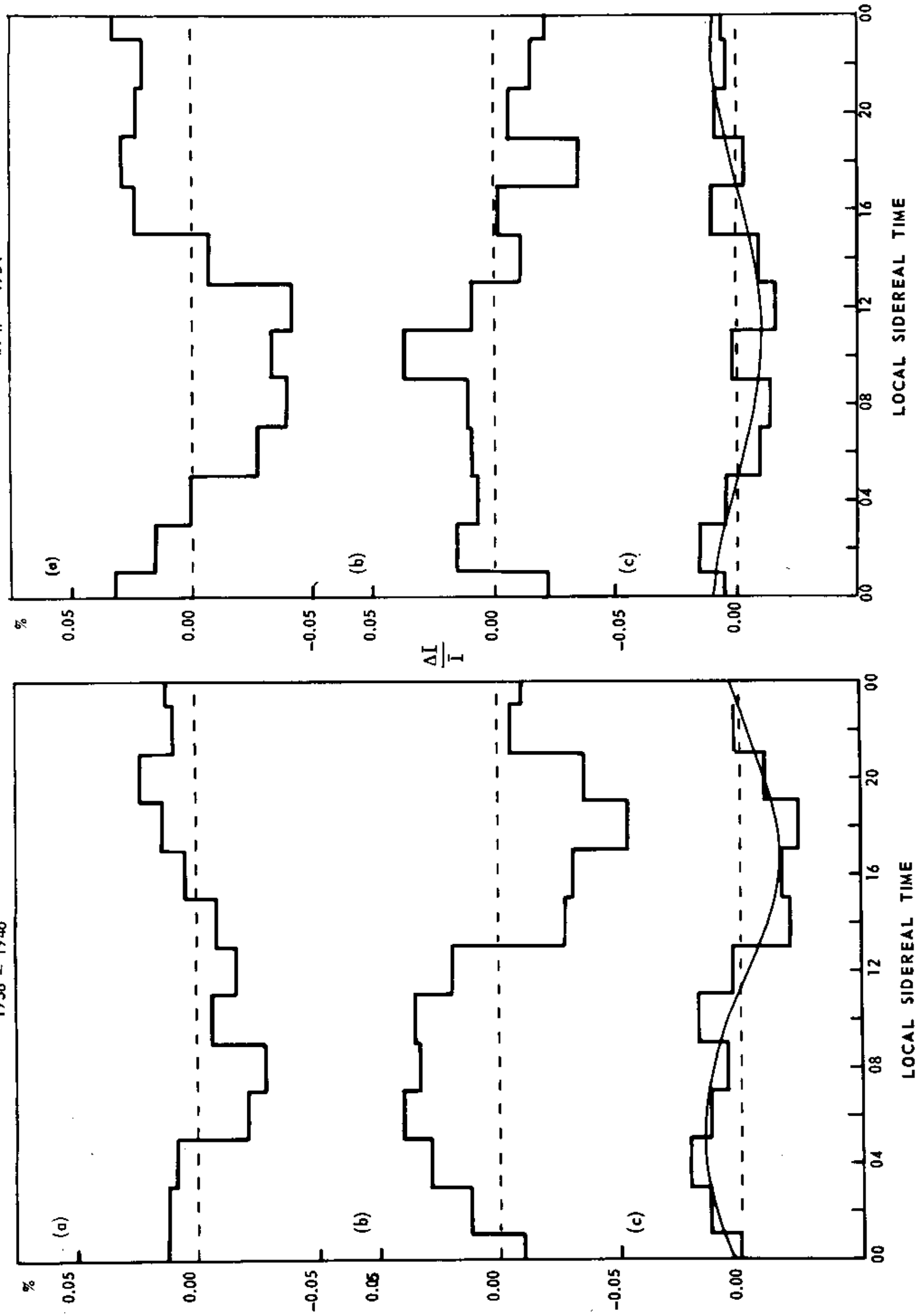



FIG. 3.1. The apparent sidereal daily variation of pressure-corrected ionization averaged over the 9 years 1938-1946 and over the 8 years 1947-1954.
(a) Cheltenham (38·7°N)
(b) Christchurch (35°S)
(c) Cheltenham and Christchurch combined, with the first harmonics of best fit.

solar time, over the previous twelve years, began to advance after 1950 and reached its earliest value, about 0700, in 1954. Thereafter the trend reversed and over a period of three years the time of maximum returned to something like its former value. Clearly, these large secular, but non-linear, displacements of phase, varying in character from year to year, would most likely be responsible for annual sideband components in sidereal and anti-sidereal time.

(2) At the solar minimum a phase anomaly occurred in the data from neutron monitors, ionization chambers and counter telescopes in the northern hemisphere. Simpson *and* Conforto (1957) noted that the phase of the solar diurnal variation in 1954 had advanced progressively at the rate of approximately two hours a month for upwards of six months in the manner of a genuine sidereal component superimposed on a smaller solar component, the sidereal maximum occurring at approximately 2000 LST. However, the figure they give is approximately 0800. It should be noted that the authors had synchronized solar and sidereal time at the March equinox instead of the September equinox, so that 12 hours must be added to sidereal times of maximum given in their paper.

Because the phase anomaly occurred in the diurnal variation of neutron intensity it could not have arisen from seasonal modulation of an atmospheric temperature component, Again, because of the magnitude and specific character of the phase anomaly it seemed unlikely to be merely an unusual seasonal variation of phase of the solar diurnal variation of the primaries, although the possibility could not be ruled out. The obvious alternative was more plausible in at least one respect: if there were a constant sidereal component in the daily variation it would be more likely to dominate the daily variation during a particularly quiet solar minimum on the expectation that the solar diurnal variation of the primaries would have greatly diminished. However, if this was what had happened, it was necessary to reconcile it with the fact that there had been large year-to-year dislacements of phase of the diurnal variation of meson intensity (see (1) above), while there was little evidence of prominent systematic changes of amplitude.

The difficulty has been studied by many workers. We refer to the more recent conclusion reached by Dorman (1963) in a review concerned essentially with the work of Glokova (1960), Kuzmin (1960) and Quenby *and* Thambyahpillai (1960), all of whom had made use of Dorman's method for determining the atmospheric temperature effect. It seemed that at many places an important and relatively constant component of the annual solar diurnal variation of meson intensity was of atmospheric origin. As the solar diurnal variation of the primaries weakened with the approach of solar maximum, so the atmospheric component began to dominate the total annual diurnal variation. This interplay between the two components was responsible for much of the secular change in phase mentioned in (1) above and to some extent masked the decrease in amplitude of the extra-terrestrial solar component. Much of the seasonal effect in the solar diurnal variation of meson intensity was also attributed to a seasonal change in the diurnal variation of atmospheric temperature. However, it would contribute only in part to the phase anomaly in the hard component of intensity and not at all to that in the nucleonic component. At this point it was concluded that there probably was a genuine sidereal component in the daily variation, but that in the hard component there would also be a spurious annual sidereal effect of atmospheric origin and perhaps of extra-terrestrial origin as well. Dorman's assessment of the temperature effect as it applied to the events at the 1954 solar minimum will be referred to again in Section 5, in connection with phase anomaly observed in the daily variation of meson intensity underground. Here, the influence from atmospheric temperature variations appears to have been very small.

In a later chapter of the same publication, Dorman (1963) goes on to outline an attempt made in collaboration with Inozemtseva (Dorman *and* Inozemtseva 1961) to discover extra-terrestrial sources of the daily variation using all available data from crossed telescopes. They hoped to be able to account for the many inconsistencies in the observations from inclined telescopes at different phases of the solar cycle over the period 1948 to 1960. They emphasized the importance of taking into account the diurnal variation of atmospheric origin and showed that the subtraction of this component had a very different effect at solar minimum than at solar maximum. Following a qualitative study of cones of viewing as functions of primary energy of response for inclined telescopes at ground level, they concluded (p. 323) that two extra-terrestrial sources of the average diurnal variation were effective at solar minimum: a low-latitude source (I) to the left of the earth-sun line, influencing mainly low-energy primaries ($\leqslant$30-40 GEV), and a high-latitude source II to the right of the earth-sun line (between 0 and 6 hours) influencing mainly particles of energy greater than 50-100 GEV. It might seem contradictory that source II should at the same time be regarded as sidereal. However, it appears that the authors' argument was as follows. Source II had a "fixed" direction with respect to the earth-sun line only in the sense that the inclined telescopes responded to it in the northern summer months, but not at other times of the year. On the other hand, the neutron monitor at Huancayo responded to it throughout the year and the observations from Huancayo were compatible with an annual anti-clockwise rotation of the source directions. Its RA would therefore appear to be about 18 hours.

It is unfortunate that confusion has arisen in the literature through the use of two definitions of sidereal time. This is particularly noticeable in Progess in Cosmic Ray Physics, Vol. VII. Table XV (p. 308) of that volume summarizes the apparent sidereal effects observed with meson telescopes at medium- and high-latitude stations from 1953 to 1957, according to the investigations of Berdichevskaya *and* Zhukovskaya (1960). The sidereal times of maximum range from 0300 to 1200 and include 0700 as the result from Huancayo in 1954. It is evident that the authors synchronized solar and sidereal time at the March equinox as had Simpson and Conforto in their treatment of the phase anomaly. Consequently, their tabulated times of maximum differ from the conclusions of Baliga *and* Thambyahpillai (1959) reported on page 307, by twelve hours. Baliga *and* Thambyahpillai had examined the low- and mid-latitude data for 1954, including the observations from Huancayo, and their times of maximum are quoted as ranging from 19 hours at the equator to 22 hours at the medium-latitude stations. These authors had synchronized solar and sidereal time correctly at the September equinox.

The ground level studies as a whole seem to suggest that the time of maximum of the apparent sidereal diurnal variation changes unpredictably, both from year-to-year and from place to place, although the amplitudes are often highly significant. Undoubtedly there may be considerable variability at any one place and this will be examined in the next Section. However, in many cases the great and apparently unsystematic differences in reported times of maximum are equally traceable to methods of treating the data. Three factors in particular seem to have led to a confused picture of the effect:

(1) Generally, experimental studies have been concerned with the search for the sidereal diurnal variation associated with a single galactic intensity maximum. Consequently, allowing for the cones of viewing, it has been expected that the same time of maximum would be observed at all places. This has led to the tendency to group the observations from the northern hemisphere (which have predominated) with those from the southern hemisphere indiscriminately.

(2) The anti-sidereal effect has been attributed almost invariably to seasonal changes in a diurnal variation of atmospheric temperature. Spurious components have been calculated accordingly and subtracted from the apparent sidereal vector. It is of interest that Baliga *and* Thambyahpillai (1959), in their discussion of the 1954 phase anomaly, have compared anti-sidereal components in the northern and southern hemispheres and have noted that they could not have entirely originated from seasonal modulation of a diurnal variation of atmospheric origin.

(3) When sidereal time has been sychronized with solar time at the March equinox, an error of 12 hours in the sidereal time of maximum has been incurred.

### 3.2. COMPARISON OF RECENT OBSERVATIONS AT GROUND-LEVEL AND UNDERGROUND IN THE SOUTHERN HEMISPHERE

The following comparison anticipates consideration of the underground experiments at Hobart, but there seem to be advantages at this point in presenting a broad picture of the response in solar, sidereal and anti-sidereal time from detectors above and below ground.

The basic material comprises pressure-corrected data from the following detectors:

(a) The vertical semi-cubical G-M telescopes (1 $m^2$ trays) at a depth of approximately 40 m.w.e. underground at Hobart (43°S, 147°E geographic).

(b) The standard vertical cubical G-M telescope ( 1 $m^2$ trays) under 10 cm lead absorber at sea level, Hobart.

(c) The standard 12-counter neutron pile at Mt. Wellington (altitude 725 m).

(d) The standard 12-counter neutron pile at sea level at Mawson (68°S, 63°E geographic).

Relatively few data are missing or have been rejected. Consequently, the observations may be considered to be simultaneous as between stations. Monthly averages of hourly data are processed in bi-hourly groups and expressed as percentage deviates from the monthly mean intensity. Considerable smoothing is effected by averaging the deviates in progressive groups of three, to form six-hour running averages. In this form the observations are expressed in solar, sidereal and anti-sidereal time and averaged over complete years centred on successive

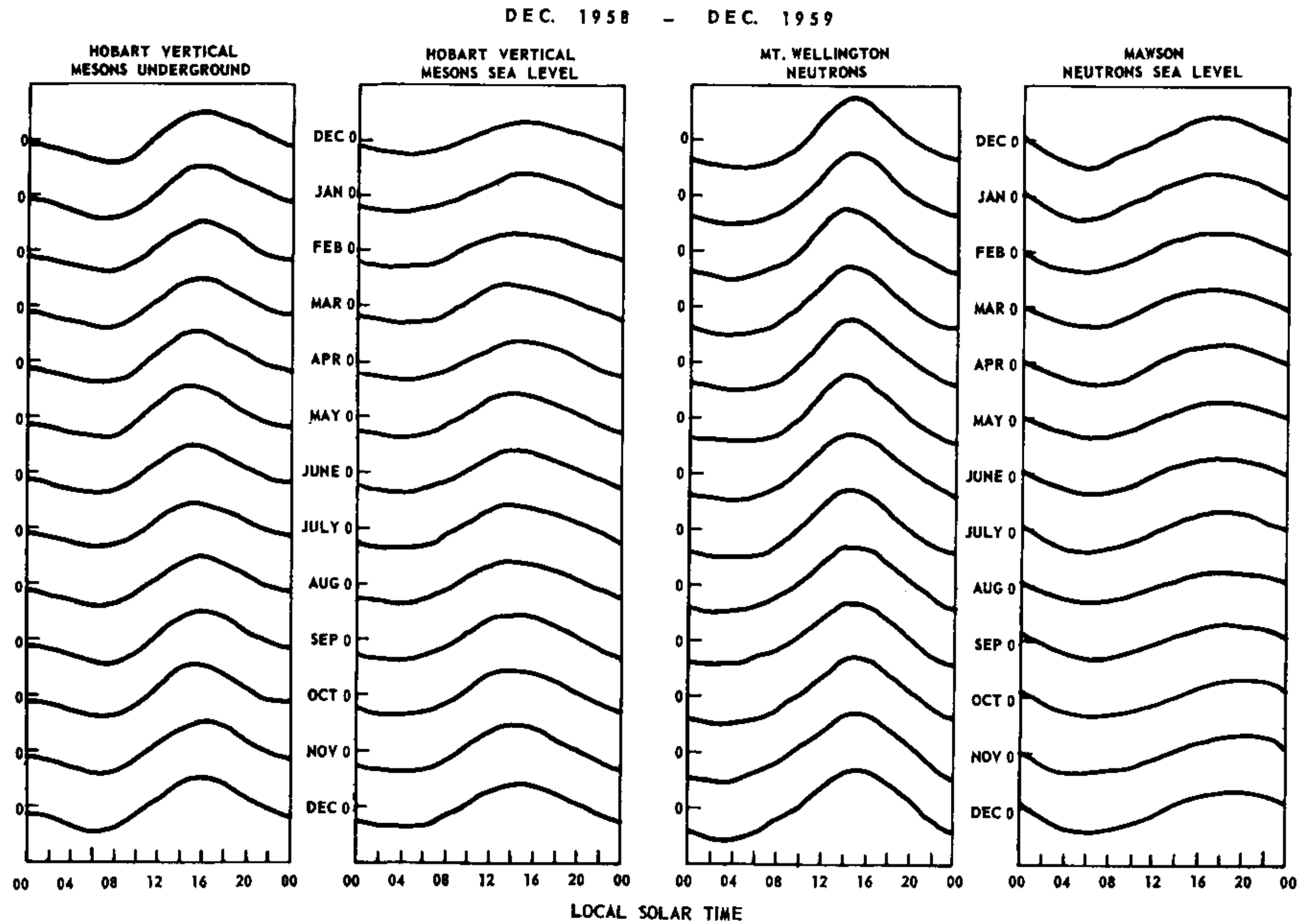



FIG. 3.2. The solar daily variation. Annual running average curves for successive months, from the year ending December 1958 to the year ending December 1959 inclusively. The ordinate scale is 0·42% per division, except for Hobart underground mesons, where the scale is 0·21% per division.

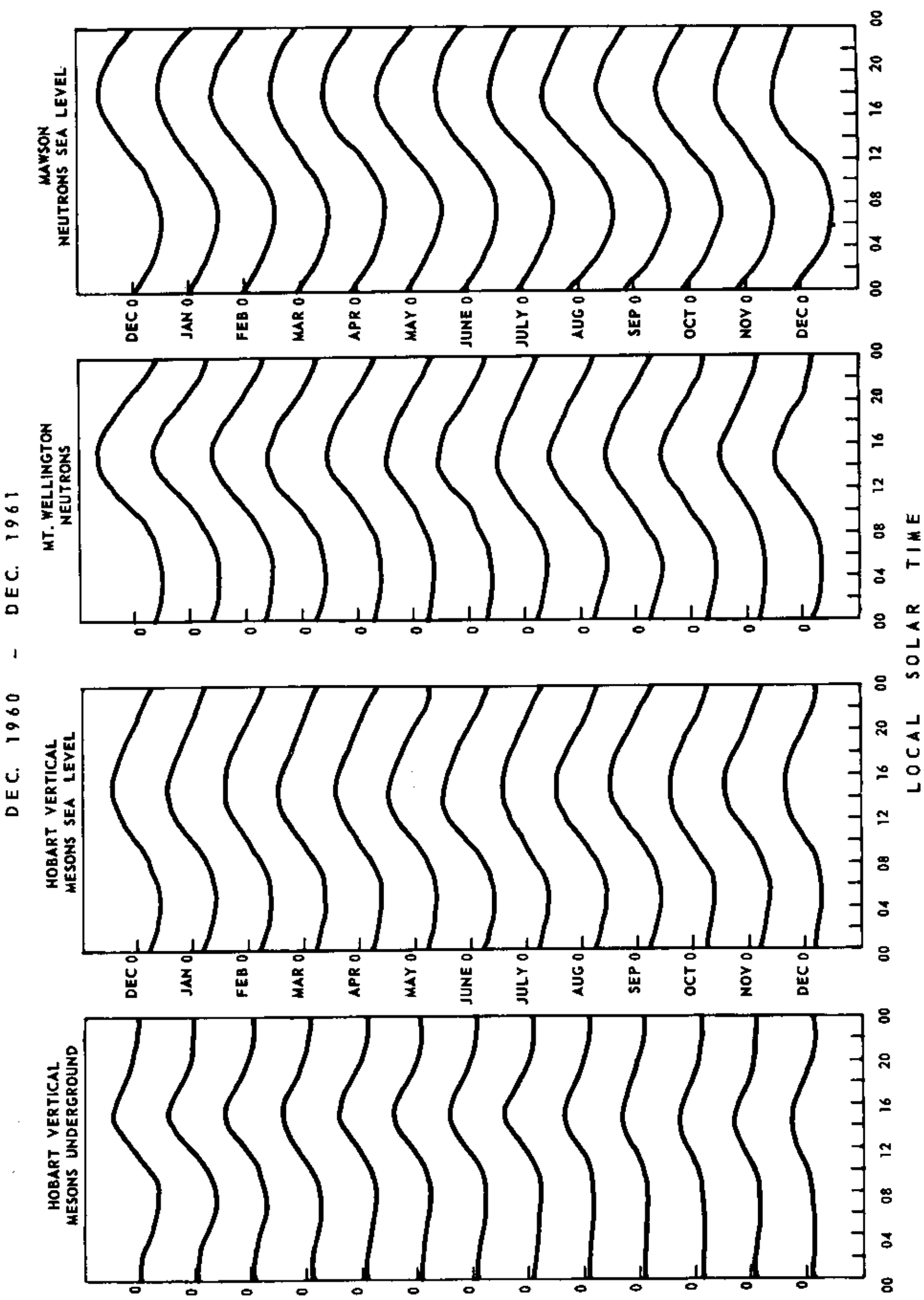



FIG. 3.3. The solar daily variation. Annual running average curves for successive months, from the year ending December 1960 to the year ending December 1961 inclusively. The ordinate scale is 0·42% per division, except for Hobart underground mesons, where the scale is 0·21% per division.

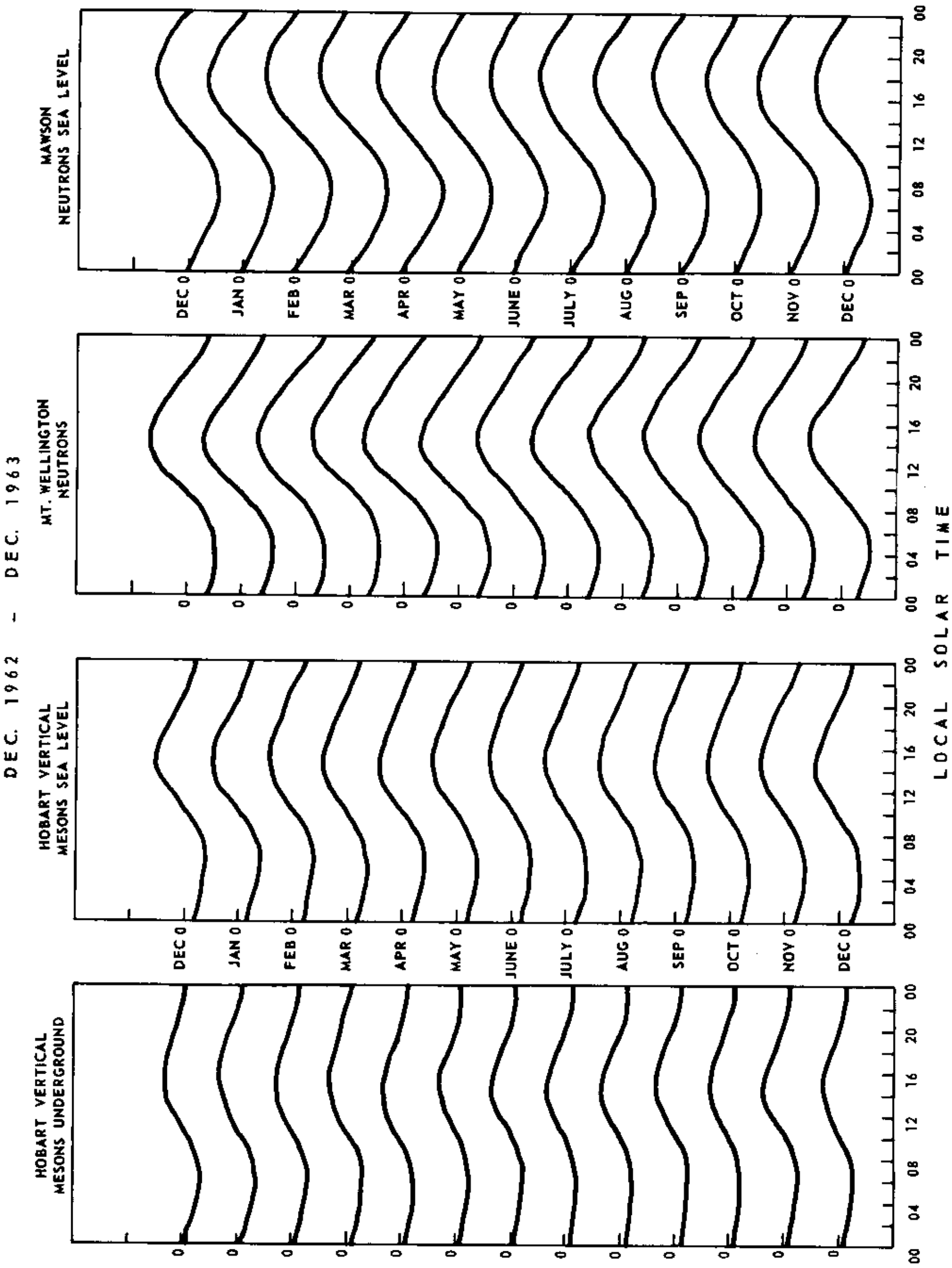



FIG. 3.4. The solar daily variation. Annual running average curves for successive months from the year ending December 1962 to the year ending December 1963 inclusively. The ordinate scale is 0·42% per division, except for Hobart underground mesons, where the scale is 0·21% per division.

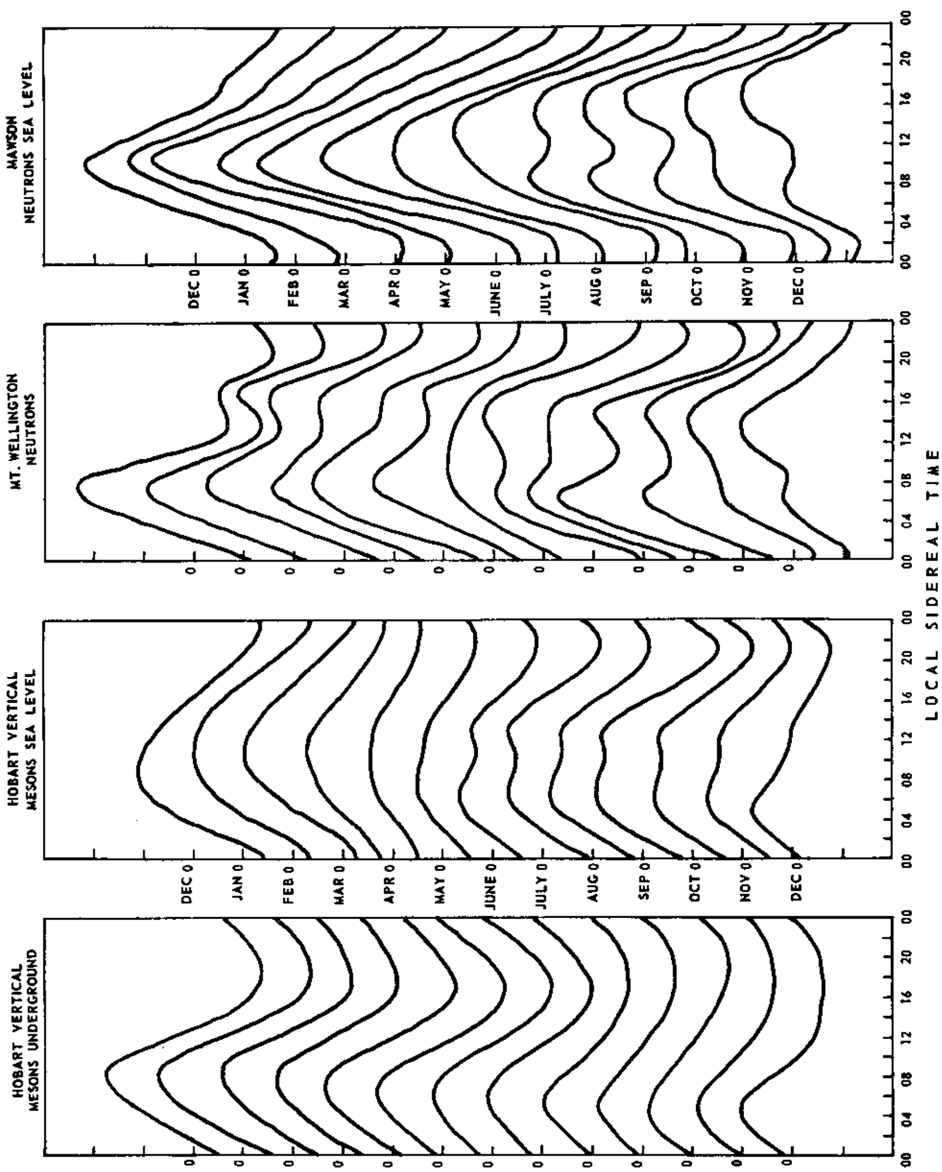



FIG. 3.5. The daily variation in sidereal time. Annual running average curves for successive months, from the year ending December 1958 to the year ending December 1959 inclusively. The ordinate scale is 0·42% per division.

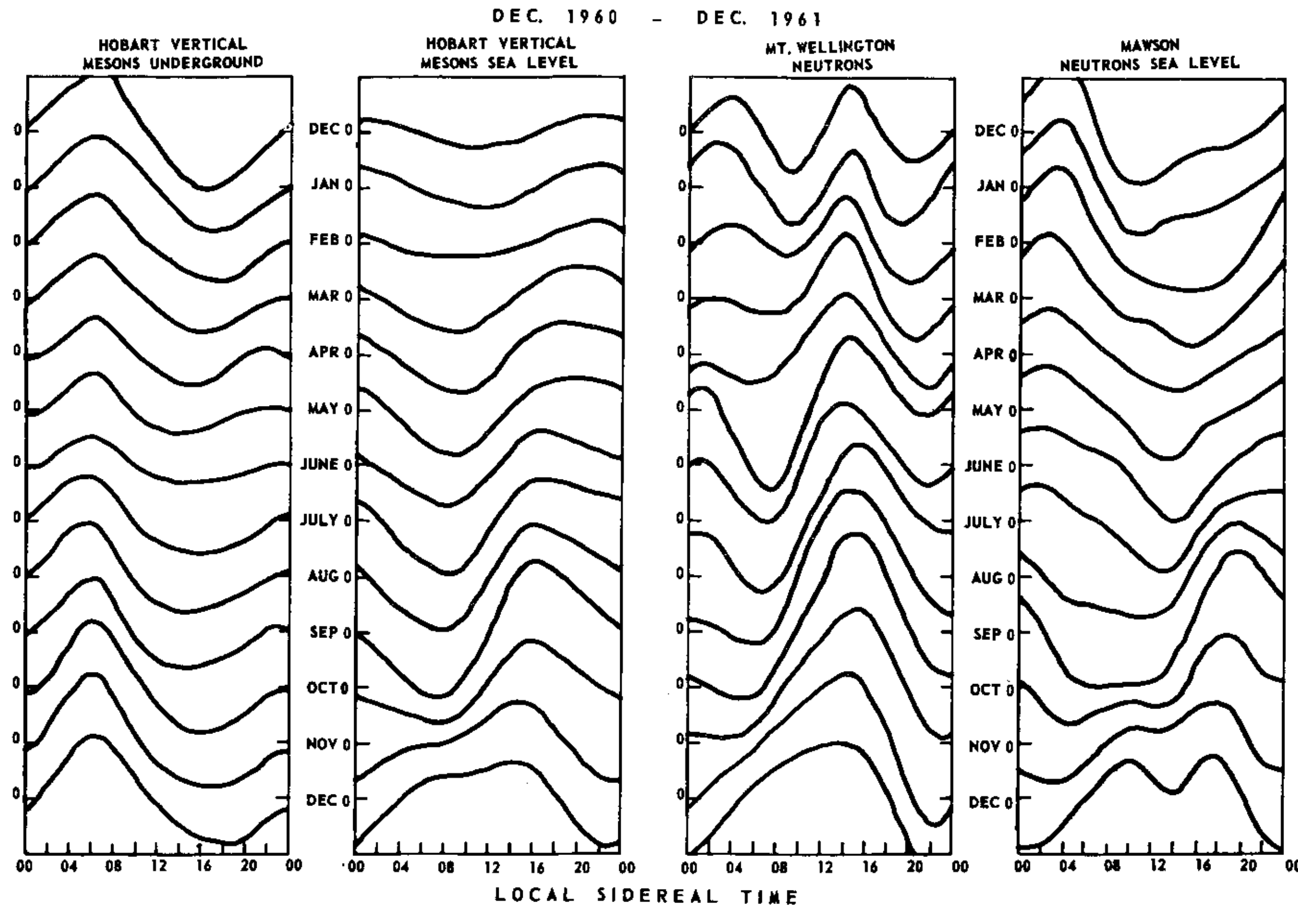



Fig. 3.6. The daily variation in sidereal time. Annual running average curves for successive months, from the year ending December 1960 to the year ending December 1961 inclusively. The ordinate scale is 0·042% per division.

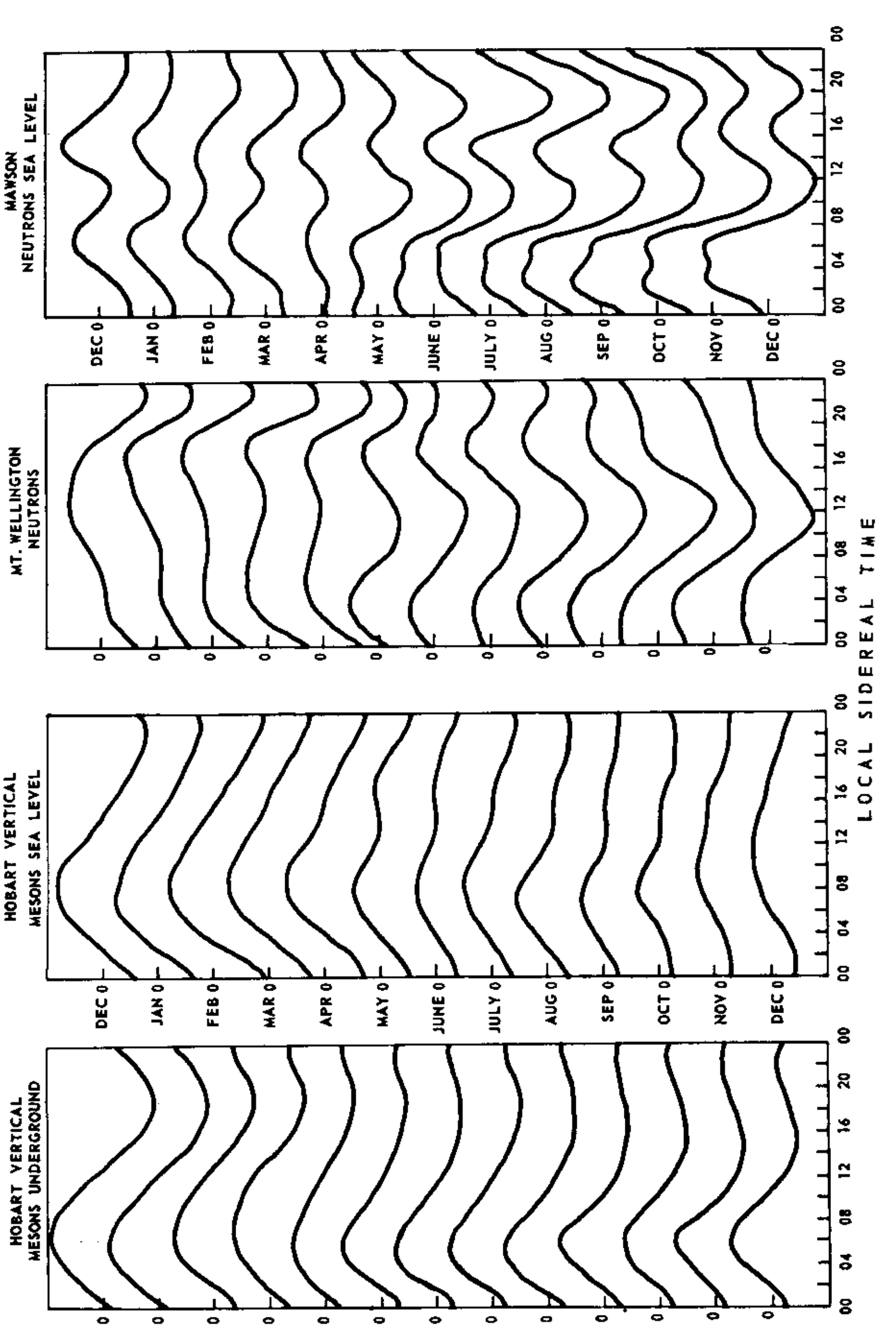



FIG. 3.7. The daily variation in sidereal time. Annual running average curves for successive months, from the year ending December 1962 to the year ending December 1963 inclusively. The ordinate scale is 0·042% per division.

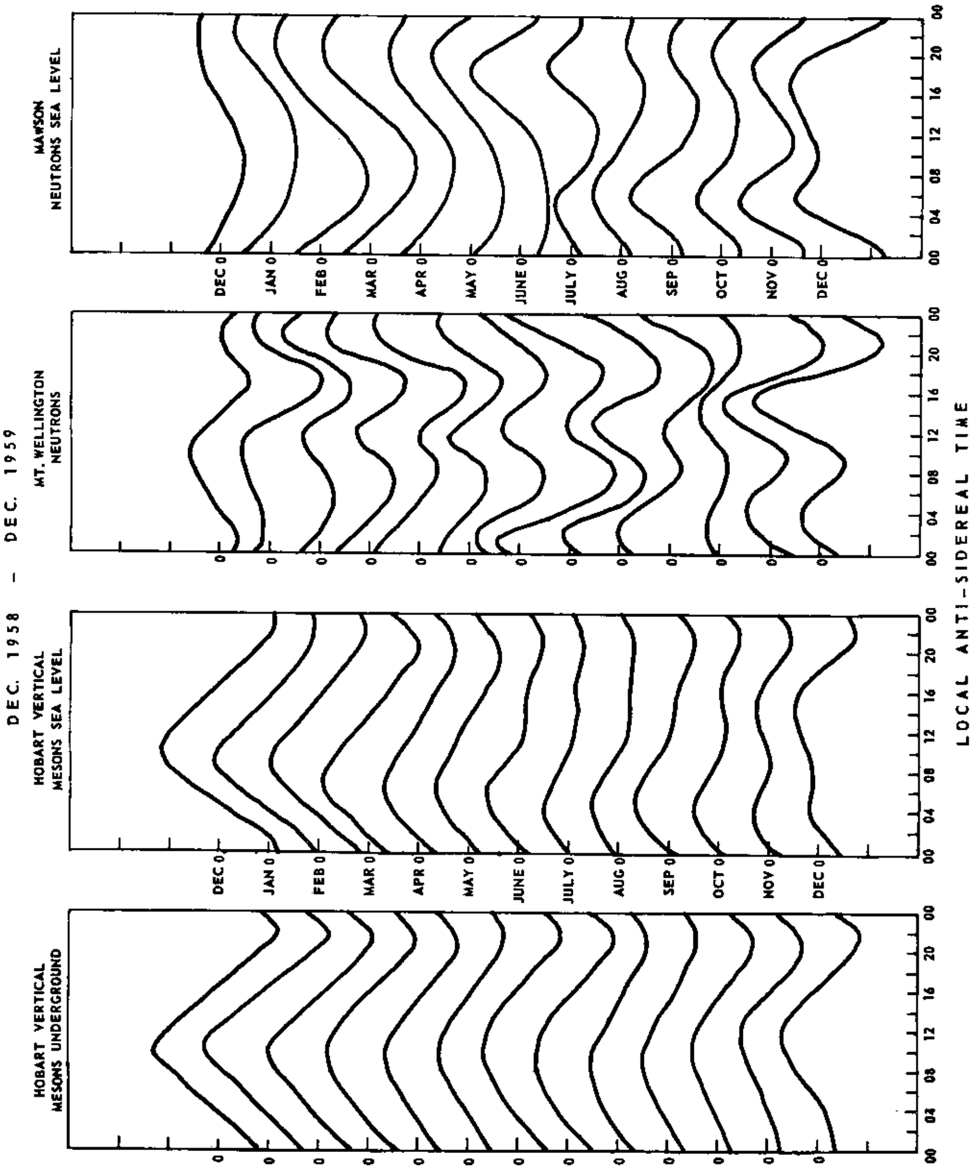



FIG. 3.8. The daily variation in anti-sidereal time. Annual running average curves for successive months, from the year ending December 1958 to the year ending December 1959 inclusively. The ordinate scale is 0·042% per division.

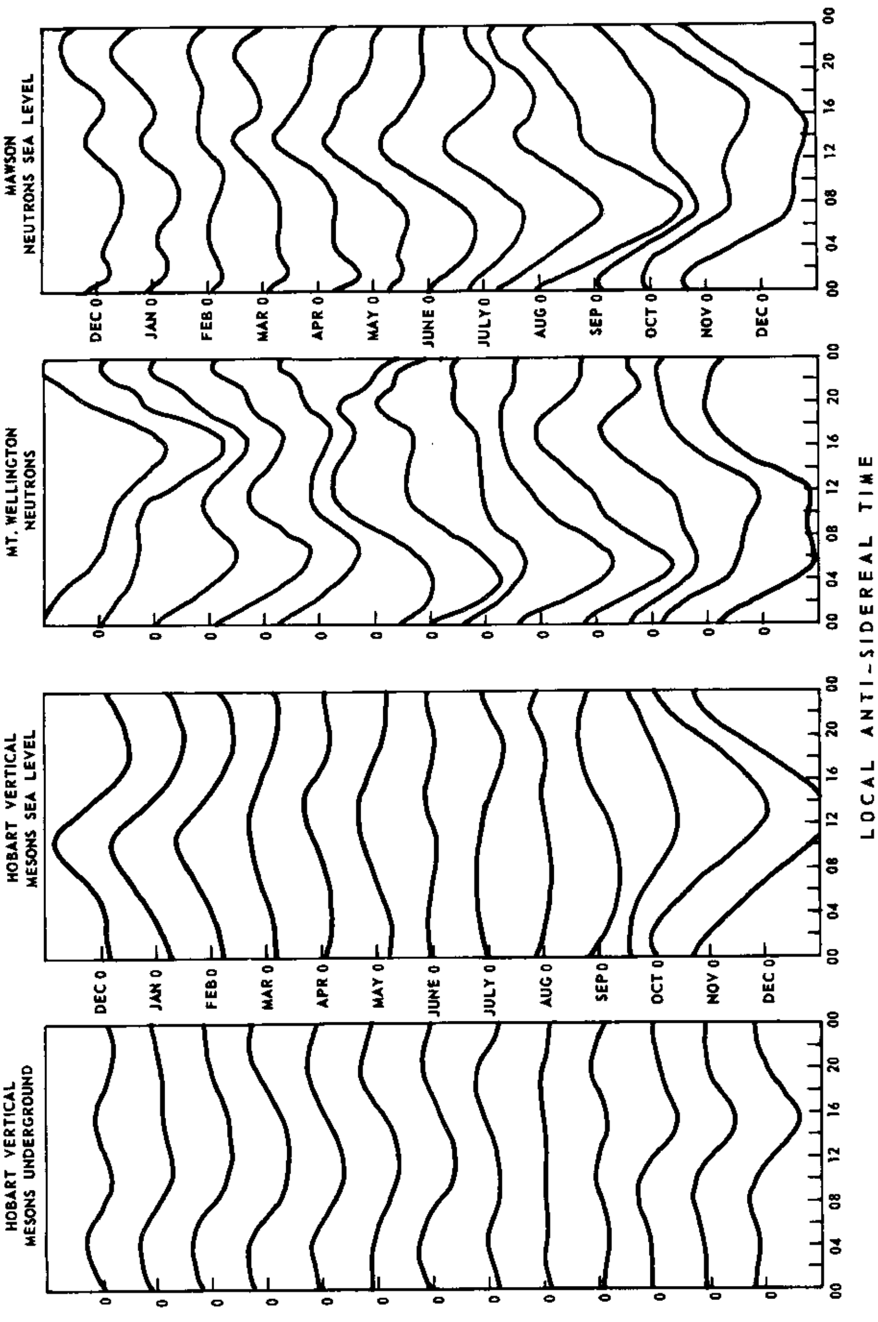



FIG. 3.9. The daily variation in anti-sidereal time. Annual running average curves for successive months, from the year ending December 1960 to the year ending December 1961 inclusively. The ordinate scale is 0·042% per division.

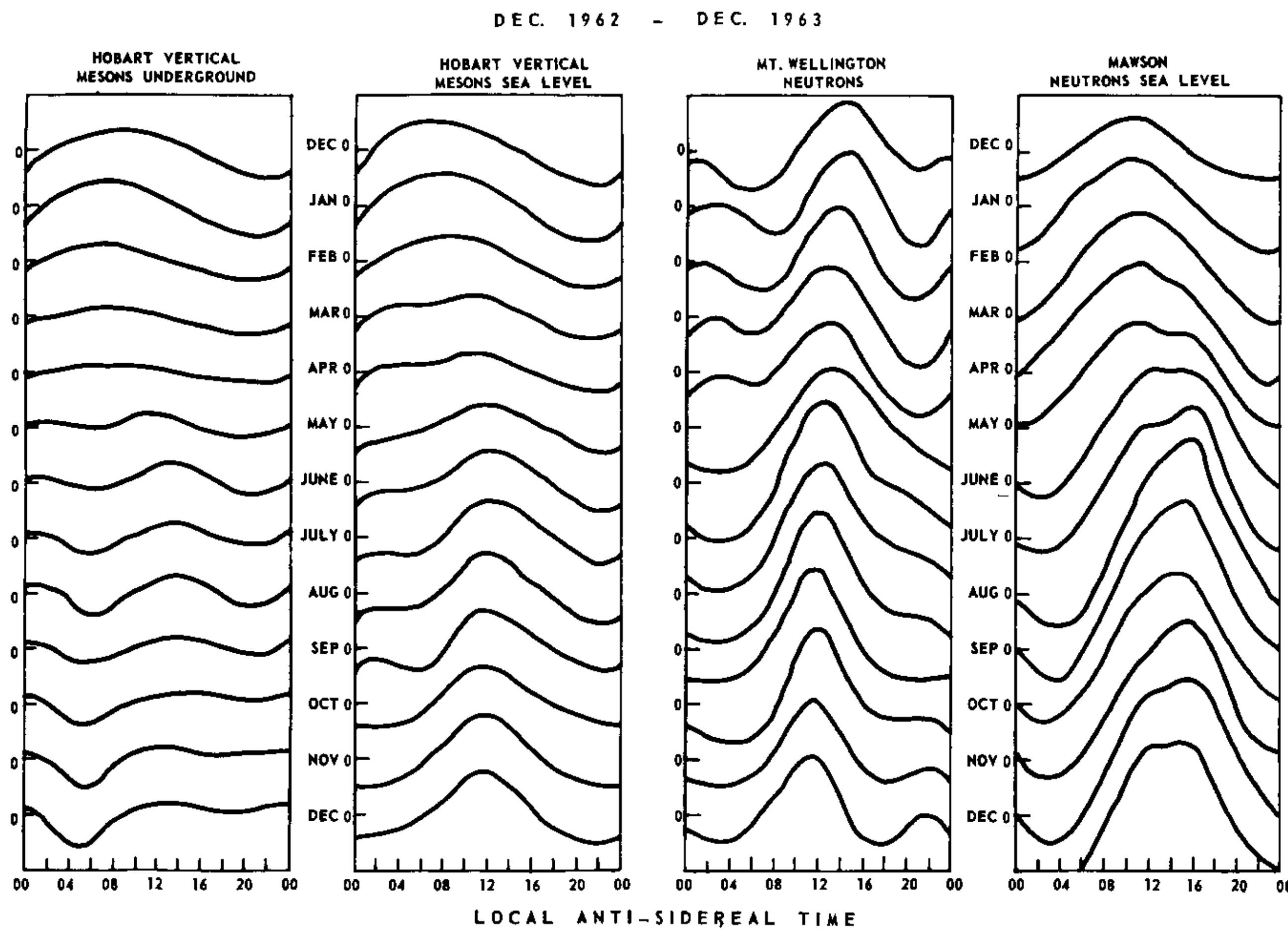



FIG. 3.10. The daily variation in anti-sidereal time. Annual running average curves for successive months, from the year ending December 1962 to the year ending December 1963 inclusively. The ordinate scale is 0·042% per division.

months, to give annual running averages. Sufficiently accurate curves of fit to the annual points are drawn by eye. In Section 5, annual running amplitudes and times of maximum obtained in this way from six-hour running average deviations are compared with values obtained from computed sums of first and second harmonics of best fit to the observed hourly deviates, and from the first harmonics of best fit. It should be explained that *annual running averages are used for qualitative comparisons only.* If they extend over a number of years they should serve the following purposes:

(a) They should distinguish between the type of daily variation in which a persistent component preponderates and the type which tends to be controlled by transient components, whether systematic or statistical.

(b) They should enable a genuine long-term trend in a parameter of the daily variation to be distinguished from an apparent trend that is influenced by transient effects of less than one year's duration. In this respect they possess an advantage over a set of independent annual averages.

The three figures for each type of daily variation relate to three independent biennial periods: January 1958 to December 1959, January 1960 to December 1961 and January 1962 to December 1963. Thus they span a period from solar maximum to within approximately 18 months of the following minimum.

*(1) The annual mean solar daily variations (Figures 3.2, 3.3 and 3.4).* Note that the solar daily variation is much smaller at 40 m.w.e. than above ground and that the ordinate scale has been adjusted to allow for this.

At all stations there is, as is to be expected, a slow variation with a single maximum. The time of maximum is approximately the same at each of the three Hobart stations and is approximately three hours in advance of that at Mawson. The phase difference between the solar daily variations of neutron intensity at Mawson and Mt. Wellington has been explained as a longitudinal difference between the mean directions of the respective asymptotic cones of response to the solar anisotropy (Rao, McCracken *and* Venkatesan 1963).

There have been no significant trends in the times of maximum over the period or any noticeable shorter-term secular changes. The figures do not allow amplitude changes to be easily detected, but on examination one important long-term trend should be apparent: the amplitude of the solar daily variation underground decreased by a factor of approximately two between the year ending December 1958 and the year ending December 1961, but without any noticeable displacement of phase. Thus there is no immediate indication of interplay between an atmospheric component and an extra-terrestrial solar component.

*(2) The annual apparent sidereal daily variations (Figures 3.5, 3.6, and 3.7).* The apparent sidereal daily variations at all stations are shown to the same scale. It should be appreciated that, in contrast with the underground observations, the amplitudes above ground are generally very much smaller than the amplitudes of the corresponding solar daily variations, so that the influence of spurious sidereal effects produced by seasonal or irregular fluctuations of the solar components should be greater accordingly, as seems to be the case.

We draw attention first to the situation in 1958. It is remarkable that the apparent sidereal effect was relatively so large at all the stations at solar maximum. The times of maximum were approximately the same, although that at Mt. Wellington was somewhat earlier than the others, and there is no evidence of the characteristic phase difference between the Mawson and Hobart observations that was seen in the solar daily variation. It would be surprising if a common mechanism did not link the observations at the four stations, and yet it is clear that there would be difficulties in attributing the result to seasonal modulation of a solar daily variation of atmospheric temperature.

The phase of the sidereal effect underground advanced somewhat during 1959, but thereafter remained fairly constant. There is also evidence of amplitude changes and some changes in the balance of harmonics, but overall it can be seen that there is a characteristic annual daily variation, reproducible from year to year and not subject to large transient fluctuations.

Above ground the effect in the meson intensity is variable and it can be seen that a complete reversal of phase may occur in a short time. Perhaps a persistent component might be detectable if the data were averaged over a number of years, but it is evident that the observations for a single year or even two years would not be sufficient.

The effect in the neutron intensity is extremely variable and after 1959 it is not obvious that there is any connection between the Mt. Wellington and Mawson results. It would seem that some of the variability must have been due to irregular fluctuations in the solar daily variation.

*(3) The annual mean daily variations in anti-sidereal time (Figures 3.8, 3.9 and 3.10).* The effects in anti-sidereal and sidereal time are all shown to the same scale.

Little need be said about the anti-sidereal effect in the neutron intensity. It is much too variable and complex to be of use in a conventional anti-sidereal analysis.

There is noticeable similarity between the relatively large anti-sidereal effects in the meson intensity above and below ground in 1958. However, the amplitudes decreased considerably during 1959 and the effect became too small and variable. In later years the amplitude of the anti-sidereal effect at the underground station did not exceed 0·02% and was statistically insignificant. It would seem, then, that modulation of components of the daily variation underground became unimportant after 1960, although very significant annual variations in solar and sidereal time were observed.

# 4. THE UNDERGROUND LABORATORY AT HOBART

## 4.1. DESCRIPTION OF THE UNDERGROUND SITE

### *(a) Location*

The underground laboratory is situated six miles north-east of Hobart in a disused railway tunnel, which passes approximately 60 feet beneath the Tunnel Hill saddle, 400 feet above sea level. The geographic position is 42° 51′ S, 147° 25′ E.

### *(b) The Tasman Highway and other roads*

Figure 4.1 is reproduced from a contour map of the tunnel area, supplied by the Department of Lands and Surveys. It shows a single-lane highway (the Tasman Highway) passing over the centre of the tunnel. The road to Mt. Rumney (1240 feet, $1\frac{1}{2}$ miles due east) branches off to the east at the top of the saddle. An access road to the farmhouse on Wiggins Hill branches off to the west and crosses above the tunnel immediately south of the cosmic ray building at O. The other road gives access from the highway through a farmyard to the northern entrance of the tunnel.

### *(c) The tunnel*

The tunnel is straight, 530 feet long and, looking north, is orientated in the direction 352° 27′ true. In cross-section it is 12 feet wide between the sandstone block walls at ground level, opening out to a maximum width of 14 feet at a height of 7 feet. Above this is a semi-circular roof of concrete, 18 inches thick. The height from floor to the centre of the ceiling is 14 feet 6 inches.

For the first 230 feet from the northern entrance the dirt floor is level and dry, an open concrete drain running flush against the western wall for approximately this distance.

The northern entrance is completely framed by a massive iron grille with a central 4 ft x 6 ft 5 in. double-barred door. An identical structure secures the other end of the tunnel, 100 feet from the southern entrance.

### *(d) The laboratory*

The equipment is housed in an oblong wooden framed hut of internal dimensions 12 ft x 28 ft x 7 ft high. The unlined walls are of hardboard and the ceiling is of caneite. The building is roofed with floorboards under a layer of malthoid. The northern wall is 95 feet from the concrete sill at the tunnel entrance. The hut is serviced electrically by the 240-volt A.C. mains supply.

## 4.2. DESCRIPTION OF THE EQUIPMENT

### *(a) The G-M counter telescopes*

There are four telescopes in operation at present, each comprising three trays of G-M counters in triple coincidence. A single tray contains 24 Maze-type external

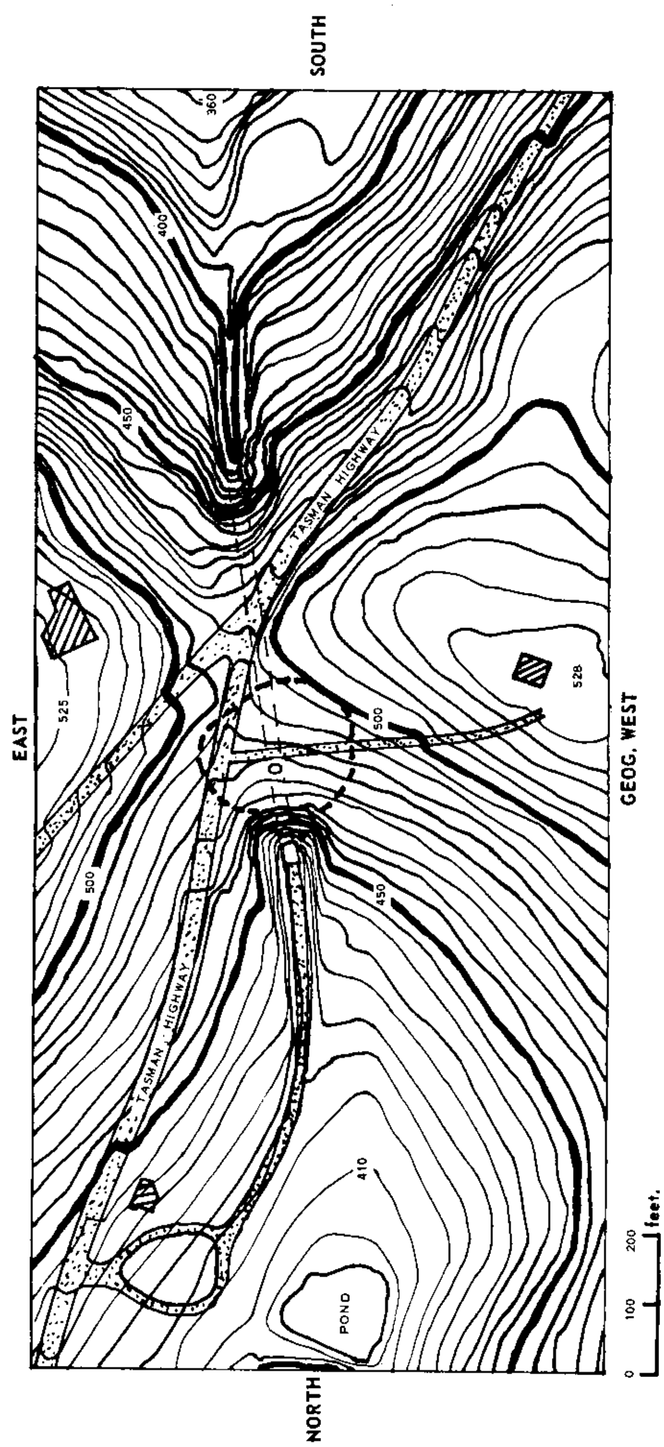



FIG. 4.1. Diagram of the Tunnel Hill saddle near Hobart, showing contour levels at intervals of five feet. The numbers give heights in feet above sea level. The cosmic ray building is situated at O inside the tunnel, which is indicated by dashed lines. The dashed circle delineates the boundary of the cone of acceptance of one of the semi-cubical telescopes at intersection with the land surface.

cathode glass counters, each of effective length 1 metre and approximately 4 cm in diameter, making up a total sensitive area of 1 sq metre.

The two vertical semi-cubical telescopes with their electronic circuitry are essentially of the same design and construction as the standard cubical telescopes operating in the cosmic ray laboratories at Hobart and Mawson. Since the latter have been fully described in *ANARE Interim Report No. 17*, entitled *The Design and Operation of ANARE Cosmic Ray Recorder "C"* (N. R. Parsons 1957), the details of construction and circuitry will not be reproduced here. The telescope trays are horizontal and are mounted above one another, care being taken to line up the ends of the counters so that the sensitive areas of the trays are correctly aligned vertically. The distance between top and bottom trays is 50 cm. The middle tray is positioned close to the top tray to allow room for a lead mount and space for 10 cm of lead. After some initial tests the lead was removed and in all of the experiments reported here the telescopes have operated without any lead absorber.

The constructional features of the two inclined telescopes will be described in the sections relating to the inclined experiments. They have the same circuitry as the vertical semi-cubical units, consisting essentially of three tray strips, a three-fold coincidence circuit and a scaler, as described by Parsons.

A photograph (Figure 4.2) taken from the northern end of the laboratory gives a general view of the four telescopes.

*(b) Voltage stabilization, variations of room temperature*

Until August 1958 an electronic-type A.C. mains stabilizer (manufactured by Stabilac Pty. Ltd., Sydney) protected the semi-cubical telescope circuits against voltage fluctuations of the mains supply. From August 1958 to July 1960 a stepwise-type voltage-regulating B.A.T. transformer placed in series with the Stabilac brought about improved stability. However, because of the risk of losing data if the Stabilac circuit failed, a more rugged type of mains stabilizer was sought. In August 1960 the Stabilac was replaced by a saturated core voltage-regulating transformer, which has proved to be efficient and trouble-free.

The mean temperature in the laboratory is approximately 55°F with maximum excursions of ± 10° over a year. The daily variation of temperature is not known, but would be expected to be small.

*(c) The recording system*

The main recording system is photographic, in conjunction with a chart trace. The scaled counts from each telescope are accumulated on three mechanical registers in series. Two of the registers are photographed and the records from them are compared from time to time to check against any losses of efficiency in the registers. The third register is a modified type, actuating a voltage pulse at the instant of every tenth scaled count. This pulse produces a deflection of one of the pens on the chart of a mechanically driven "Evershed" operations recorder. Six chart-recorder channels are used, four of them recording scale-of-ten pips from the four telescopes. The two flanking channels record hourly time pips from a

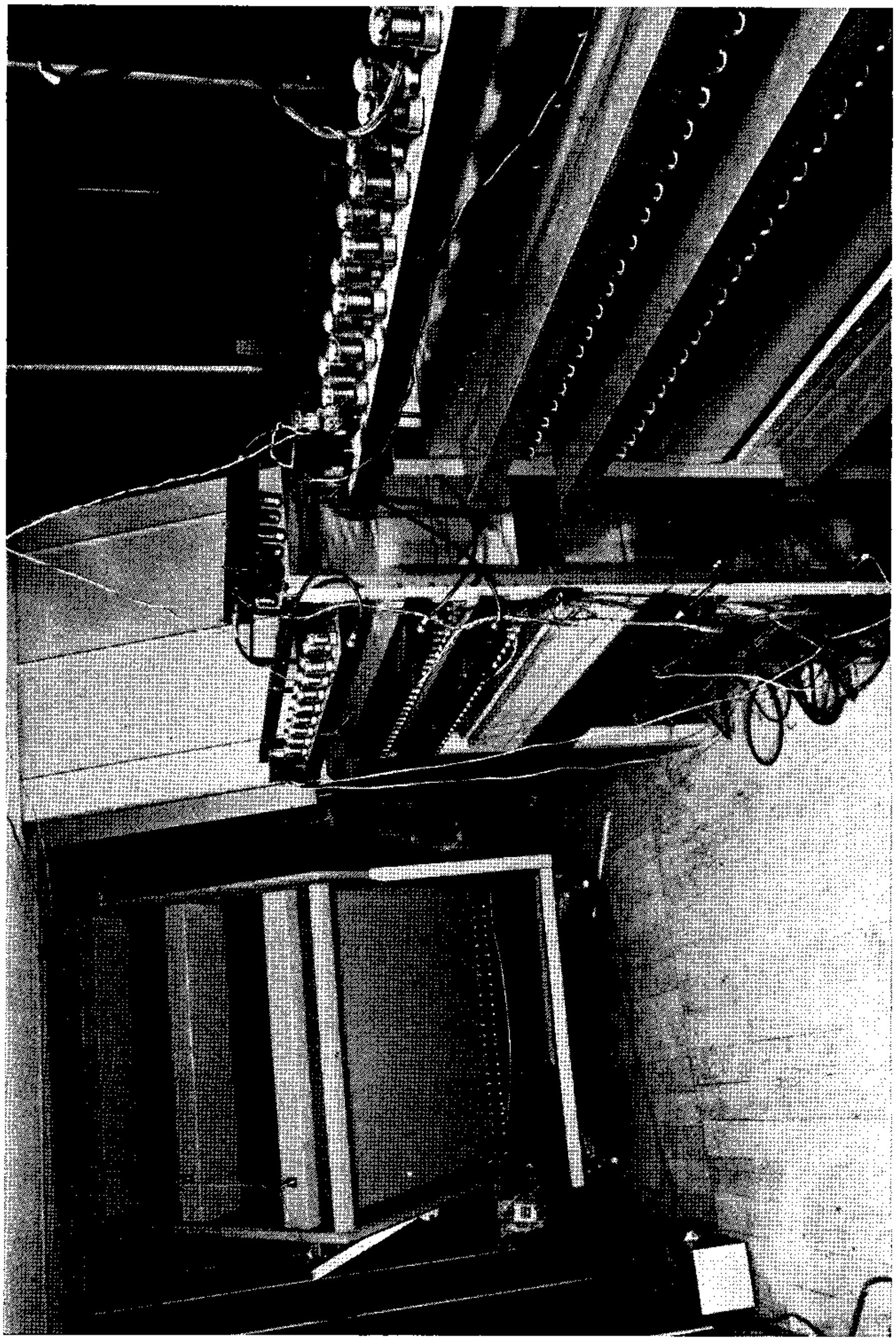


FIG. 4.2. The underground laboratory at Tunnel Hill.

self-winding chronometer (manufactured by Thomas Mercer and Sons, St. Albans, England).

At the end of each GMT hour the chronometer contacts actuate the camera drive, shutter mechanism and the lights in the photographic recording panel. Thereby, hourly photographs are taken of the four pairs of registers and of an electric clock, which enable the approximate time-sequence of frames to be monitored.

The photographic unit and the electro-mechanical system for winding the chronometer operate from a 12-volt accumulator and the chart recorder solenoids are in parallel with a 50-volt A.C.-D.C. supply.

At weekly intervals the film and the chart have to be replaced and the Evershed recorder rewound. Consequently, general maintenance of the equipment is carried out on a weekly basis.

*(d) Timing*

Before the Mercer chronometer was installed in July 1960, timing was effected by a sychronous one-hour Venner motor. Power breaks excepted, timing would have been accurate to within approximately 5 seconds. However, because of the possibility that average daily variations of timing might occur (amplitudes of the order of 1 second could not be tolerated) the system was inherently unsatisfactory. The battery-wound Mercer chronometer operates virtually independently of interruptions to the mains supply. It has performed with great reliablity and for a long period has maintained a steady rate of gain approvimately 2·5 seconds per day. The time is checked periodically against the National Bureau of Standards (U.S.A.) time signal WWV or the time signals transmitted by the Australian Broadcasting Commission.

A small, regular, 2-hour periodicity in timing has been found since the paragraph above was written. It is described in Section 4.11 (b).

4.3. THE MATERIAL ABSORBER

The results of a survey of the Tunnel Hill region made by the Geology Department of the University show that the rock structure of the tunnel area is a uniform Triassic siltstone of dry density 2·41 ± 0·04 gm/cc. The nearest dolerite is approximately 1000 feet to the east. It is assumed, therefore, that in all directions above the tunnel the material is essentially siltstone of density 2·41 gm/cc, plus water that might be held by the rock and the layer (two to four feet) of soil above it.

A map showing contour levels at intervals of five feet became available in 1965. This has allowed the mean absorber within the viewing cone of any one of the telescopes to be calculated rather more accurately than had been possible earlier. It has also become possible to assess, with reasonable accuracy, other effects connected with the distribution of the absorber in azimuth and zenith angle, and the influence of traffic and the parking of vehicles within the viewing cones of the telescopes. Up to the present these more accurate calculations relate only to the vertical semi-cubical telescopes.

The profiles of the absorber in various azimuth planes are shown in Figures 4.3 and 4.4. The viewing cone of the vertical equipment is delineated in cross-section by 5° bearings in zenith angle. Note where the boundary of the cone intersects the land surface at the zenith angle $Z = 60°$. Seen from above, the points of intersection form a rough circle, shown in Figure 4.1. This circle is the approximate boundary of the area of the surface penetrated by the particles which are detected by the vertical telescopes.

### 4.4. THE LOCAL CONE OF ACCEPTANCE OF THE VERTICAL TELESCOPES

The calculations of the sensitivity of a vertical semi-cubical telescope to the incoming secondary radiation, as a function of azimuth and zenith angle of arrival, derives from a theoretical treatment given by Parsons (1957) for telescopes at sea level. Parsons defines the sensitivity of a telescope to the radiation from a given direction as the product of the cross-sectional area presented by the telescope in that direction and the intensity of radiation arriving from that direction (particles per unit area, per unit solid angle, per unit time), and he calculates it in two stages. First, the influence of the cross-sectional area presented in a given direction is determined, on the assumption that the radiation above the top tray is uniform in all directions. This gives the *geometric sensitivity (G.S.)*. Then, the geometric sensitivity is multiplied by a factor specifying the observed zenith angle dependence of intensity, to give the *radiation sensitivity (R.S.)*.

*(a) The geometric sensitivity (GS)*

He considers an incident parallel beam of radiation inclined at an angle $Z$ to the zenith (the direction of the telescope axis) from azimuth $a$. Then the effective cross-sectional area presented by the semi-cubical telescope normal to the beam is given by

$$A = \cos z \left(1 - \frac{\tan z}{2}|\cos a|\right) \left(1 - \frac{\tan z}{2}|\sin a|\right) \qquad (4.1)$$

He shows that the relative geometric sensitivity of the telescope to radiation from directions within a volume element ($z$ to $z + dz$, $a$ to $a + da$) is given by

$$GS_{z,\,a}\, dzda = A \sin z\, dzda \qquad (4.2)$$

That is to say, *the geometric sensitivity per square degree,* $GS_{z,\,a}$, is given by

$$GS_{z,\,a} = A \sin z = \sin z \cos z \left(1 - \frac{\tan z}{2}|\cos a|\right) \left(1 - \frac{\tan z}{2}|\sin a|\right) \qquad (4.3)$$

The value of $GS_{z,\,a}$ for a given value of $Z$ varies somewhat with azimuth, the more so as $Z$ increases. The total sensitivity for a given $z$ is $GS_z = \int A \sin z\, da$, the integration being taken over all values of $a$ for which the cross-sectional area $A$ is positive. It may also be regarded as the average geometric sensitivity for the given $z$.

In later Sections the integrated responses to solar and sidereal anisotropies will be estimated. Whether $GS_{z,\,a}$ was used in the calculations or was replaced by the values $GS_z$, integrated over all azimuths and appropriately normalized, the final result was found to be essentially the same. The large azimuthal variations at the high zenith angles are outweighed by the reduced overall sensitivities at these angles. Consequently, it is sufficient for our purposes to consider the semi-cubical telescope as having circular cross-section and to replace $GS_{z,\,a}$ by $GS_z$.

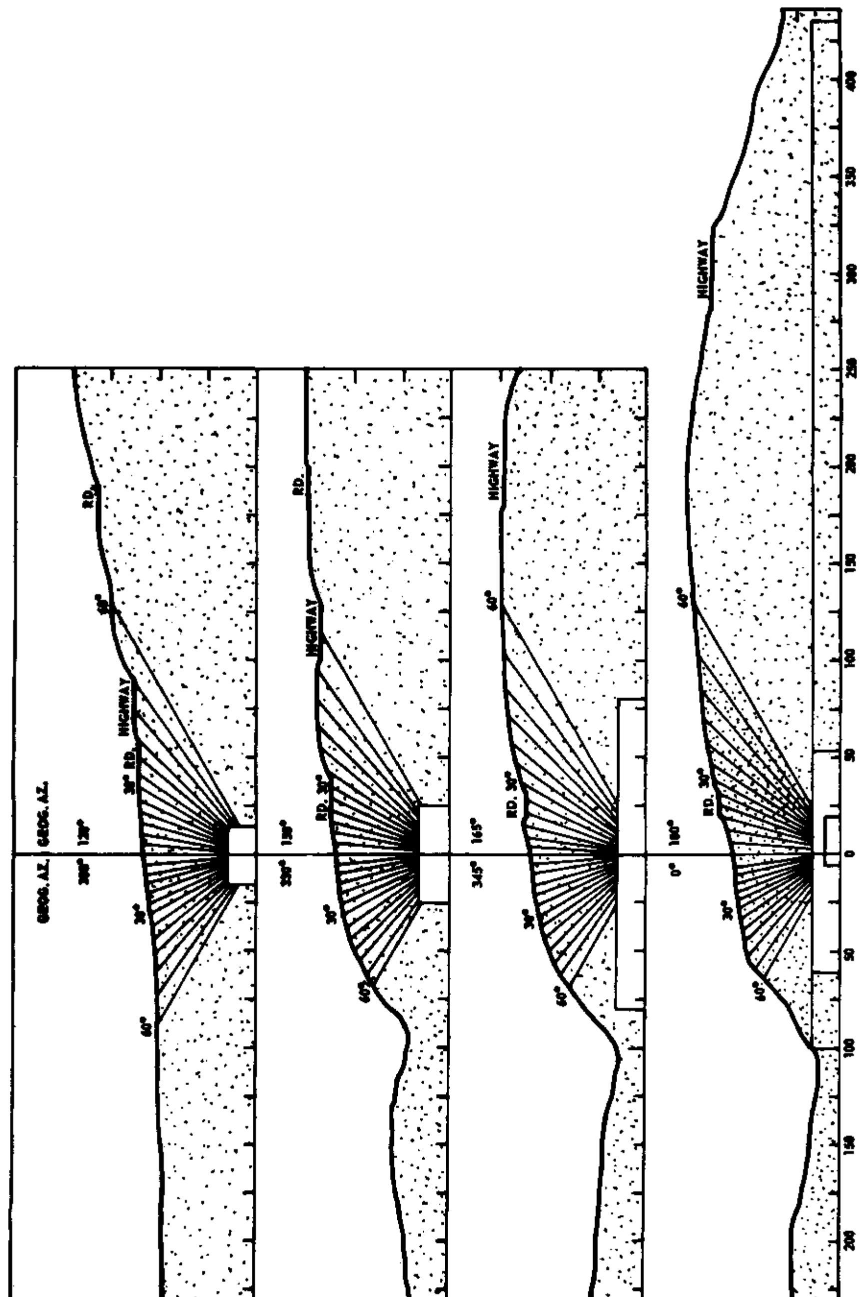



FIG. 4.3. Hobart underground station. Profiles of the material absorber in the azimuthal planes. 300° — 120°, 330° — 150°, 345° — 165° and 0° — 180°. The dimensions of the tunnel, and of the cosmic ray laboratory in elevation, are shown to scale in the bottom diagram. Cross-sections of the tunnel in the various planes are indicated by undotted areas. The bearings in zenith, at 5° intervals, delineate the local viewing cone of one of the vertical semi-cubical telescopes in relation to the absorber, a private access road, the Tasman highway and the road to Mt. Rumney.

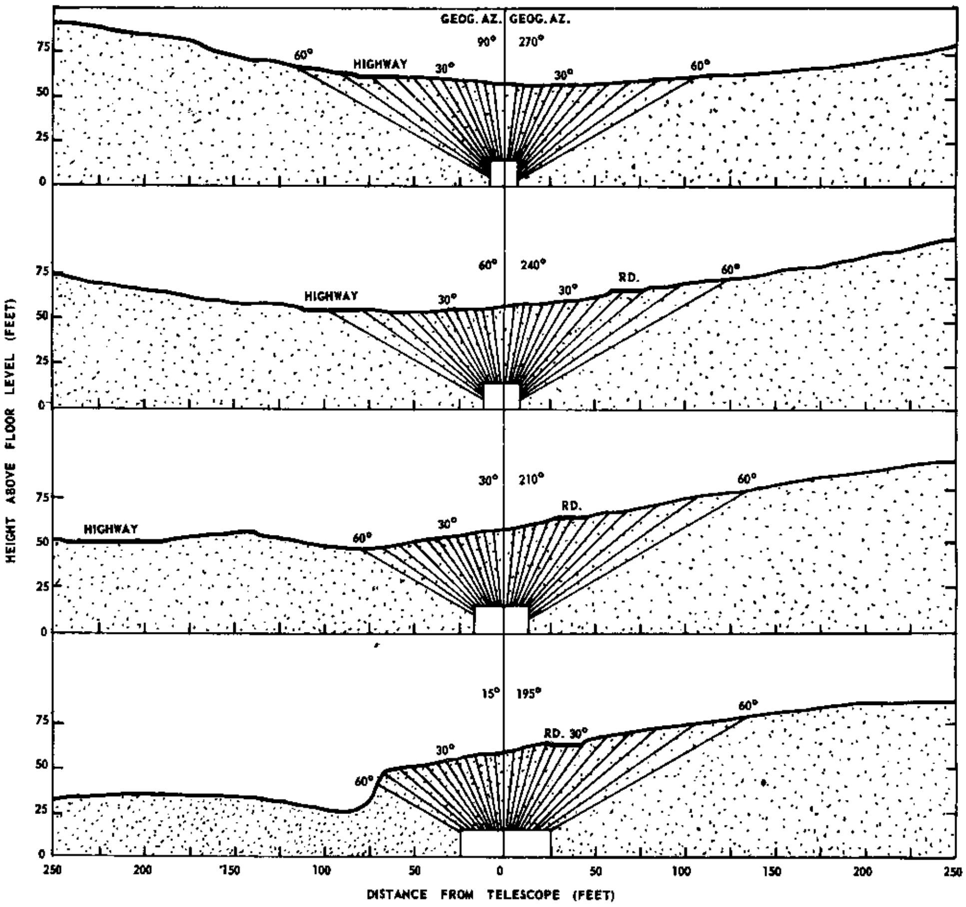



FIG. 4.4. Hobart underground station. Profiles of the material absorber in the azimuthal planes 90° — 270°, 60° — 240°, 30° — 210° and 15° — 195°.

*(b) The radiation sensitivity (RS)*

It is well known that the $\mu$-meson intensity at sea-level varies with zenith angle $z$ as $\cos^{\lambda}z$, so that

$$Iz = I_o \cos^{\lambda} z$$

where $I_o$ is the vertical intensity.

Parsons finds that the zenith angle dependence of intensity at sea-level is well approximated if $\lambda = 2{\cdot}2$, at least up to $z = 70°$.

Allowing for the zenith angle dependence of intensity, the total sensitivity of the telescope to the incoming radiation as a function of zenith angle is known as the *radiation sensitivity (RS)* and is given by

$$RS_z = GS_z \cos^{2\cdot 2} z \tag{4.5}$$

At moderate depths underground the zenith angle dependence of intensity appears to be of the same form as at sea level, although $\lambda$ seems to be closer to $2{\cdot}0$

than 2·2 (Cousins, Nash *and* Pointon 1957; Cousins *and* Nash 1962). The asymptotic cones of response (Section 6) were calculated for both these values of λ and the difference in the final response to anisotropies was found to be negligible.

It is concluded that, as far as present studies are concerned, the radiation sensitivity underground would be sufficiently well approximated by that calculated for sea-level telescopes, if we were to disregard directional inequalities in the material absorber.

*(c) The modified radiation sensitivity (RS′)*

In fact, as Figures 4.3 and 4.4 show, the amount of material absorber underground varies considerably with direction of viewing within the cone of acceptance of the telescope. It is therefore necessary to calculate a modified radiation sensitivity, $RS'$, of the form

$$RS'_{za} = \left(\frac{I}{I_o}\right)_{z,\,a} RS_z$$

where $\left(\frac{I}{I_o}\right)_{z,\,a}$ is the ratio of the intensity ($I$) in the direction ($z$, $a$) to the intensity ($I_o$) in the vertical direction, due only to the difference in amount of material absorber. The ratios for all the directions of interest have been estimated from the intensity-depth curve relating to the vertical direction, published by Thorndike (1952) and based on the measurements of Clay and others (Clay 1939). For example $\left(\frac{I}{I_o}\right)_{50,\,180}$ is calculated as follows. The depth of material absorber directly above the vertical equipment is 42 feet (Figures 4.3 and 4.4). We assume a mean density of 2·41 gm/cc as for dry siltstone The amount of absorber is therefore approximately 30·9 m.w.e. From the intensity-depth curve the relative intensity at a depth of 30·9 m.w.e. below ground is 0·165. This is $I_o$. At $z = 50°$, $a = 180°$, the depth of absorber is 88 feet (Figure 4.4) or 64·5 m.w.e. The relative intensity at this depth in the vertical direction is 0·054. This is $I$. Hence $\left(\frac{I}{I_o}\right)_{50,\,180}$ is 0·327.

Since the rate of decrease of intensity with depth below ground is somewhat less at a given depth in an inclined direction than in the vertical direction (e.g., see Clay, 1939, Figure 2) the use of the vertical intensity-depth curve tends to overestimate effects due to the directional inequalities of the material absorber.

*(d) Specification of the local cone of acceptance*

In the various applications that will follow, the solid angle of acceptance of a vertical semi-cube is divided up into 576 elements ($\omega_r$) of dimensions 7·5° in azimuth centred on 0°, 7·5°, 15°, . . . 352·5°) and 5° in zenith angle (centred on 5°, 10°, 15°, . . . 55°), and the radiation sensitivity $RS'$ is calculated for each of these elements. This provides an assembly of directional sensitivities that is used in the determination of the mean absorber and its directional effects and in the calculation of the asymptotic cones of acceptance (Section 6).

## 4.5. CHARACTERISTICS OF THE ABSORBER VIEWED BY THE VERTICAL TELESCOPES

### (a) *Directional inequalities*

1. The effect in zenith angle. The dashed curve in Figure 4.5 shows the zenith angle dependence of *RS*, summed over all azimuths, for a semi-cubical telescope at sea level, as calculated by Parsons. The material absorber above the underground telescopes reduces the sensitivity at the higher zenith angles. The full line shows the modified zenith angle dependence of *RS'*, summed over all azimuths, so that it takes into account the average inequalities of the absorber in zenith directions.

2. The effect in azimuth. If the values of $RS'_r$ at a given azimuth are averaged over all zenith angles within the cone of acceptance at that azimuth, the average response of the vertical equipment to inequalities of the absorber in azimuth may be plotted at intervals of 7·5°. The result is shown in Figure 4.6, where the relative intensity of response has been normalized to 100% at the azimuths where the absorber is a maximum. It can be seen that inequalities of the absorber, as they influence the counting rate, are rather symmetrically distributed about the N-S plane. In fact, it is found from the curve that 49·4% of the counting rate is due to particles arriving east of the meridian plane, 50·6% arriving from the west.

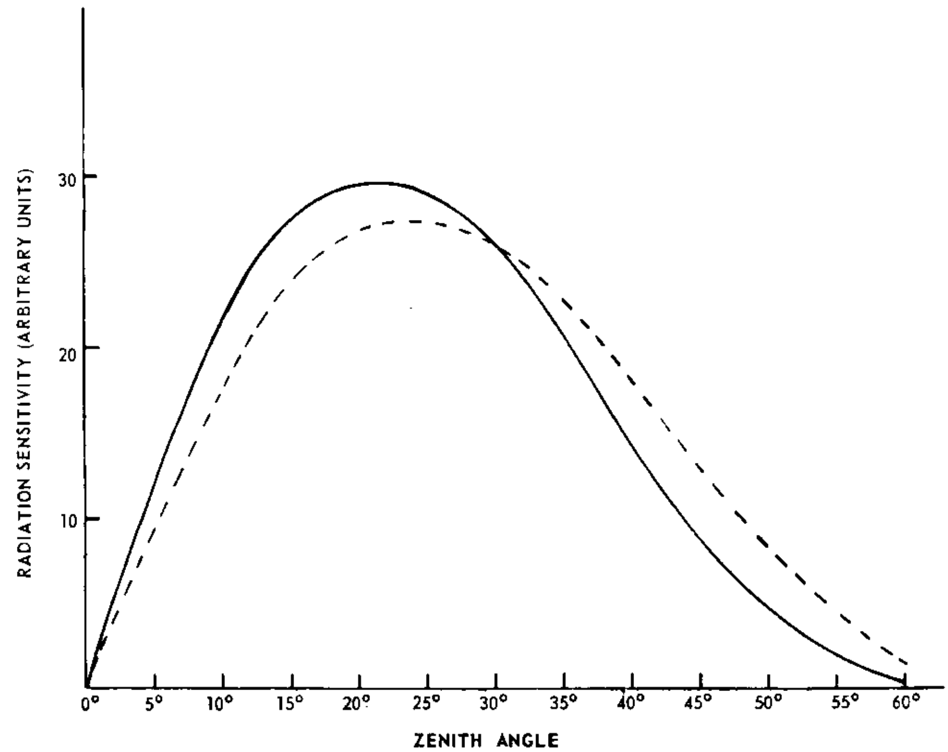



FIG. 4.5. Mean radiation sensitivity as a function of zenith angle of arrival for a vertical semi-cubical telescope. The dashed line gives the calculated effect at sea level (from Parsons). The full line gives the calculated effect for a semi-cube at the underground station at Hobart, taking into account the distribution of material absorber above the telescope.

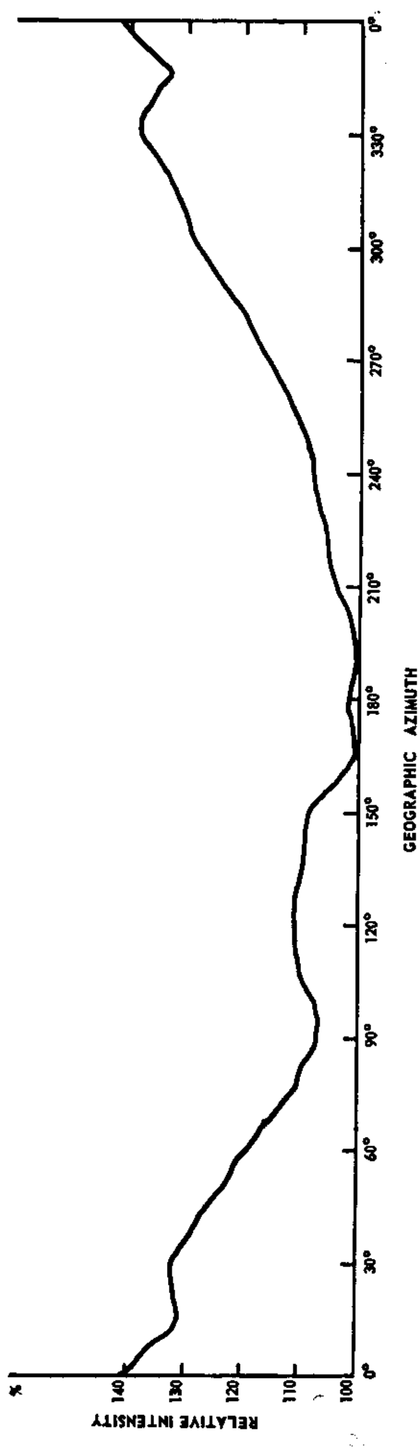



Fig. 4.6. Radiation sensitivity of a vertical semi-cubical telescope at the Hobart underground station. Azimuthal variation due to the distribution of material absorber. The differential intensities have been estimated relative to the minimum value, at 165° azimuth.

Consequently, no significant bias of the local cone of acceptance either east or west of the meridian accrues from the distribution of the absorber.

Figures 4.5 and 4.6 indicate that the distribution of the absorber must be taken rather carefully into account in studies of the observed daily variation underground. If not, the tilting and other distortions of the local cone of acceptance that may be produced by the absorber could lead to serious errors in the calculation of the response to an anisotropy. On the other hand, if the influence of the absorber is included in the cone of sensitivity, any directional bias which results is eventually incorporated in the asymptotic cone of acceptance (Section 6)).

A plaster model (Figure 4.7) demonstrates the characteristics of the local cone of sensitivity of the semi-cubical vertical telescope underground at Hobart. Ideally, the solid cone should meet the flat base at the point where the converging inside and outside surfaces join the axis of the telescope which is envisaged as emerging from the centre of the cone normal to the base. Zero sensitivity in the vertical (axial) direction (see also Figure 4.5) follows from the fact that the distribution of *sensitivity per square degree* is being represented here, not the sensitivity to parallel radiation. Per degree of azimuth, per degree of zenith angle, the area of surface of a volume element penetrated by the radiation becomes vanishingly small as the vertical direction is approached. On the other hand, at high zenith angles the effective cross-sectional area $A$ (equation 4.1) presented to the incoming radiation, becomes vanishingly small as the boundary of the telescope is approached. These competing influences cause maximum sensitivity to occur at a zenith angle of approximately 25°. The reduced amount of absorber in the northerly azimuths (Figures 4.3 and 4.5) is reflected in the distortion of the model.

In the treatment of asymptotic cones of acceptance (Section 6) the determination of the asymptotic constants is based on the distribution of $RS$, rather than $RS'$, since it is found that the constants would be very nearly the same in both cases. For instance, it will be seen in Table 6.1 that the asymptotic constants for the lowest rigidity ($R = 50\ GV$), at which the greatest discrepancies should occur, are ($A_R = 0{\cdot}87$, $\delta_{1R} = -34°$, $\phi_{1R} = 30°$). Based on the distribution of $RS'$, the constants are ($A_R = 0{\cdot}90$, $\delta_{1R} = -34°$, $\phi_{1R} = 29°$).

*(b) The amount of dry material absorber above the vertical telescopes*

If the radiation sensitivity appropriate to the volume element $\omega_r$ is $RS'_r$, and the amount of material absorber in that direction is $m_r$, then the average amount of absorber within the viewing cone of the telescope is

$$\bar{m} = \frac{\sum_r m_r RS'_r}{\sum_r RS_r} \qquad (4.6)$$

where the summations are taken over the 576 elements of the local acceptance cone. By this method, the weighted average amount of dry absorber above the semi-cubes up to ground level was calculated to be 35·9 m.w.e., or 48 feet of material.

Material above the rock, consisting of two to three feet of clay and fragmented rock and one foot of topsoil, has not been distinguished from the rock itself in the calculation, so that it would be necessary to subtract perhaps 0.3 m.w.e. to allow

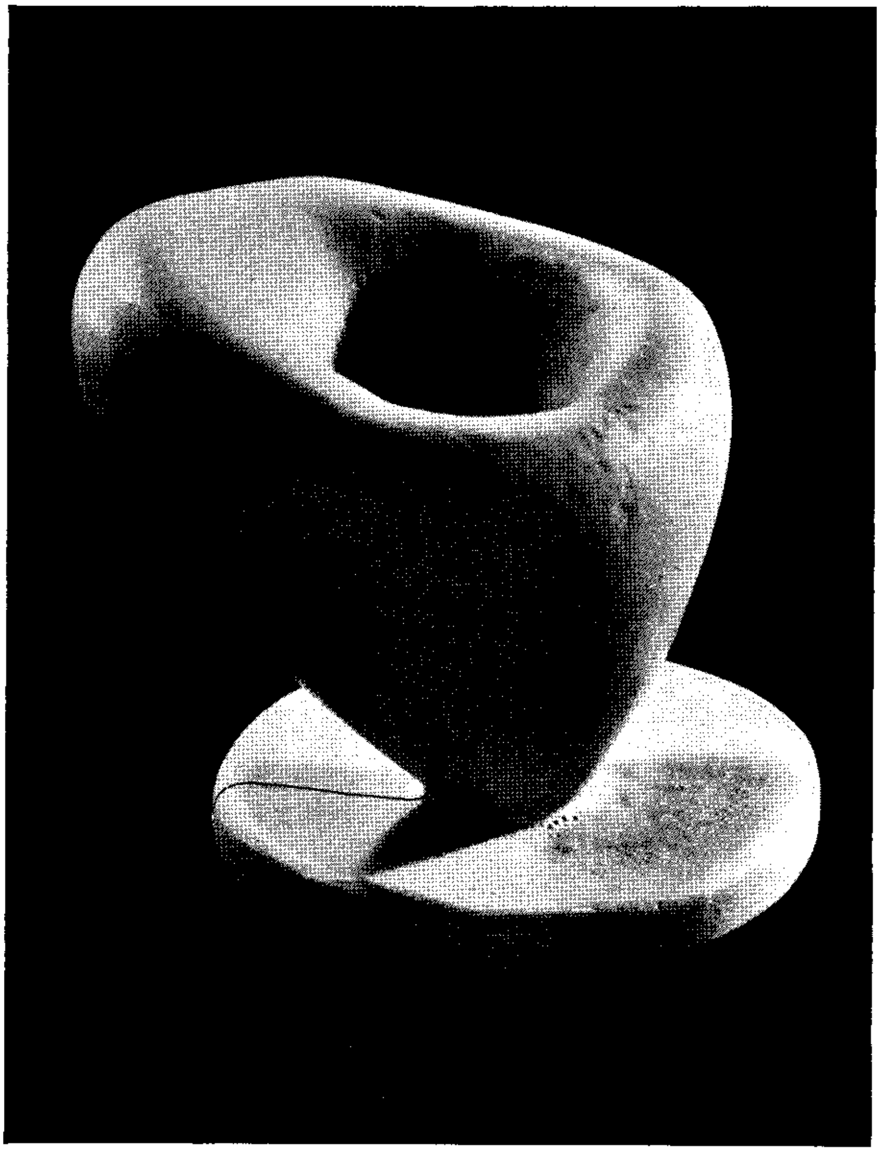


FIG. 4.7. Model of the local cone of sensitivity of a vertical semi-cubical telescope at the Hobart underground station.

for the difference in density. Therefore, the estimate of the amount of dry absorber above the telescopes is

$$\overline{m} = 35{\cdot}6 \text{ m.w.e.}$$

*(c) The average water content*

It is very difficult to estimate precisely the average amount of water held in the rock and in the soil above it. Only a rough guess will be offered here.

Murdock, Ogilvie *and* Rathgeber (1959) have estimated that, on the average, the water content of the sandstone above their underground laboratory in Sydney increased the density of the rock by 0·05 gm/cc, equivalent to 0·015 m.w.e./foot. It is known that the porosity of Triassic siltstone is low, very much less than that of sandstone. When we consider also that an annual average rainfall of 1·2 metres in Sydney compares with 0·6 at Hobart, it is concluded that the average depth of rock above the vertical telescopes, somewhat less than 49 feet, holds considerably less than 0·7 metres of water. It is suggested that half that figure would be an upper limit and would include moisture held in fissures in the rock.

Some indication of the average moisture content in the layer of crumbled rock, clay and topsoil above the siltstone may be gained from the following. As reported in Paper 3, phenomenal rainfalls (approximately 40% of the annual total), which succeeded a long period of dry weather, were experienced over a few days in April 1960 and appeared to be responsible for a decrease of approximately 0·8% in the counting rate underground. This was equivalent to an increase of approximately 0·25 metres of water in the material absorber. It is suggested that this roughly represented the transition from the dry to the saturated state of the material above the rock, so that on the average it would not be expected to hold more than about half of that amount of water.

It is concluded that the average water content of the material absorber above the equipment is not likely to be greater than approximately 0·5 m.w.e.

*(d) The total material absorber*

Adding the water content to the amount of dry absorber, the mean absorber above the vertical telescopes to ground level is now taken to be approximately

$$\overline{m} = 36 \text{ m.w.e.}$$

Successive estimates of the mean absorber have followed a downward trend. In the earlier papers (3 and 4), the figure given was 42 m.w.e. Overestimation of the depth of the material was the main source of error. Later papers (5, 6 and 7) specify 40 m.w.e. as the better approximation. This was deduced from an inter-comparison of counting rates above and below ground, using the intensity-depth relationship. Normalization for the different telescope geometries was perhaps the greatest factor of uncertainty here. Since most of the calculations described in this paper had been made before the contour map of the tunnel area became available, it will be found that the mean absorber is usually taken to be 40 m.w.e.

*(e) Road traffic and parking of vehicles above the equipment*

The Tasman Highway (Figures 4.1, 4.3 and 4.4) lies within the viewing cone of one of the vertical telescopes for a distance of approximately 170 feet, from $a = 60°$, $z = 60°$ and distance 105 feet from the telescope, to $a = 150°$, $z = 60°$ and distance 115 feet. The zenith angle decreases to 50° at $a = 90°$. Accordingly, the sensitivity to traffic will not be underestimated if the segment of highway is considered to lie in a quadrant of azimuth at a zenith angle of 50° from the telescope axis.

*Influence of the daily variation of the volume of traffic.* The Traffic Engineering Section of the Transport Commission has kindly made available the hourly records, from an automatic counter, of the volume of traffic that passed over Tunnel Hill from midnight, 6th November 1963, to midnight 15th November. The average flow was 113 vehicles/hour. The average daily variation is shown in Figure 4.8. The amplitude of the first harmonic is 73% of the mean, or 82 vehicles/hour. Allowing for heavy vehicles, the average speed is assumed to be 30 ft/sec and the average weight 5700 lbs (2600 Kg). In the calculations that follow, a vehicle is thought of as a sheet of lead 14 feet long, 6 feet wide and 3 cm high. At the peak of the first harmonic:

Time interval between excess vehicles = 44 sec.
Therefore average spacing between vehicles = 1320 feet, and
the number of excess vehicles within view of telescope = 0·13

Distributing the mass of 0·13 vehicles over 170 feet of highway, 6 feet wide, the equivalent lead thickness ($T$) = 0·0322 cm, or 0·354 cm water equivalent. Per quadrant of azimuth, the average radiation sensitivity ($RS'_z$) at $z = 50°$ is 0·13%/deg.z. The ribbon of lead subtends an angle of approximately 3·5° in zenith angle at the telescope. Consequently, 0·45% of the total particle flux ($I$) would pass through the area covered by the lead. From the intensity–depth relationship (Thorndike 1952),

$$\frac{dI}{dT} = -0{\cdot}0303\%/\text{cm water equivalent.}$$

Therefore the amplitude of the first harmonic of the solar daily variation due to the daily variation of the volume of traffic is

$$\left(\frac{\Delta I}{I}\right)_{max} = 0{\cdot}0303 \times 0{\cdot}354 \times 0{\cdot}0045\%$$
$$= 0{\cdot}049 \times 10^{-3}\%$$

and the time of maximum intensity is 0236 solar time. A similar calculation gives $0{\cdot}027 \times 10^{-3}\%$ as the amplitude of the second harmonic, the maxima occurring at 1324 and 0134 solar time.

Clearly, the calculated effect is too small by three orders of magnitude to be of any significance, since the amplitudes of the observed annual mean daily variations underground are of the order of 0·05%. It may be concluded that the influence of the daily variation of the volume of traffic is negligible, while conceding a wide margin of error in estimation of the various constants.

*Parking of vehicles.* The average motor vehicle parked on the side of the highway within view of the vertical telescopes subtends an angle of approximately 8° in azimuth and 3·5° in zenith angle at the equipment. Representing the vehicle as we have done, by a slab of lead, 3 cm thick, the absorber is 33 cm water equivalent. The radiation sensitivity per 8° of azimuth, per 3·5° of zenith angle at $z = 50°$, is $RS' = 0{\cdot}0416\%$. Consequently, the decrease of intensity produced by the vehicle is

$$\frac{\Delta I}{I} = (0{\cdot}0303 \times 33 \times 0{\cdot}00042)\%$$
$$= 0{\cdot}042 \times 10^{-2}\%.$$

If a single vehicle is parked near the highway for eight consecutive hours of the

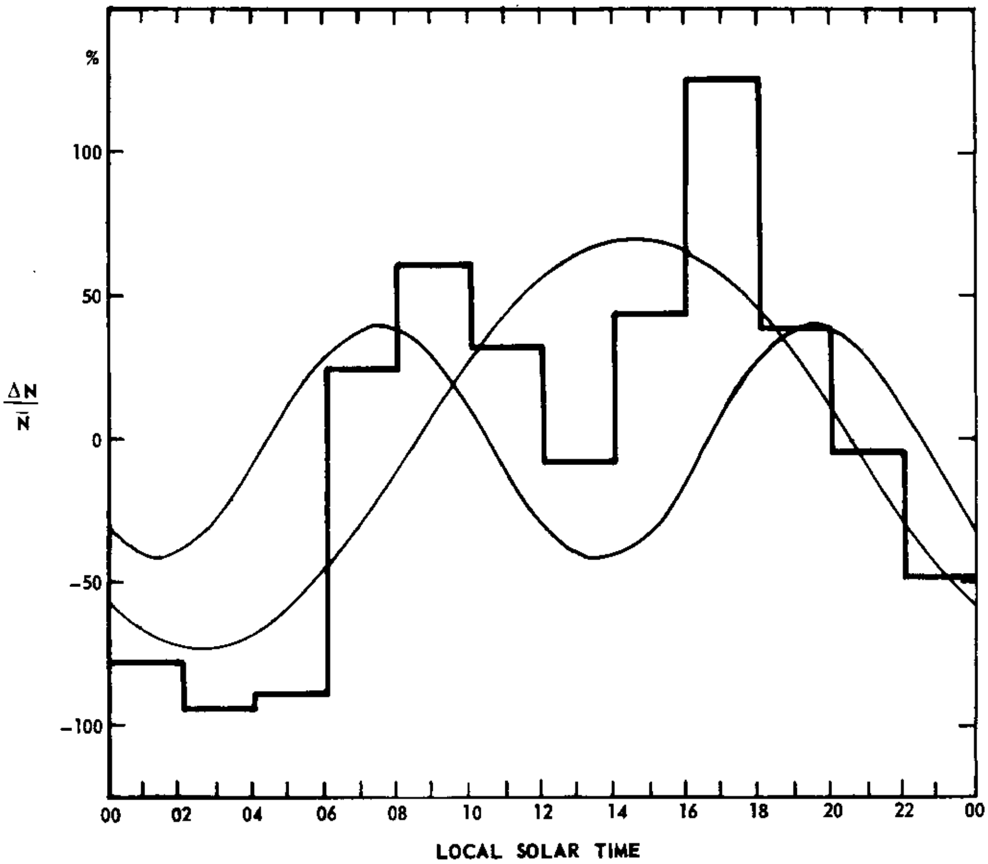



FIG. 4.8. The daily variation of vehicular traffic over Tunnel Hill, near Hobart, during November 1963. The percentage deviations refer to a mean rate $(\overline{N})$ = 113 vehicles per hour. The curves are the first and second harmonics of best fit.

day, the amplitudes of the first and second harmonics of the daily variation it produces at the equipment will be approximately

$$\frac{0{\cdot}00042}{2} = 0{\cdot}021 \times 10^{-2}\%.$$

Parking in the zone of maximum sensitivity (e.g., on the access road to Wiggins' farm) could increase the response by as much as a factor of 25, but it is evident that even then the daily variation observed at the equipment would be imperceptible. It is concluded that the occasional parking of vehicles that does occur near the junction of the highway and Wiggins' access road would have negligible influence on the daily variation underground. Nevertheless, parking is potentially the more serious of the two effects we have considered.

### 4.6. THE MEAN CUT-OFF ENERGY AT PRODUCTION. $\bar{E}_c$

The cut-off energy at production is the initial energy of the $\mu$-meson at the production level (assumed to be 100 mb) that is just sufficient to allow the meson to penetrate the atmosphere and material absorber and be recorded by the vertical equipment. The value that matters is not the cut-off in the vertical direction, where the sensitivity per square degree ($RS'$) is zero, but the weighted mean value, $\bar{E}_c$, averaged over the cone of sensitivity:

$$\bar{E}_c = \frac{\sum_{z,a} (E_c)_{z,a} RS_{z,a}}{\sum_{z,a} RS'_{z,a}} \tag{4.7}$$

A sufficiently accurate value for the present can be obtained by disregarding the azimuthal variations of $RS'$ due to the absorber and using representative values for each zenith angle, as can be obtained from the full curve in Figure 4.5. Then

$$\bar{E}_c = \frac{\sum_z (E_c) RS'_z}{\Sigma RS'_z} \tag{4.8}$$

The summations are taken over 12 values of zenith angle, selected at 5° intervals.

The cut-off energy $(E_c)_z$ at the zenith angle $z$ must be estimated in its two parts, that due to the material absorber and that due to the atmosphere. In column 2 of Table 4.1 the material absorber at each zenith angle is the value averaged over all azimuths. In column 3 the corresponding cut-off energy has been calculated on the assumption that the average $\mu$-meson energy loss for low energies of arrival underground is 2·1 Mev gm$^{-1}$ cm$^2$. This has been estimated from the work of (a) Murdoch, Ogilvie *and* Rathegeber (1959), who calculate the momentum losses versus momentum for $\mu$-mesons, arriving under 7000 gm cm$^{-2}$ of sandstone, increasing from 1·9 Mev/c gm$^{-1}$ cm$^2$ near zero momentum of arrival to e.g., 2·2 Mev/c gm$^{-1}$ cm$^2$ at 10 Gev/c; and (b) Cousins *and* Nash (1962) who give calculated average energy losses in quartz, increasing from 1·6 Mev gm$^{-1}$ cm$^2$ at the lowest energies of arrival at their depth of 36 m.w.e. (dry) to e.g., 2·3 Mev gm$^{-1}$ cm$^2$ at an arrival energy of 20 Gev.

The energy loss in the atmosphere at various zenith angles and for the different cut-off energies at the surface (column 4) has been estimated from a set of tables of range in air versus initial momentum, compiled by K. B. Fenton (1951), based on the Bethe/Bloch expression for energy loss by ionization and excitation (Bethe 1932; Bloch 1935). A flat, exponential atmosphere has been employed in the calculation of the mass of air to 100 mb at the different zenith angles.

Columns 5 and 6 give the pairs of values of cut-off energy at production $(E_c)_z$ and radiation sensitivity $RS'_z$ from which the weighted mean value of the cut-off energy, $E_c$, is calculated. Using equation 4.8, the value obtained was

$$\bar{E}_c = 10{\cdot}6 \text{ Gev.}$$

This figure is accurate to within 1 Gev if we allow a margin of error of ± 0·5

TABLE 4.I.

Meson cut-off energies and radiation sensitivity ($RS'_z$) versus zenith angle within the cone of acceptance of a vertical semi-cubical telescope underground. The values are averaged over all azimuths.

| Zenith angle degrees | Depth of material absorber (m.w.e.) | Cut-off energy at surface (Gev) | Energy loss in atmosphere (Gev) | Cut-off energy at production $(E_c)_z$ (Gev) | Radiation sensitivity $RS'_z$% |
|---|---|---|---|---|---|
| 0 | 30·9 | 6·5 | 2·5 | 9·0 | 0·00 |
| 5 | 31·6 | 6·6 | 2·5 | 9·1 | 0·062 |
| 10 | 31·6 | 6·6 | 2·5 | 9 1 | 0·111 |
| 15 | 33·1 | 6·9 | 2·6 | 9·5 | 0·140 |
| 20 | 33·8 | 7·1 | 2·7 | 9·8 | 0·149 |
| 25 | 35·3 | 7·4 | 2·8 | 10·2 | 0·147 |
| 30 | 36·0 | 7·6 | 2·9 | 10·5 | 0·130 |
| 35 | 38·9 | 8·2 | 3·2 | 11·4 | 0·103 |
| 40 | 43·3 | 9·1 | 3·5 | 12·6 | 0·073 |
| 45 | 47·7 | 10·0 | 3·9 | 13·9 | 0 046 |
| 50 | 53·6 | 11·3 | 4·7 | 16·0 | 0·025 |
| 55 | 61·7 | 12·9 | 5·7 | 18·6 | 0·012 |
| 60 | 74·9 | 15·7 | 6·7 | 22·4 | 0·002 |

m.w.e. in estimation of the depth of absorber and $\pm$ 0·2 Mev/c gm$^{-1}$ cm$^2$ for uncertainty in estimation of the average $\mu$-meson momentum loss.

A knowledge of the cut-off energy at production is of importance in the determination of the effective primary spectrum responsible for the mesons that are able to be detected underground. The differential coupling coefficient specifying the fractions of the counting rate, due to the primary protons per interval of primary energy, have been derived by Fenton from his calculated effective primary spectrum (Fenton 1963) and enter critically into the computations of the asymptotic cones of acceptance, to be described in Section 6. In his determination of the spectrum, Fenton used 15 Gev as the best estimate available at the time for the cut-off energy $E_c$. An important refinement of the treatment of the asymptotic cones of acceptance given here may result from the re-evaluation of the coupling coefficients, using the latest value of $E_c$ given above.

## 4.7. THE ATMOSPHERIC EFFECT

It has been assumed in the calculation of the atmospheric correction coefficients that the meteorological effect on the meson intensity underground is adequately described by Duperier's regression equation (Duperier 1949)

$$\frac{dI}{I} = \beta_{1B\cdot HT}\, dB + \beta_{1H\cdot BT}\, dH + \beta_{1T\cdot BH}\, dT \tag{4.10}$$

where $B$ is the barometric pressure at sea level, $H$ is the height of the assumed mean pressure level for production (100 mb) and $T$ is the temperature in the vicinity of the production level (taken to be the mean temperature in the interval 100 — 200 mb).

An account of the evaluation of the coefficients and a discussion of their significance is given in Paper 3 (Fenton *et al.* 1961). The observed coefficients, derived from the analysis of 432 sets of daily mean values of the variables, are, with the standard errors of estimate,

Total barometer coefficient, $\frac{dI}{dB} = \beta = -0{\cdot}65 \pm 0{\cdot}02\%/\text{cm}$
Partial barometer coefficient, $\beta_{1B \cdot HT} = -0{\cdot}59 \pm 0{\cdot}02\%/\text{cm}$
Negative temperature coefficient, $\beta_{1H \cdot BT} = -0{\cdot}46 \pm 0{\cdot}13\%/\text{km}$
Positive temperature coefficient, $\beta_{1T \cdot BH} = +0{\cdot}02 \pm 0{\cdot}005\%/°\text{C}$
Positive 3-fold temperature coefficient, $\beta_{1T \cdot B} = +0{\cdot}03 \pm 0{\cdot}005\%/°\text{C}$.

The only other known determinations at approximately the same depth relate to the vertical semi-cubical telescopes at the Budapest underground laboratory. Sandor, Somagyi *and* Telbisz (1962) report the following values for the partial regression coefficients:

$$\beta_{1B \cdot HT} = -\ 0{\cdot}78 \pm 0{\cdot}04\%/\text{cm}$$
$$\beta_{1H \cdot BT} = -1{\cdot}42 \pm 0{\cdot}08\%/\text{km}$$
$$\beta_{1T \cdot HB} = -0{\cdot}19 \pm 0{\cdot}06\%/°\text{C}.$$

The errors given are total values with contributions from count rate statistics and from instability in the equipment.

Dutt *and* Thambyahpillai (1965), in a recent treatment of the barometric effect, have indicated that the Hobart values of the total and partial barometer coefficients are in accord with observations at London at 60 m.w.e., while the Budapest values are too high. It is not known whether the average amounts of absorber at Hobart and Budapest are strictly comparable or whether the absorber is distributed in approximately the same manner, but it would seem from the close agreement between the respective values of the observed solar daily variation (Table 6, Jacklyn *and* Humble 1965b) that the effective absorber is almost the same at both places.

The differences between the Hobart and Budapest coefficients may have come about through the short-term variability of the atmospheric effect. In an earlier paper (Sandor *et al.* 1959), the authors showed that, over the period of 13 months of their analysis, month to month fluctuations in the coefficient at Budapest were far in excess of the variability due to counting rate statistics. For instance, their average total barometer coefficient, $\beta = -0{\cdot}86 \pm 0{\cdot}01\%/\text{cm}$, was considerably influenced by a very large value ($-1{\cdot}72 \pm 0{\cdot}04\%/\text{cm}$) in August 1958. In its absence, the mean value of $\beta$ would have been $-0{\cdot}76\%/\text{cm}$.

It is likely that the coefficient $\beta$ at Hobart would also vary considerably over short periods of time, but the value given here, averaged over two years of data, would seem to be reliable, judging by the values obtained for the two individual years:

1958 $\beta = -0{\cdot}69 \pm 0{\cdot}05\%/\text{cm}$ (single telescope)
1959 $\beta = -0{\cdot}65 \pm 0{\cdot}02\%/\text{cm}$ (duplex telescope).

Since observations of the daily variation of atmospheric structure are not available at Hobart, no attempt has been made to correct the observed daily variation of intensity for atmospheric temperature changes. Even if it were possible to do this, it appears that the method of Dorman or Maeda would be more accurate than that of Duperier (e.g., see Bercovitch 1963). We note that the observed negative temperature coefficient underground (—0·46 ± 0·13%/km) is very much less than at sea level at Hobart (—4·9 ± 0·9%/km). As Dutt *and* Thambyahpillai have observed (1965), after comparison of the widely differing experimental values at London, Budapest and Hobart with the theory, the decay effect appears to be very weak at these depths underground.

If there was an out-of-phase contribution to the solar daily variation underground due to variations of atmospheric temperature, it should have revealed itself over the six years of observation, because of the large changes that have occurred in the annual mean amplitude. It will be shown in the following Sections that there is no indication of a significant effect of this kind.

On the other hand, a daily variation due to the positive temperature effect would be expected to be more nearly in phase with the solar daily variation of extra-terrestrial origin and would be difficult to detect from examination of long-term trends. However, a study of the daily variation underground in relation to the solar anisotropy (Section 6) indicates that if there was any contribution of atmospheric origin it would have been small compared with the total solar daily variation.

### 4.8. DATA PROCESSING

#### (a) *Barometric correction*

The data, in the form of daily values of intensity, are corrected for pressure variations only, using the total barometer coefficient $\beta = -0{\cdot}65\%/\text{cm}$ ($-1{\cdot}65\%/\text{inch}$). Corrections to the nearest scaled count introduce maximum errors of ± 0·1%. Hourly mean values of surface pressure are obtained from daily barographs at the Hobart office of the Bureau of Meteorology. To correct for instrumental errors, the barograph records are checked against spot readings taken with a mercury column barometer at 0600, 0900, 1200, 1500, 1800 and 2100 local time, each day.

#### (b) *Corrections for changes in counting efficiency*

A continuous record is kept of the daily mean values $C_1/C_2$, $C_1/C_3$, $C_3/C_4$ where $C_1$ and $C_2$ are the counting rates of the two vertical telescopes underground and $C_3$ and $C_4$ are the counting rates of the inclined telescopes. Sudden changes or short-term drifts in the ratios can normally be traced to a change in the counting efficiency of a particular telescope. In doubtful cases, where it appears that more than one of the counting efficiencies has changed or where continuity has been interrupted because of lack of data from one of the telescopes, graphs of the pressure-corrected daily mean intensities are examined and from these it is almost always possible to detect a sustained change in counting level in either of the vertical telescopes.

Whenever sustained efficiency changes of magnitude approximately 0·1% or greater are detected in the vertical intensities, a normalizing factor is computed for the combined intensity $C = C_1 + C_2$ and two other factors are computed which will give the equivalent of C if $C_1$ or $C_2$ is missing. When the hourly values of the counting rate C (or $C_1$ or $C_2$ if only one telescope is operating) have been corrected for pressure, they are multiplied by the normalizing factor that prevails at that time and are thereby adjusted to a standard level of counting efficiency. The adjustments are made to the nearest scaled count and for that reason fluctuations of $\pm$ 0·1% in the final counting levels are liable to occur.

The overall stability of the normalized vertical intensity is difficult to assess. Small long-term drifts in the counting efficiencies cannot very well be taken care of by inter-comparison of counting rates with only two telescopes in the vertical direction. It would probably be fair to say that the counting level has been adjusted for short-term changes to within 0·2% of the standard level in most cases.

*(c) Selection of data for daily variation studies*

The data are arranged in groups of one month in the form of bi-hourly percentage deviates from the mean value. A single day's data comprise the twelve bi-hourly values plus the first bi-hourly value of the following day. Thereby a linear adjustment for secular change can be effected when the data are averaged over a month.

Unusable data are rejected when the normalizing factors are determined. Individual days are rejected from the tabulated data if more than two bi-hourly values are missing.

When a major Forbush-type intensity decrease occurs, the day of minimum intensity is rejected as is a number of days on either side of it, depending on the magnitude and duration of the decrease.

### 4.9. HARMONIC ANALYSIS

In the derivation of the Fourier coefficients the least squares method of Whittaker *and* Robinson (1944) is followed. The particular treatment given here is based very largely on that given by Parsons (1959).

*(a) The Fourier coefficients*

We have $m + 1$ experimental values $U_o$, $U_1$, . . . $U_{12}$, . . . $U_m$ spaced at equal intervals of time ($t$) over the daily period $T$, the corresponding times of occurrence being $O$, $t$. . . . $kt$ . . . $mt$. The values $U_k$ are in the form of percentage deviations from the mean intensity, so that

$$U_k = \frac{(I_k - \bar{I})}{\bar{I}} \%.$$

A progressive linear adjustment for secular (non-periodic) change causes $U_o$ to become equal to $U_m$.

After the adjustment for secular change, the curve of best fit to the observed daily variation of the $m$ values $U_o$, $U_1$, . . . $U_k$ . . . $U_{m-1}$ is expressed as the sum of harmonics of period $T/i$ where i = 1, 2, 3 etc. Our *estimate* of $U_k$ is then specified by $\Delta I_k$ where

$$\Delta I_k = \sum_{i=1}^{\infty} \left( a_i \frac{\cos 2k\pi it}{T} + b_i \frac{\sin 2k\pi it}{T} \right)$$
$$= \sum_{i=1}^{\infty} \left( a_i \frac{\cos 2k\pi i}{m} + b_i \frac{\sin 2k\pi i}{m} \right) \qquad (4.11)$$

$$= \sum_{i=1}^{\infty} R_i \cos i\ (\theta - f_i) \qquad (4.12)$$

where $\theta = 2\pi k/m$.

*(b) Determination of the constants $a_i$, $b_i$, $R_i$, $f_i$*

The average daily variation of intensity under consideration here is adequately described by the sum of the first two harmonics of best fit. The first harmonic (diurnal component) is specied by the Fourier coefficients for $i = 1$, and we have

$$(\Delta I_k)_1 = a_1 \cos\theta + b_1 \sin\theta$$
$$= R_1 \cos(\theta - f_1).$$

Likewise, putting $i = 2$, the second harmonic (semi-diurnal component) is

$$\Delta(I_k)_2 = a_2 \cos 2\theta + b_2 \sin 2\theta$$
$$= R_2 \cos 2(\theta - f_2).$$

The curve of best fit is accordingly described by

$$\Delta I_k = a_1 \cos\theta + b_1 \sin\theta + a_2 \cos 2\theta + b_2 \sin 2\theta.$$

The constants $a_1$, $b_1$, $a_2$ and $b_2$ are determined in such a way that the mean of the squares of the deviations of the observed values $U_k$ from the estimates $\Delta I_k$ is a minimum, i.e.,

$$\frac{\Sigma(\Delta I_k - U_k)^2}{m}$$

is minimized.

This requirement is fulfilled if

$$a_1 = \frac{2}{m}\sum_{k=0}^{m-1} U_k \cos\frac{2\pi k}{m} \qquad b_1 = \frac{2}{m}\sum_{k=0}^{m-1} U_k \sin\frac{2\pi k}{m}$$
$$a_2 = \frac{2}{m}\sum_{k=0}^{m-1} U_k \cos\frac{4\pi k}{m} \qquad b_2 = \frac{2}{m}\sum_{k=0}^{m-1} U_k \sin\frac{4\pi k}{m}.$$

The amplitude of the first harmonic is

$$R_1 = \sqrt{a_1^2 + b_1^2}$$

and the phase $f_1$, including the sign, is determined from

$$\cos f_1 = a_1/R_1 \text{ and } \sin f_1 = b_1/R_1$$

Similar considerations apply to the determinations of $R_2$ and $f_2$.

In the present investigation all observed daily variations of intensity are arranged in the form of bi-hourly deviates from the mean value. Consequently, the harmonic coefficients are calculated on a 12-ordinate scheme ($m = 12$). For this purpose, computing sheets (Tables 4.2 and 4.3) have been drawn up to facilitate the calculations when desk machines are used.

TABLE 4.2.

HARMONIC COEFFICIENTS $a_1$, $b_1$, $a_2$, $b_2$ FOR 12-ORDINATE SCHEME.

| Multiplier M | $U_k$ selection | Tunnel Vertical Solar 1959 $\Sigma U_k$ | Tunnel Vertical Solar 1959 $M\Sigma U_k$ | $\Sigma U_k$ | $M\Sigma U_k$ | $\Sigma U_k$ | $M\Sigma U_k$ | $\Sigma U_k$ | $M\Sigma U_k$ | $\Sigma U_k$ | $M\Sigma U_k$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0·16667 | 1 − 13 | −104 | −17·334 | | | | | | | | |
| 0·14434 | 3 − 11 − 15 + 23 | −215 | −31·033 | | | | | | | | |
| 0·08333 | 5 − 9 − 17 + 21 | −152 | −12·666 | | | | | | | | |
| | $a_1$ | | −61·033 | | | | | | | | |
| 0·08333 | 3 + 11 − 15 − 23 | −183 | −15·249 | | | | | | | | |
| 0·14434 | 5 + 9 − 17 − 21 | −314 | −45·323 | | | | | | | | |
| 0·16667 | 7 − 19 | −149 | −24·834 | | | | | | | | |
| | $b_1$ | | −85·406 | | | | | | | | |
| 0·16667 | 1 − 7 + 13 − 19 | 107 | 17·834 | | | | | | | | |
| 0·08333 | 3 − 5-9 + 11 + 15 − 17 − 21 + 23 | 123 | 10·249 | | | | | | | | |
| | $a_2$ | | 28·083 | | | | | | | | |
| 0·14434 | 3 + 5 − 9 − 11 + 15 + 17 − 21 − 23 | 289 | 41·714 | | | | | | | | |
| | $b_2$ | | 41·714 | | | | | | | | |
| | $a_1 + b_1 + a_2 + b_2$ | | −76·642 | | | | | | | | |
| CHECK | | | | | | | | | | | |
| 0·3334 | 1 − 19 | −73 | −24·334 | | | | | | | | |
| 0·45534 | 3 | −54 | −24·588 | | | | | | | | |
| 0·28868 | 5 − 21 | −75 | −21·651 | | | | | | | | |
| 0·16667 | −9 − 17 | −43 | −7·167 | | | | | | | | |
| 0·12202 | −11 | 9 | 1·098 | | | | | | | | |
| | $a_1 + b_1 + a_2 + b_2$ | | −76·642 | | | | | | | | |

*N.B.* *EXAMPLE* TUNNEL VERTICAL SOLAR 1959

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| HOUR (CENTRE OF INTERVAL) | 0100 | 0300 | 0500 | 0700 | 0900 | 1100 | 1300 | 1500 | 1700 | 1900 | 2100 | 2300 |
| $\Delta I/I$ (% × $10^3$) | −23 | −54 | −92 | −99 | −98 | −9 | 81 | 145 | 141 | 50 | −17 | −25 |
| $U_k$ number | 1 | 3 | 5 | 7 | 9 | 11 | 13 | 15 | 17 | 19 | 21 | 23 |

$\phi$ = (0100 − 0000) in degrees = 15°

TABLE 4.3.

FIRST AND SECOND HARMONICS $R_1$, $T_{1\ max}$, $R_2$, $T_{2\ max}$.

$\Delta I/I = R_1 \cos(\theta - f_1) + R_2 \cos 2(\theta - f_2) = a_1 \cos\theta + b_1 \sin\theta + a_2 \cos 2\theta + b_2 \sin 2\theta \qquad (\Delta I/I)^* = R^*_1 \cos(\theta - \theta_1 \max) + R^*_a \cos 2(\theta - \theta_2 \max)$

| | Tunnel Vertical Solar 1959 | | | | | | |
|---|---|---|---|---|---|---|---|
| $a_1$ | −61·033 | | | | | | |
| $b_1$ | −85·406 | | | | | | |
| $R_1^2 = a_1^2 + b_1^2$ | 11019 | | | | | | |
| $R_1$ | 104·97 | | | | | | |
| $R_1^* = 1{\cdot}012\ R_1$ | 106·23 | | | | | | |
| $a_2$ | 28·083 | | | | | | |
| $b_2$ | 41·714 | | | | | | |
| $R_2^2 = a_2^2 + b_2^2$ | 2529 | | | | | | |
| $R_2$ | 50·29 | | | | | | |
| $R_2^* = 1{\cdot}047\ R_2$ | 52·65 | | | | | | |
| $\cos f_1 = a_1/R_1$ | −0·5814 | | | | | | |
| $f_1$<br>$\phi$ | 234° 27′<br>15° | | | | | | |
| $\theta_1 \max = f_1 + \phi$ | 249° 27′ | | | | | | |
| $T_1 \max$ | 1638 | | | | | | |
| $\cos 2f_2 = a_2/R_2$ | 0·5584 | | | | | | |
| $2f_2$ | 56° 03′ | | | | | | |
| $\theta_2 \max = f_2 + \phi$ | 43° 03′ | | | | | | |
| $T_2 \max$ | 0252<br>1452 | | | | | | |

*(c) Adjustments for smoothing*

As the number of ordinates in the ordinate scheme decreases, the amplitude of a given harmonic suffers increasingly from a smoothing error so that the true harmonic of best fit is underestimated. To adjust for smoothing of the $i^{th}$ harmonic in an $m$-ordinate scheme, the amplitude $R_i$ must be multiplied by $\omega/\sin\omega$ where $\omega = i\pi/m$. The multiplying factors for $R_1$ and $R_2$ in a 12-ordinate scheme are 1·012 and 1·047 respectively.

*(d) Adjustments for change of time scale*

The procedures for determining the harmonic coefficients assume that the first intensity value $u_0$ refers to time 0000. However, where bi-hourly values are used, for instance, the first value usually refers to the mean of hours 01 and 02 and is taken as the value for the time 0100. The phase angle $f_i$ must be shifted forward accordingly, by the addition of a constant $\phi$. In the case referred to, $\phi = 15°$. In the accompanying form it can be seen at what stage $\phi$ must be added to $f_2$.

4.10. STATISTICAL ERRORS OF AMPLITUDE AND PHASE

Standard errors of estimate of amplitude and phase of the first and second harmonics of the observed daily variations are calculated on the assumption that, in the absence of systematic changes, the number of particles detected by the equipment in a given interval of time is a Poisson variable.

*(a) The standard error (SE) of $u_k$*

A given $u_k$ value is derived from $N$ particles and is expressed as a percent deviation from the mean $\bar{N}$. The SE of estimate, $\sigma(u_k)$, is

$$\sigma(u_k) = \frac{100\sqrt{N}}{\bar{N}}\ \%.$$

The daily variation is very small, so that $N \approx \bar{N}$ and we can put

$$\sigma(u_k) = \frac{100\sqrt{\bar{N}}}{\bar{N}} = \frac{100}{\sqrt{\bar{N}}}.$$

If the mean counting rate is $\bar{C}$, derived from $n$ hourly values and containing a scale factor $F$,

$$\bar{N} = n\bar{C}F$$

and

$$\sigma(u_k) = \frac{100}{\sqrt{n\bar{C}F}}.$$

*(b) The standard error of $a_1$*

$$a_1 = \frac{2}{m}\sum_{0}^{m-1} u_k \cos\frac{2\pi k}{m}.$$

Since all the $u_k$'s have the same variance $\sigma^2(u_k)$,

$$\sigma^2(a_1) = \left(\frac{2}{m}\right)^2 \sum_0^{m-1} \cos^2 \frac{2\pi k}{m} \sigma^2(u_k) = \left(\frac{2}{m}\right) \sigma^2(u_k);$$

therefore

$$\sigma(a_1) = \sigma(b_1) = \sqrt{\frac{2}{m}}\, \sigma(u_k).$$

*(c) The standard errors of $R_1$ and $R_2$*

We use another expression for the variance of a function $F(a,b)$.

$$\sigma^2[F(a, b)] = \left(\frac{dF}{da}\right)^2 \sigma^2(a) + \left(\frac{dF}{db}\right)^2 \sigma^2(b);$$

since

$$R = \sqrt{a^2 + b^2}$$

$$\sigma^2(R) = \left(\frac{a^2}{R^2} + \frac{b^2}{R^2}\right)\sigma^2(a) = \sigma^2(a),$$

therefore

$$\sigma(R_1) = \sigma(a_1) = \sqrt{\frac{2}{m}}\, \sigma(u_k)$$

$$= 100 \sqrt{\frac{2}{mn\bar{C}F}}$$

$$= \sigma(R_2).$$

*(d) The standard error of time of maximum*

The error circle attached to the end-point of $R_1$ or $R_2$ on the harmonic dial subtends an angle at the origin equal to twice the SE of estimate of time of maximum. If small, $\frac{\sigma(R)}{R}$ approximates the SE of phase, in radians.

4.11 (a) *Observed versus calculated variability of the counting rate*

After correction for pressure changes, the systematic time-averaged variations of intensity underground are relatively small. For instance, the amplitude of the daily variation is usually less than 0·1%, Forbush-type decreases rarely exceed 1%, and the change in the level of the daily average intensity from maximum to minimum solar activity is only about 2·5%. Moreover, time-dependent processes that appear to produce rather small short-term irregular fluctuations that are of significance at ground level may not be nearly so important at the higher energies of response. Consequently, it seemed that fluctuations of the underground intensity from one hour to the next might obey Poisson statistics, at least during periods of minimum solar activity, if the influence of the solar daily variation were removed. Since direct evidence of this was desirable, so that the applicability of the usual Standard Error tails could be assessed, it was decided to determine experimentally the variance of the count rate due to random fluctuations, using the pressure-corrected vertical intensity that related to the two quiet years 1964 and 1965. Table 4.4 is a typical monthly data sheet.

TABLE 4.4.

COSMIC RAY DATA—PHYSICS DEPARTMENT, UNIVERSITY OF TASMANIA

*STATION*: CAMBRIDGE TUNNEL (HOBART), TASMANIA (42° 51′S, 147° 25′E) Altitude 110 metres. DECEMBER, 1964
*RECORDER*: VERTICAL SEMI-CUBICAL MESON TELESCOPE Absorber 40 m.w.e. Barometer Coefficient −0·65%/cm Hg.
*TABULATED*: Hourly count totals corrected to standard pressure of 28·0 inches of Hg. REAL COUNTS = 128 × tabulated counts

| Hour GMT | Date 01 | 02 | 03 | 04 | 05 | 06 | 07 | 08 | 09 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 01 | 558s | 557 | 559 | 560 | 558 | 558 | 558 | 561 | 561 | 562 | 560 | 558 | 557 | 559 | 563 | 561 | 560 | 565 | 558 | 559 | 558 | 557 | 556 | 559 | 554 | 555 | 557 | 560 | 560 | 558 | 558s |
| 02 | 560s | 559 | 559 | 560 | 560 | 563 | 562 | 560 | 563 | 561 | 557 | 560 | 564 | 554 | 561 | 559 | 558 | 563 | 559 | 561 | 555 | 558 | 560 | 555 | 5 9 | 558 | 554 | 559 | 556 | 554 | 554s |
| 03 | 565s | 561 | 560 | 561 | 563 | 558 | 560 | 564 | 564 | 562 | 554 | 558 | 562 | 557 | 560 | 558 | 562 | 561 | 54 | 560 | 556 | 554 | 557 | 560 | 555 | 556 | 558 | 560 | 562 | 556 | 559 |
| 04 | 558s | 555 | 559 | 561 | 554 | 562 | 557 | 561 | 561 | 563 | 559 | 556 | 561 | 557 | 561 | 557 | 556 | 559 | 558 | 556 | 562 | 555 | 558 | 559 | 558 | 557 | 558 | 560 | 557 | 559 | 556 |
| 05 | 565s | 561 | 562 | 564 | 562 | 559 | 557 | 558 | 563 | 562 | 558 | 559 | 557 | 558 | 560 | 559 | 555 | 563 | 560 | 553 | 555 | 555 | 556 | 556 | 556 | 557 | 556 | 558 | 561 | 558 | 558 |
| 06 | RF | 562 | 558 | 558 | 560 | 561 | 561 | 561 | 563 | 560 | 561 | 563 | 556 | 561 | 558 | 562 | 559 | 562 | 557 | 561 | 557 | 555 | 557 | 560 | 557 | 560 | 561 | 558 | 559 | 558 | 556 |
| 07 | 558s | 559 | 559 | 558 | 561 | 559 | 557 | 558 | 561 | 561 | 562 | 561 | 558 | 556 | 557 | 560 | 560 | 565 | 560 | 555 | 558 | 556 | 556 | 561 | 555 | 557 | 558 | 556 | 559 | 559 | 556 |
| 08 | 561s | 557 | 555 | 559 | 561 | 559 | 562 | 559 | 561 | 562 | 563 | 559 | 556 | 557 | 561 | 558 | 561 | 560 | 563 | 557 | 559 | 553 | 557 | 559 | 553 | 559 | 554 | 561 | 554 | 553 | 563 |
| 09 | 561s | 558 | 562 | 561 | 558 | 557 | 558 | 562 | 561 | 559 | 558 | 560 | 558 | 561 | 558 | 562 | 562 | 559 | 555 | 560 | 558 | 557 | 558 | 556 | 555 | 557 | 553 | 564 | 559 | 556 | 555 |
| 10 | 555s | 558 | 557 | 560 | 559 | 558 | 559 | 564 | 566 | 556 | 558 | 561 | 560 | 562 | 563 | 560 | 558 | 560 | 564 | 558 | 557 | 559 | 554 | 557 | 558 | 557 | 559 | 560 | 554 | 556 | 556 |
| 11 | 569s | 554 | 559 | 559 | 558 | 559 | 558 | 565 | 561 | 560 | 557 | 560 | 558 | 554 | 558 | 561 | 558 | 562 | 560 | 558 | 556 | 557 | 559 | 562 | 556 | 554 | 556 | 560 | 560 | 557 | 558 |
| 12 | 562s | 561 | 558 | 557 | 560 | 557 | 559 | 561 | 562 | 565 | 560 | 560 | 556 | 558 | 560 | 565 | 560 | 558 | 557 | 558 | 560 | 559 | 556 | 555 | 556 | 557 | 556 | 560 | 553 | 560 | 562 |
| 13 | 562s | 557 | 564 | 559 | 561 | 561 | 563 | 562 | 557 | 559 | 559 | 559 | 558 | 558 | 556 | 561 | 564 | 560 | 554 | 555 | 555 | 562 | 561 | 560 | 561 | 559 | 559 | 559 | 557 | 559 | 558 |
| 14 | 558s | 563 | 562 | 561 | 562 | 559 | 561 | 562 | 560 | 558 | 563 | 562 | 558 | 561 | 561 | 558 | 557 | 562 | 561 | 558 | 556 | 557 | 558 | 559 | 560 | 561 | 556 | 554 | 558 | 558 | 558 |
| 15 | 558s | 563 | 561 | 562 | 560 | 558 | 557 | 557 | 557 | 564 | 559 | 558 | 560 | 559 | 562 | 562 | 566 | 561 | 559 | 558 | 557 | 554 | 556 | 557 | 556 | 562 | 557 | 556 | 559 | 556 | 556 |
| 16 | 556s | 558 | 561 | 553 | 559 | 559 | 562 | 564 | 563 | 563 | 557 | 563 | 558 | 560 | 561 | 562 | 561 | 561 | 561 | 556 | 558 | 558 | 556 | 559 | 560 | 560 | 559 | 559 | 556 | 556 | 564 |
| 17 | 560s | 561 | 558 | 559 | 561 | 562 | 559 | 559 | 562 | 563 | 559 | 556 | 558 | 558 | 560 | 562 | 562 | 561 | 561 | 560 | 553 | 556 | 559 | 560 | 559 | 558 | 557 | 555 | 556 | 560 | 559 |
| 18 | 560s | 562 | 559 | 563 | 562 | 557 | 563 | 562 | 560 | 562 | 558 | 562 | 554 | 556 | 559 | 561 | 563 | 561 | 558 | 559 | 560 | 554 | 560 | 560 | 555 | 559 | 554 | 555 | 556 | 557 | 560 |
| 19 | 558s | 556 | 560 | 562 | 558 | 556 | 560 | 560 | 557 | 562 | 559 | 560 | 559 | 557 | 561 | 563 | 561 | 559 | 558 | 556 | 559 | 559 | 554 | 557 | 558 | 559 | 559 | 560 | 559 | 555 | 560 |
| 20 | 552s | 560 | 562 | 560 | 560 | 563 | 561 | 566 | 556 | 561 | 554 | 558 | 558 | 563 | 561 | 563 | 563 | 561 | 560 | 559 | 560 | 558 | 562 | 556 | 556 | 562 | 556 | 560 | 560 | 558 | 560 |
| 21 | 550s | 561 | 557 | 563 | 562 | 557 | 561 | 563 | 560 | 561 | 565 | 556 | 559 | 556 | 561 | 559 | 558 | 556 | 557 | 554 | 559 | 553 | 556 | 560 | 552 | 553 | 557 | 555 | 557 | 558 | 556 |
| 22 | 562s | 557 | 560 | 558 | 561 | 561 | 564 | 561 | 561 | 561 | 560 | 560 | 560 | 563 | 564 | 564 | 560 | 562 | 557 | 553 | 557 | 558 | 557 | 560 | 556 | 558 | 556 | 558 | 555 | 561 | 556 |
| 23 | 558s | 562 | 559 | 558 | 558 | 560 | 562 | 563 | 558 | 561 | 563 | 560 | 558 | 559 | 560 | 556 | 559 | 559 | 564 | 558 | 562 | 561 | 556 | 559 | 555 | 555 | 560 | 556 | 559 | RF | 562 |
| 24 | 564s | 562 | 564 | 559 | 562 | 558 | 559 | 561 | 558 | 556 | 560 | 558 | 560 | 560 | 559 | 563 | 561 | 560 | 559 | 555 | 560 | 558 | 557 | 557 | 558 | 555 | 557 | 559 | 557 | 557 | 559 |
| Σ | (12870) | 13424 | 13434 | 13435 | 13440 | 13421 | 13440 | 13474 | 13456 | 13464 | 13423 | 13427 | 13403 | 13404 | 13445 | 13456 | 13444 | 13460 | 13414 | 13377 | 13387 | 13363 | 13376 | 13403 | 13358 | 13385 | 13367 | 13402 | 13383 | (12819) | 13399 |
| *M* | | 559·3 | 559·7 | 559·8 | 560·0 | 559·2 | 560·0 | 561·4 | 560·7 | 561·0 | 559·3 | 559·5 | 558·5 | 558·5 | 560·2 | 560·7 | 560·2 | 560·8 | 558 9 | 557·4 | 557.8 | 556·8 | 557.3 | 558·5 | 556.6 | 557·7 | 557.0 | 558·4 | 557·6 | | 558·3 |

s = Estimated from single telescope RF = Recording failure

The observed solar daily variation averaged over the two years is shown in Figure 4.9 (a), in the form of bi-hourly percentage deviates from the mean intensity. It is clear that the solar daily variation on the average produces scarcely any change in the counting rate from about 1900 hours to 0900 hours. Therefore, since it seemed that day-to-day fluctuations of counting rate during these hours would be least influenced by day-to-day changes in the solar daily variation, two neighbouring bi-hourly periods from this relatively undisturbed part of the day were selected for the statistical test. It was decided to examine fluctuations of the difference of pressure-corrected intensity between the two bi-hourly periods (05, 06) and (07, 08). By taking differences, the influence of day-to-day changes in the average level of intensity would be minimized.

The data were arranged in successive groups of 10-day averages, so that a sample value of intensity would be the average of 20 hours of counting. We represent the scaled counting rate for the bi-hourly period (05, 06), averaged over the $i^{\text{th}}$ group of 10 days, by $C_{1i}$ and that for the period (07, 08) by $C_{2i}$, while the difference is given by $d_i = C_{1i} - C_{2i}$. Regarding $C_{1i}$ as a Poisson variable, the estimated variance is $V(C_{1i}) = C_{1i}/nF$, where $n$ is the number of hours over which the average is taken and $F$ is the scale factor. Averaged over the two years, $\overline{C}_1$ was found to be approximately 559 scaled counts while departures of the sample values from this did not exceed 4 counts (see Figure 4.9 (b)). Therefore it is adequate to assign a constant estimated variance to each sample value, representing $C_1$ by its mean, $\overline{C}_1$, in the calculation of the variance.

Putting $C_{1i} = 559$, $n = 20$ and $F = 128$, we get $V(C_{1i}) = 0{\cdot}218$. The correction of the count rate to a standard pressure of 28 inches of mercury artificially raises the counting level by 2·75% on the average, so that the estimated variance must be multiplied by $(1{\cdot}0275)^2$, giving us a revised $V(C_{1i}) = 0{\cdot}230$. Then the estimated variance of the difference is $V(d_i) = 0{\cdot}460$ and the standard estimate of error is $SE(d_i) = \pm\ 0{\cdot}68$.

The 60 observed sample values of $d_i$ the difference, each relating to $\sim 2{\cdot}9 \times 10^6$ detected particles, are shown in Figure 4.9 (c) in their order of occurrence. The observed standard deviation is

$$SD(d_i) = \sqrt{\overline{(d_i^2)} - (\overline{d_i})^2}.$$

It was found that $\overline{d_i}$ was zero and that

$$SD(d_i) = \pm\ 0{\cdot}75.$$

The ratio $\dfrac{\text{observed } SD}{SE \text{ of estimate}}$ was therefore 1·10. In the figure the observed $SD$ and $SE$ of estimate are shown as full lines and dashed lines respectively.

It would seem that the $SE$ tails that are given with the individual harmonics of best fit to the observations adequately represent the errors of estimate arising from random fluctuations of the observed pressure-corrected intensity.

4.11 (b) *A bi-hourly periodicity in timing*

When the influence of the solar daily variation is minimized, the annual bi-hourly deviates in solar time appear to obey Poisson statistics. However, the same

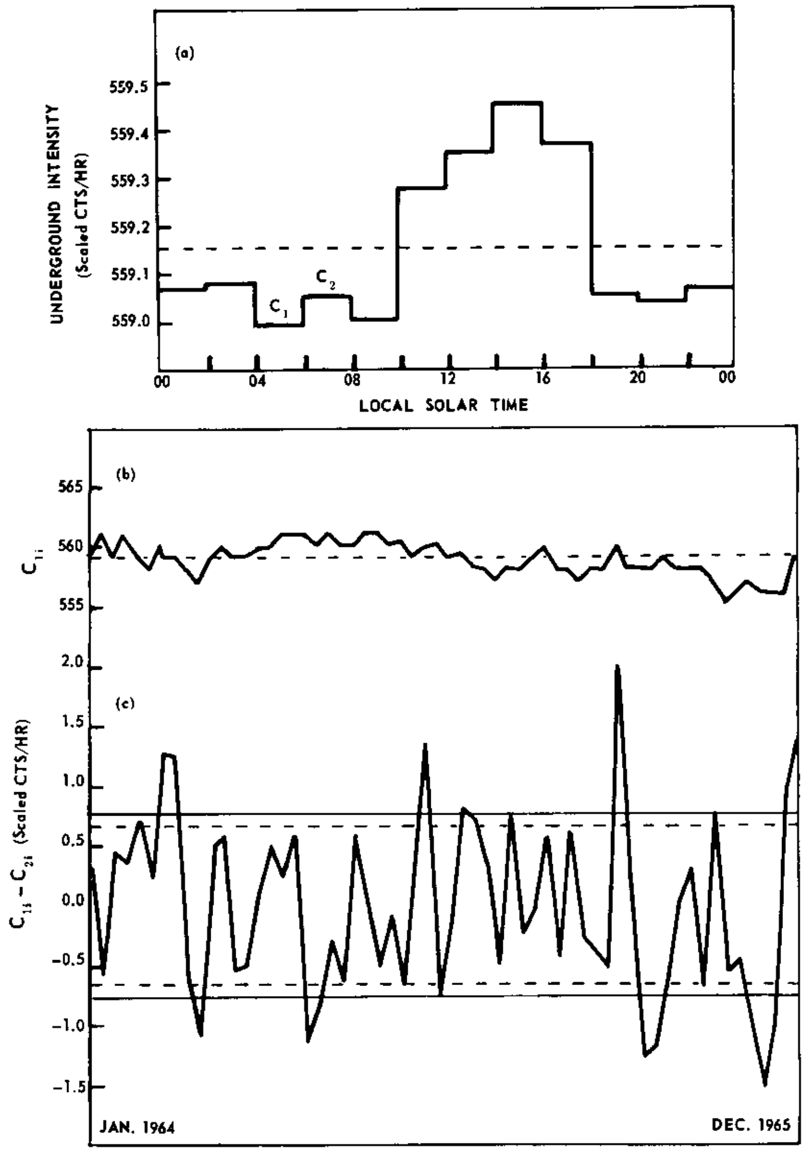



FIG. 4.9. Variability of the pressure-corrected bi-hourly counting rate in the vertical direction underground.

(a) The solar daily variation averaged over 1964 and 1965. The average bi-hourly counting rates centred on 0500 and 0700 are shown as $C_1$ and $C_2$.

(b) Successive 20-day averaged values of $C_1$, from January 1964 to December 1965. The dashed line is the biennial mean value of $C_1$, as shown in (a).

(c) Successive 20-day averaged values of $C_1 - C_2$. The full lines represent the observed standard deviation and the dashed lines the S.E. of estimate, assuming the difference to follow a Poisson distribution.

cannot be said of the hourly deviates. Very recently, an examination of the annual hourly deviates that had been computed showed that the deviates would tend to be consistently negative on odd hours G.M.T. and positive by about the same amount on even hours, if the daily variation were removed. That is to say, successive pairs of hourly deviates would exhibit approximately a constant difference and in the same sense.

It was concluded, from a closer examination of this effect, that over the period since July 1960, when the Mercer Chronometer was installed, until May 1965, when it was overhauled, odd hours had been very regularly shorter than even hours by approximately 2·5 seconds, and that after the overhaul even hours were shorter than odd hours by that same amount. This latter difference was confirmed by taking successive hourly photographs, initiated by the chronometer, of the position of its own second hand. It became evident, by taking photographs with other timing devices, that the fault originated in the chronometer and not in the timing control circuit.

When bi-hourly deviates of intensity are used, as has been almost exclusively the case in the analyses of the underground data described in this paper, a 2-hour periodicity in timing is averaged out. Care has been taken to point out in the text (Section 5) the two occasions on which hourly data have been used, but it should be noted that even if hourly data had been used throughout,

(1) a $12^{th}$ harmonic in the daily variation, such as that introduced by the periodicity in timing, would not have compromised determinations of first and second harmonics;

(2) if diurnal modulation of the $12^{th}$ harmonic had occurred it would have given rise not to an apparent *diurnal* variation of intensity, but to $11^{th}$ and $13^{th}$ harmonics;

(3) if seasonal modulation of the $12^{th}$ harmonic had occurred it would not have given rise to sideband components at the sidereal and anti-sidereal frequencies, since the frequency of the carrier is much too high (4380 cycles/year).

To confirm that the 2-hour periodicity would not affect calculations of first and second harmonics based on hourly deviates, the harmonic components of the annual daily variations were compared when the same underground data had been arranged in both hourly and bi-hourly form. The year 1961 and the biennial periods 1962-1963 and 1964-1965 were considered. For each of these periods the results obtained were the same, whether one used hourly or bi-hourly deviates.

A modification of the method of timing, which it is hoped will eliminate the timing periodicity, is at present under test.

# 5. THE UNDERGROUND EVIDENCE FOR A TWO-WAY SIDEREAL ANISOTROPY

## 5.1. INTRODUCTION

In this Section a summary is given of the qualitative evidence from underground, acquired up to the end of 1965. Some of the evidence has been published, and the following relevant papers will be referred to in the text:

Paper 4 (Jacklyn 1963b). "The apparent sidereal daily variation of cosmic ray intensity at 42 m.w.e. underground at Hobart, Tasmania. I. Results of observations in 1958".

Paper 5 (Jacklyn 1965a). "The apparent sidereal daily variation of cosmic ray intensity at $\sim$ 40 m.w.e. at Hobart, Tasmania. II. Results of observations 1958-1962".

Paper 8 (Jacklyn 1966). "Evidence for a two-way sidereal anisotropy in the charged primary cosmic radiation".

It is proposed to arrange the summary under headings which tend to relate to particular papers and which follow the order of the list of observational requirements given in Section 1.3. These are considered to be requirements that must be satisfied by the present observations if a sidereal component produced by a two-way anisotropy is to be identified—as against a spurious effect arising from seasonal modulation of a solar component of the daily variation. The headings are as follows:

(a) the evidence from annual averages, relating to seasonal modulation as detected by the anti-sidereal technique;

(b) the evidence from phase anomalies;

(c) the evidence from the 12-hour phase difference between the hemispheres;

(d) the evidence from the observed harmonics. Under (d) only the general character of the harmonics is noted in this Section. Employing the asymptotic cones of response (Section 6), we go on in Section 7 to examine the latitude dependence of the deduced free-space harmonics. This leads to an estimation of the free-space direction of the anisotropy and provides a useful test of the coherence of the evidence, obtained as it was from several detectors with differing response characteristics.

Since the evidence is not being presented in strict chronological order it might be helpful to begin with a brief description of the experimental arrangements, elaborating where necessary in later Sections.

## 5.2. THE UNDERGROUND EXPERIMENTS AT HOBART

Observations have been obtained from four directions in the plane of the geographic meridian. Figure 5.1 shows the axial inclinations to the zenith, namely 70°N, 30°N, 0° and 45°S. Four counter telescopes have been used, each essentially comprising three metre-square trays in triple coincidence. However, only two telescopes have been available for the measurements in the three inclined directions, the other two being permanently tied up in the vertical experiment.

*(a) The vertical semi-cubical experiment*

The two vertical semi-cubical telescopes were installed in July 1957 by R. Taylor, and continuous observations date from October of that year.

*(b) The cubical telescopes inclined 30°N and 45°S of zenith*

Towards the end of 1960, two additional telescopes were put into operation. The tray assemblies and electronic circuits were copies of the vertical semi-cubical construction in all but minor detail. The cubical framework comprised two square side-frames of welded angle-iron, bolted to each other at the four corners by cross-bars. This provided a rigid structure to which the trays could be fastened. In Figure 5.2 one of these cubical telescopes can be seen on the left, mounted on a rotatable platform and tilted at an angle of 45°S of zenith in the geographic meridian plane. One of the two pivoting points at which the telescope was attached to the platform is seen more clearly in Figure 5.2 (b). Two vertical supports, cut to the appropriate length, bolted the telescope securely to the platform fore and aft, at the given inclination to the zenith.

A spirit level attached to the frame of the telescope allowed variations of the zenith setting, that might result from equipment tests and from movements in the floor level, to be read off to an accuracy of 10 seconds of arc. Occasional slight adjustments were effected by raising the platform with a car jack (Figure 5.2 (b)) and placing pieces of shim under the wheels.

A cylindrical metal sleeve projecting downwards under the centre of the platform was fitted over a vertical peg screwed into the floor, thereby fixing the axis of rotation of the telescope. The rotatable platform was intended for investigations of azimuthal effects and was of no particular relevance for the experimental studies of the sidereal effect.

The two directional telescopes were equipped with independent HT and EHT supplies and were connected to the mains through a common voltage-regulating transformer. They shared with the vertical telescopes the same photographic recording panel, timing system and chart recorder (Section 4).

Over the two complete calendar years 1961 and 1962 one telescope was set at an inclination 30°N of zenith and the other at 45°S of zenith so that simultaneous results could be obtained from low-, mid- and high-latitude scans. The calculation of the asymptotic cones of viewing will be described in Section 6. It suffices to say here that the mean asymptotic latitudes of viewing in the three directions are equally spaced, being approximately 17°S, 39°S and 60°S geographic.

Of the two inclined telescopes, the south-pointing had much the lower counting rate due both to its greater inclination to the zenith and to the greater amount of material absorber within its cone of viewing.

*(c) The narrow-angle telescope inclined 70°N of zenith*

The experiment at the zenith angle setting 30°N was discontinued in 1963. The cubical telescope was converted to a narrow-angle detector of dimensions

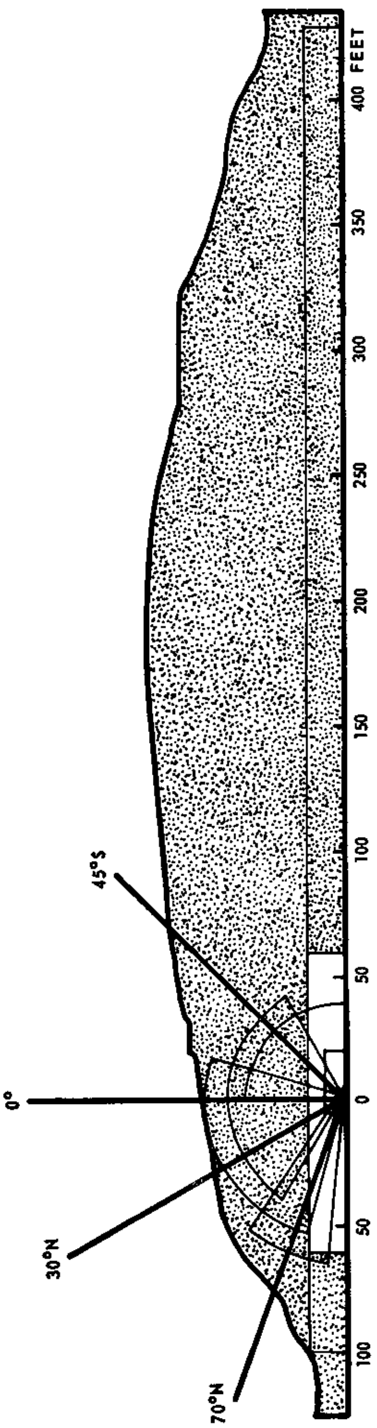



FIG. 5.1. Cross-section of tunnel in the meridian plane, showing a xial headings and geometric apertures of the detectors in relation to the material absorber.

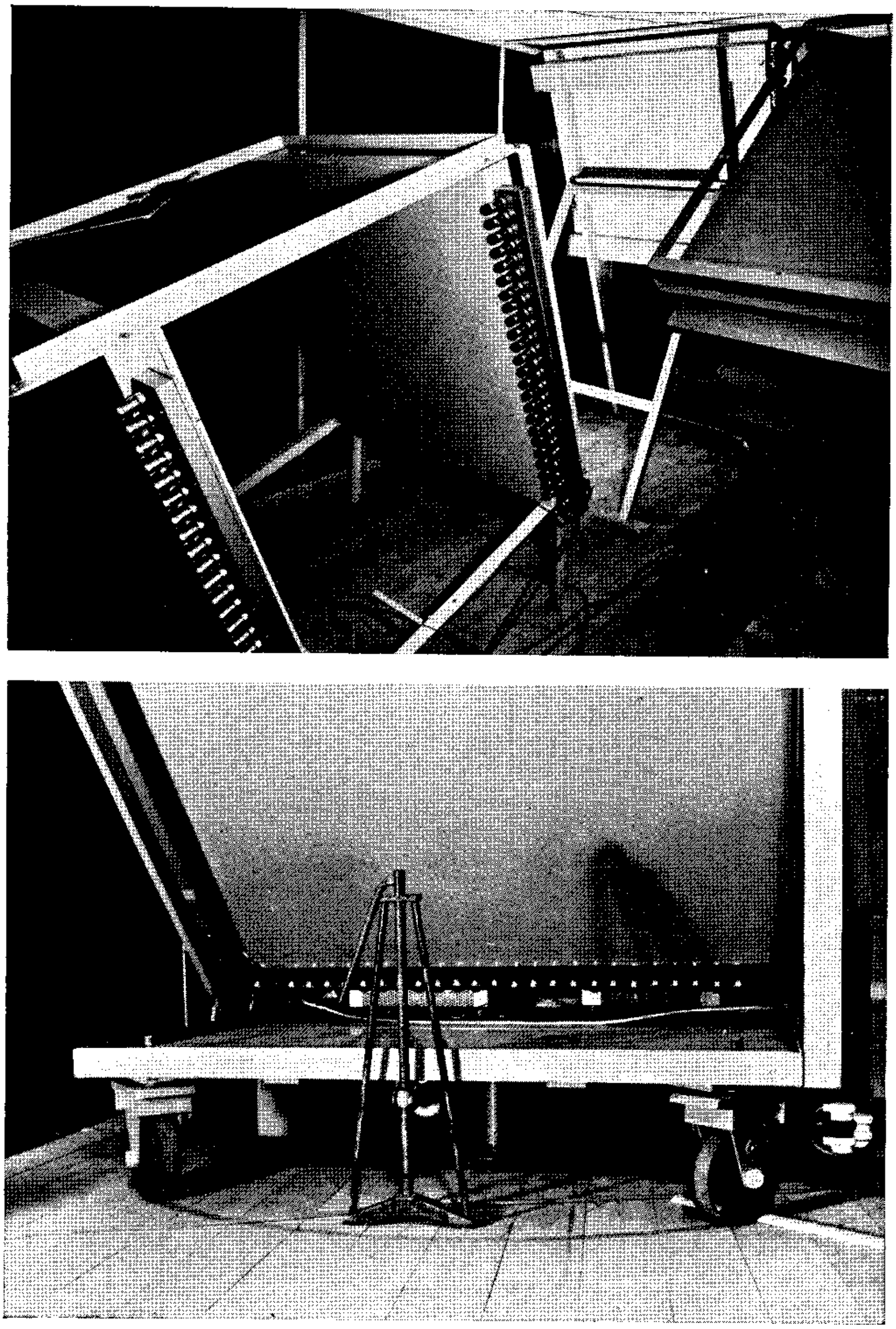


FIG. 5.2. The narrow angle inclined telescope and the cubical inclined telescope at the underground station.

3m x 1m², at the zenith angle setting 70°N. To effect this, the telescope was removed from the turntable and set up vertically on the floor. It was then tilted over on one edge to 70°N of zenith and supported in that position by two angle-iron uprights at the corners. The middle tray was removed and placed on a wooden easel, in line with the other two trays, and at a distance of 3 metres from the bottom tray. The arrangement is shown in Figure 5.2 (a) and can also be seen in Figure 4.2, to the right of the south-pointing telescope and beyond the two semi-cubes. Further details of this experiment are given in Section 5.5.

Some constants of interest relating to the four experiments are listed in Table 5.1. The absorber within the viewing cone of the narrow-angle telescope has been approximated by the absorber in the direction of the axis, since a detailed cone of acceptance has not yet been calculated for this detector. In the case of the inclined cubical telescopes, the technique of the local cone of acceptance (see Section 4.4) has enabled the effective absorber to be calculated as a weighted average of contributions from a large number of elements of solid angle, according to the radiation sensitivity of each element.

TABLE 5.1.

| Telescope Geometry | Single cube (1 × 1 × 1m) | Duplex semi-cube (1 × 1 × 0·5m) | Single cube (1 × 1 × 1m) | Narrow-angle (1 × 1 × 3m) |
|---|---|---|---|---|
| Inclination of axis | 30°N | 0° | 45°S | 70°N |
| Approximate mean asymptotic latitude of viewing | 17°S | 39°S | 60°S | ~20°N |
| Average material absorber (m.w.e.) | 31 | 36 | 44 | ~33 |
| Average absorber, including atmosphere to 100 mb (m.w.e.) | 42 | 46 | 56 | ~60 |
| Particles/hour | 16,700 | 70,000 | 8,700 | 1,200 |

## 5.3. THE EVIDENCE FOR A SIDEREAL COMPONENT FROM THE ANNUAL AVERAGE EFFECT IN THE VERTICAL DIRECTION

The evidence presented under this heading relates to the interpretation of seasonal modulation of the daily variation underground. As explained in Section 2, the treatment differs from that given by others, in that the possibility of seasonal modulation of a sidereal component is examined. The reason for considering this possibility, in the first place, was that the observed behaviour of the anti-sidereal effect appeared to be incompatible with seasonal modulation of a solar component. Naturally, it is not suggested that the sidereal anisotropy itself would exhibit seasonal changes—what is of concern here is the response at the earth, where a sidereal anisotropy is expected to be viewed through the interplanetary medium, the spatial and temporal properties of which are not at all well known.

It will not be presumed to present answers to the complex problem of seasonal changes here. Rather, although it is proposed to outline the distinctive nature of seasonal modulation that may be inferred from the underground observations and to suggest a possible explanation that conforms with the present observations, the main concern here is to decide whether or not the observed annual apparent sidereal effect contains a significant spurious component.

Two aspects of seasonal modulation are examined. There are the seasonal changes that are effective over a particular year and there are the long-term trends in seasonal modulation that may become evident over a number of years of observation. These trends are observable to a marked degree in the underground data from Hobart. Consequently, an hypothesis should not only account for seasonal modulation that may apply over a given year, but should also accommodate the year to year trends.

*(a) Hobart, 1958*

The pressure-corrected daily variations averaged over the first complete year of operation of the vertical telescopes underground were notable for the large amplitudes in solar, sidereal and anti-sidereal time—0·126%, 0·090% and 0·052% respectively. This was the only year when the anti-sidereal effect was so pronounced that a quantitative treatment of seasonal modulation seemed to be worth attempting. In an analysis of this kind, it is necessary to determine rather precisely the constants specifying the seasonal variations of amplitude and phase of the total daily variation and to be able to apply realistic tests of theory against observation month by month. The analysis of observations presented in Paper 4 is now briefly reviewed.

It was found that the seasonal characteristics were more easily recognized if the total daily variation in solar time ($V_r$) for each month $r$ was subtracted from the daily variation for month $r + 6$, giving an average difference variation

$$Dr = \frac{V_r - V_{r+6}}{2}$$

as a monthly unit of information applicable to the first six months of the year (Paper 4, Section 2).

The annual average daily variations in solar, sidereal and anti-sidereal time, the monthly values of amplitude and phase of the total daily variation in solar time and the monthly values of amplitude and phase of the $D_r$ variation constituted a set of seven quantities characterizing the daily variation phenomena in 1958 (Figure 5.3). As mentioned in Paper 4, considerable smoothing was achieved with 6-hour running average deviations. The use of running averages arose from the need to obtain a large number of curves of fit to observed daily variations at a time when the facilities of a high-speed computer were not available. The monthly parameters have since been obtained from the harmonics of best fit to the daily variations of observed bi-hourly deviates. In Figure 5.4 the parameters obtained from the first harmonics and from the running averages are compared [(d) and (e)]. Sums of harmonics differ in giving rather better agreement with the phase changes in solar time and rather worse agreement with the phase changes in sidereal time. In (a), (b) and (c) of Figure 5.4 the annual daily variation curves drawn through the running averages are shown against the observed bi-hourly deviates. It appears that the method of running averages specifies the chosen characteristics of the daily variation in 1958 as accurately for our purposes as would other methods. It was proposed to attempt to reconstruct the situation in 1958, month by month, using facsimile daily variations to express various hypotheses which would predict markedly better than any other, the seven characteristics

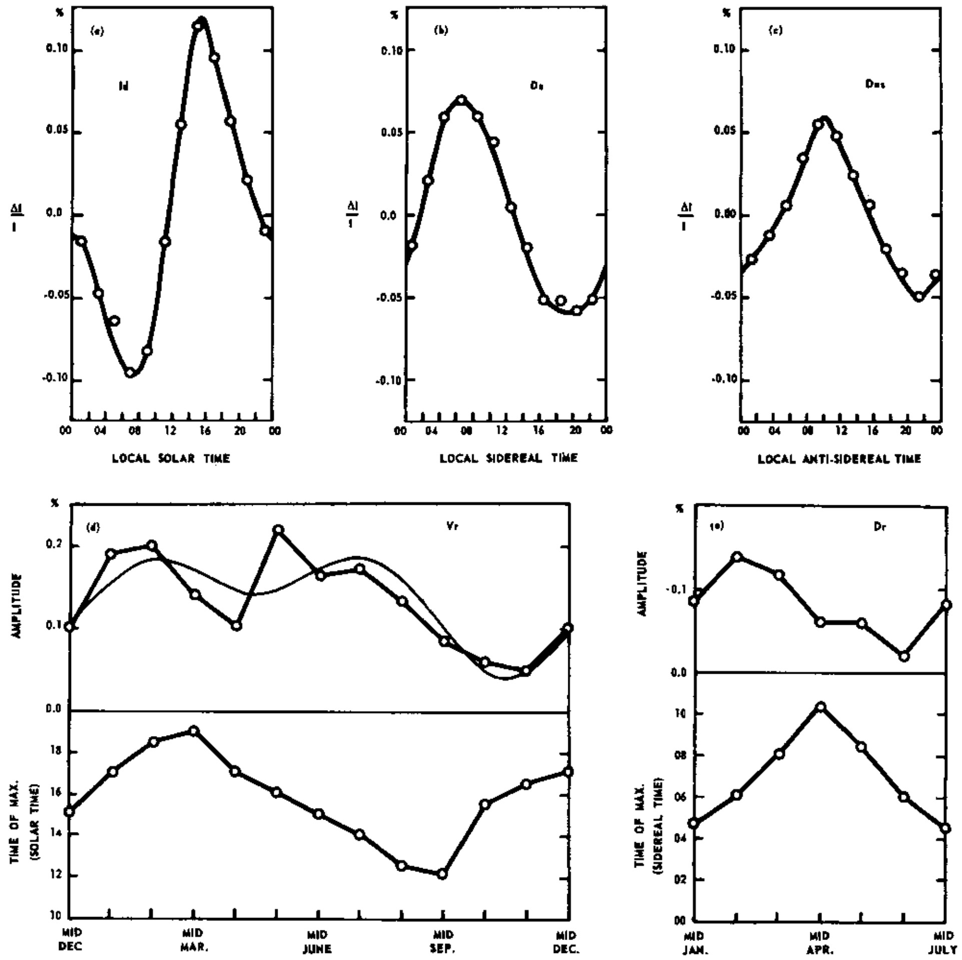



FIG. 5.3. Characteristics of the daily variation at 40 m.w.e. underground, Hobart, in 1958. The circled points are obtained after conversion of the observed bi-hourly mean deviations of intensity to 6-hour running means.

(a) The annual mean daily variation in solar time.

(b) The annual mean difference, $D_r$, variation in sidereal time.

(c) The annual mean $D_r$ variation in anti-sidereal time.

(d) Month by month mean values of amplitude and phase of the solar daily variation. A curve of best fit to the amplitude values is also shown.

(e) Month by month mean values of amplitude and phase of the $D_r$ daily variation in sidereal time.

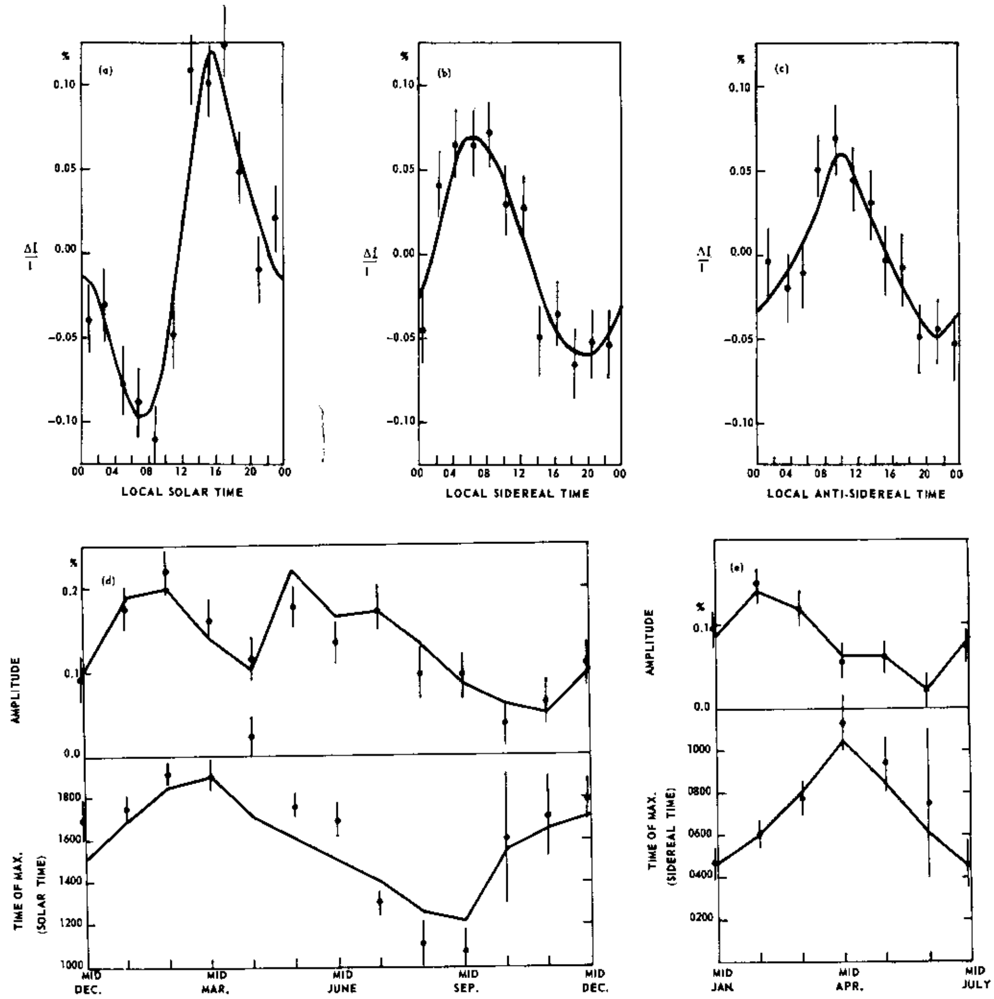



FIG. 5.4. Hobart, 1958. Characteristics of the daily variation of vertical intensity underground. Full lines are drawn through values obtained after the conversion of bi-hourly deviations of intensity to 6-hour running averages. For comparison, values obtained from the unsmoothed deviates are shown, together with the S.E.'s of estimate.

(a) The annual mean daily variation in solar time
(b) The annual mean difference, $D_r$, variation in sidereal time
(c) The annual mean $D_r$ variation in anti-sidereal time
(d) Month by month mean values of amplitude and phase of the solar daily variation. A curve of best fit to the amplitude values is also shown
(e) Month by month mean values of amplitude and phase of the $D_r$ daily variations in sidereal time.

shown in Figure 5.3. The hypotheses were of three types according as the observed sidereal effect was considered to be entirely spurious (I), partly spurious and partly genuine (II), or entirely genuine (III).

The particular hypotheses, describing the components of the total daily variation, were:

I. *Spurious.* I (a). A solar daily variation with combined annual sinusoidal variations of amplitude and phase.

I (b). A solar diurnal variation with combined annual sinusoidal variations of amplitude and phase, plus a constant solar daily variation.

II. *Partly spurious.* II (a). A solar daily variation with an annual sinusoidal variation of amplitude, plus a constant sidereal daily variation.

II (b). A solar diurnal variation with a sinusoidal annual variation of amplitude, plus a constant solar daily variation, plus a constant sidereal diurnal variation.

III. *Genuine.* III (a). A sidereal diurnal variation with semi-annual variation of amplitude and phase, plus a constant solar daily variation.

III (b). A sidereal diurnal variation with combined annual and semi-annual variation of phase, plus a constant solar daily variation.

Although only selected data were used (Section 4.8), the value of the analysis was limited by the observational uncertainties, due not only to statistical fluctuations, but to possible irregularities and secular changes that may have been produced by processes of various kinds. Again, the possibility of a second harmonic in the hypothetical carrier had to be considered, leading to the use of models (Paper 2.3) based on the observed solar daily variation, which contains an important second harmonic. Because of the presence of the second harmonic, the sidebands may differ considerably from those generated by modulation of a simple diurnal variation (e.g., see Figure 8 of Paper 4, relating to hypothesis II (a)).

Considering the above limitations, the fact that *hypothesis* III (a), relating to modulation of a sidereal component, accounted for the observations rather better than any other hypothesis was perhaps not very significant. It is the fact that it conformed with the observed characteristics, both seasonal and annual, as well as it did, that is important. The agreement may be seen in Figure 5.5, where the observational quantities shown in Figure 5.3 are compared with the predictions. Not shown is the annual mean solar daily variation, since it is not affected by this type of modulation. The solved characteristics of the phase- and amplitude-modulated sidereal diurnal component were:

unmodulated amplitude: 0·094%;
unmodulated time of maximum: 0700 LST;
degree of semi-annual modulation of amplitude: 80%;
maximum semi-annual displacement of phase: 3 hours;
months of minimum amplitude: December and June;
months of maximum advancement of phase: October and April.

Also of interest is *hypothesis* II (b), which would be invoked if the anti-sidereal effect were attributed to a diurnal variation of atmospheric temperature exhibiting an annual variation of amplitude. Best fit to the observations is obtained (Figure 5.6) if the amplitude-modulated solar component has the following characteristics:

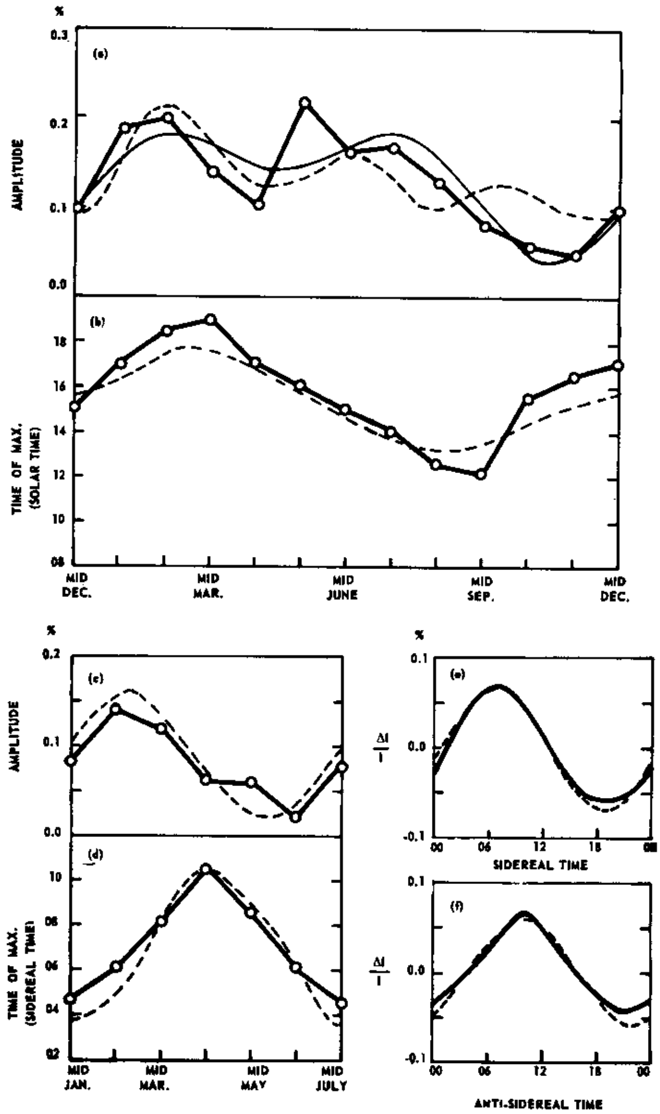



FIG. 5.5. *Hypothesis III.* The results of model analyses (dashed lines) compared with the observed characteristics of the daily variation. The values of the sidereal constants $k$ and $n$ used in the model are estimated from observations and the model is designed to give the correct values for the amplitudes and times of maximum of $D_s$ and $D_{as}$ and to reproduce $I_d$ identically. In (a) the curve of best fit to the amplitude variations is also shown.

amplitude: 0·120/k%, where k is the constant of seasonal modulation;
time of maximum: 2000 local solar time;
date of maximum amplitude: mid-February (Tasmanian summer).

As mentioned in Section 2, it seems to be generally agreed that maximum amplitude due to atmospheric temperature effects occurs in summer (e.g., Dorman 1965) and that the maximum of the cosmic ray diurnal variation due to the integrated temperature effect occurs at about 0600 local solar time (e.g., Quemby *and* Thambyahpillai 1960). As far as the positive temperature effect (connected with $\pi - \mu$ decay) is concerned, Thambyahpillai *et al.* (1965) report that the induced cosmic ray diurnal maximum would be expected to occur at $\sim$ 1300 local solar time in Britain. While amplitudes might be expected to vary considerably from place to place, the times of maximum of diurnal temperature effects should be relatively constant. Thus, there is a difficulty with the phase of the carrier if the anti-sidereal effect observed at Hobart is to be attributed to seasonal changes in a diurnal variation of atmospheric temperature. Perhaps the discrepancy should not be regarded too seriously, since as yet we have no direct knowledge of daily variations of upper atmospheric structure in Tasmania.

A more general difficulty connected with an interpretation based on hypothesis II (b) concerns the magnitude of the amplitude-modulated component. It is clear that, on the annual average, its amplitude (0·12/k%) would be greater than that of the total solar daily variation (0·13%) if seasonal modulation of amplitude were less than 100% (i.e., $k < 1$). Thus there would be a tendency for this component to have a controlling influence over the daily variation as a whole. This will be shown to be at variance with the very significant trends of the daily variation over the eight years 1958-1965 (Section 4 (b) below), and with the apparent influence of the solar anisotropy at this depth underground (Section 6).

*(b) Hobart, 1958-1965*

A description of the observations in the vertical direction over the years 1958-1962 has been given in Paper 5. Results from the subsequent three years have confirmed that very substantial changes in the component daily variations have occurred, apparently connected with changes in solar activity.

Figure 5.7 gives an impression in cross-section of some of the most important changes. The observations, in three biennial groups, represent in succession the situation near solar maximum (1958-59), during the declining phase (1960-61) and at the minimum of activity (1964-65). Curves of fit to the histograms of bi-hourly deviates are shown. Sums of harmonics, rather than first harmonics, are compared because of the evidence, to be discussed in later Sections, that both solar and sidereal anisotropies produce significant second harmonics. We consider first the solar daily variation.

In Paper 7 (Jacklyn *and* Humble 1965b; see also Section 6) the decrease in amplitude of the solar daily variation is largely attributed to the diminishing effectiveness of the solar anisotropy at high rigidities of response as solar activity declines. In a quantitative treatment it will be shown that the observed first harmonics in the vertical and inclined directions represent a realistic underground response to the free-space solar diurnal variation that was formulated by Rao *et al.*

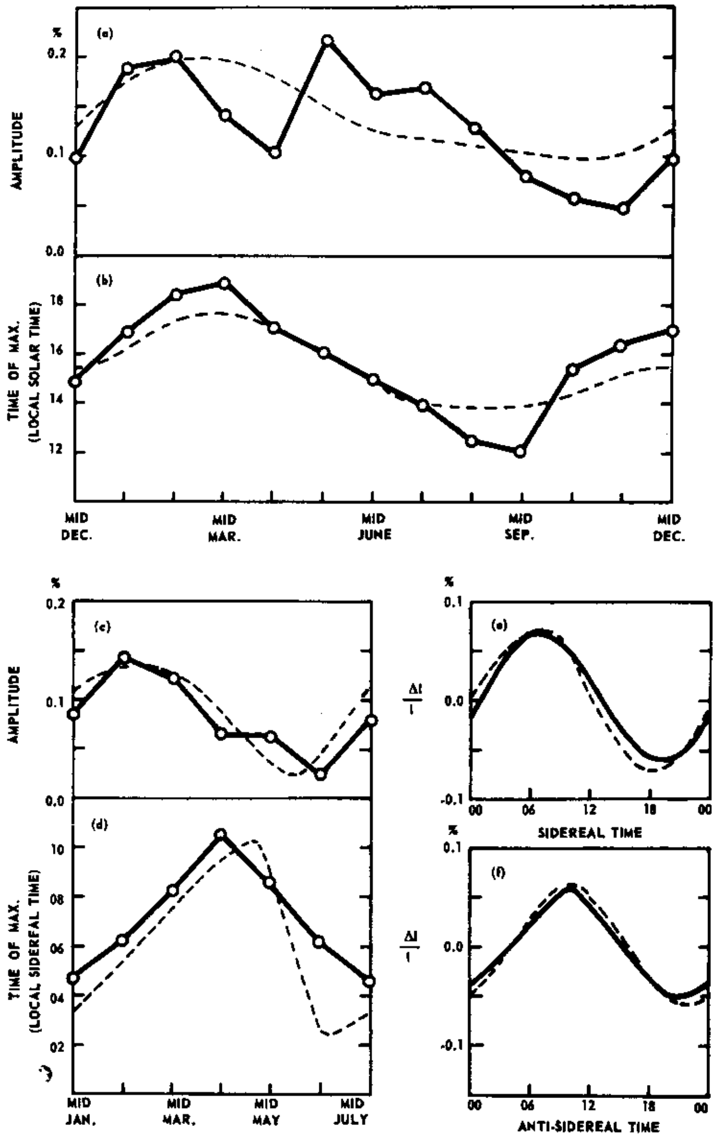



FIG. 5.6. *Hypothesis II (b).* The results of model analyses (dashed lines) compared with the observed characteristics of the daily variation. The model is designed to give the correct amplitudes and times of maximum $D_s$ and $D_{as}$ to reproduce $I_a$ identically.

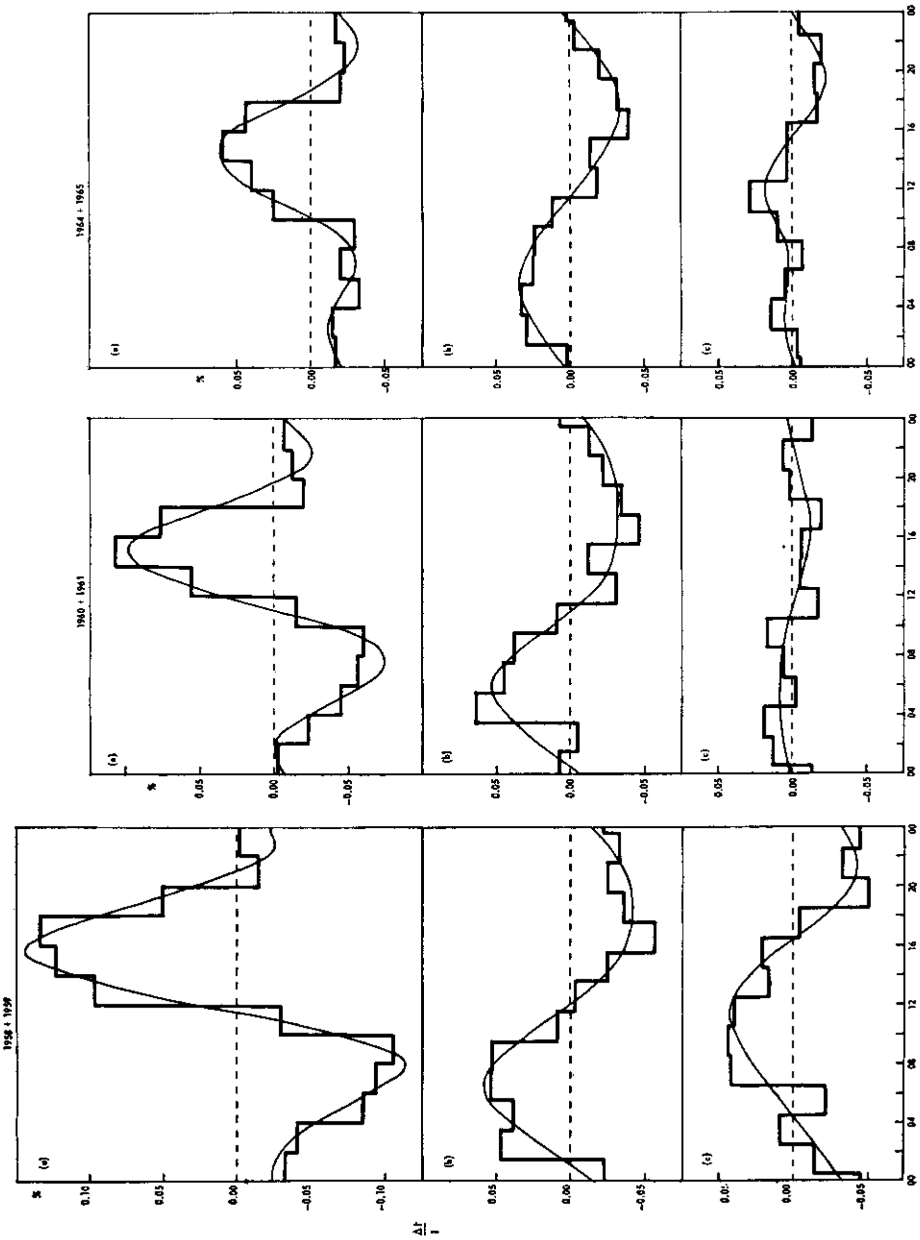



FIG. 5.7. Observed biennial daily variations of vertical intensity underground, Hobart, relating to three periods of interest between 1958 and 1965, in
(a) solar time
(b) sidereal time
(c) anti-sidereal time.
The curves of best fit to the histograms are the sums of first and second harmonics.

(1963) from observations with neutron monitors. It has also become apparent from the analyses of neutron intensity (McCracken *et al.* 1965; J. E. Humble, private communication) that the average direction of the anisotropy has not changed appreciably over the eight years 1958-65. Consequently, there should be no secular displacement of phase of the diurnal response underground except that which is due to the contraction of the range of primary rigidities over which the response is integrated. This produces a large change in amplitude but only a small displacement in the time of maximum, as Figure 2 of Paper 6 shows. With the proviso that the role of the second harmonic is not yet fully understood, the long-term behaviour of the solar daily variation indicated in Figure 5.7 can be explained without difficulty in terms of changes to be expected in the solar anisotropy between the maximum and minimum of activity. In the light of this, the observations are incompatible with the presence of a large diurnal temperature component whose seasonal changes might have explained the anti-sidereal effect in 1958. The temperature component vector would have to be balanced off by a residual diurnal vector whose amplitude progressively increases and phase angle decreases as solar activity declines, the time of maximum of the total daily variation remaining constant. Figure 5.8 shows the first harmonic vector triangles that would have to be satisfied for a minimal temperature effect.

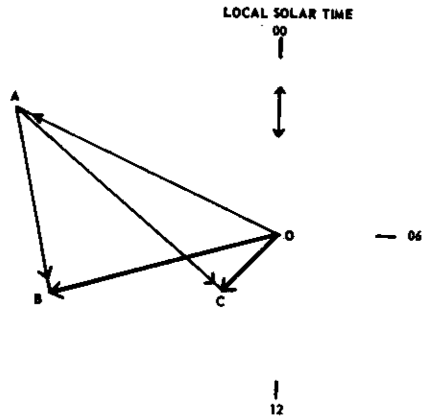



FIG. 5.8. Vector triangle showing the residual first harmonic vectors $\overline{AB}$ and $\overline{AC}$ that are obtained by subtracting an assumed temperature vector $\overline{OA}$ from the solar diurnal components of vertical intensity underground averaged over the respective biennial periods 1958-59 $(\overline{OB})$ and 1964-65 $(\overline{OC})$.

The discussion of the temperature effect will not be carried any further here, since we now wish to consider other evidence from Figure 5.7 that tends to refute the possibility of a significant spurious sidereal effect, whether this comes from a temperature-induced diurnal variation or from any other kind of modulated solar component.

The sidereal and anti-sidereal effects observed in 1958-59 and in 1960-61 are compared. We note first that a very significant anti-sidereal component was observed in 1958-59 (the SE of amplitude of individual harmonics was ± 0·005%), whereas in the average of the years 1960-61 the effect was almost completely absent. The change was by no means erratic. The amplitude decreased quite rapidly after 1958 and by 1960 had reached the low level shown for the two years 1960-61, and, as the result for 1964-65 indicates, has remained at approximately that level ever since. We next note that the amplitudes of the apparent sidereal and anti-sidereal components were comparable over the two years 1958-59, the ratio anti-sidereal/sidereal being 0·75, whereas in 1960-61 the ratio was approximately 0·2. The point we now wish to make is this: if the apparent sidereal daily variation in 1958-59 contained a significant spurious component, comparable to the anti-sidereal effect, it should have been evident from the parameters of the sidereal daily variation in 1960-61 that the spurious component has been removed. There is no indication whatever of this. Except for a slight difference of amplitude, it is clear that the sidereal effect observed in each of the two periods was essentially the same. There is no difference either in time of maximum or in the relationship between the harmonics (the individual harmonics are discussed in Section 5.6 below and are shown in Figure 5.16). In fact it is evident that, if the spurious sidereal effect in 1958-59 is obtained as the difference between the two observed sidereal effects, it must be negligible.

In seeking an explanation of the anti-sidereal effect in 1958-59 that would be compatible with later observations, the only possibility that seems worth considering seriously at present is that of semi-annual modulation of a sidereal component. Unfortunately, there is a lack of useful data from other latitudes of viewing at the peak of the effect. One possible mechanism for semi-annual modulation of amplitude will be suggested at the end of this Section (5.7) but little can be said with confidence until a more complete description of the effect is obtained. It is considered that this must be an important objective in a continuing experimental study of the sidereal daily variation.

A general impression of the changes that have occurred in the component daily variations is given in the month to month sequence of annual running averages of amplitude (Figure 5.9) and phase (Figure 5.10). An individual pair of values of amplitude and phase is obtained from the curve drawn through the set of twelve 6-hour running average deviates that represents an annual daily variation. Some of these curves are shown in Figures 3.2 to 3.10.

It may be of interest to compare annual running averages obtained from smoothed annual daily variations with those derived from harmonic analyses of unsmoothed hourly deviates. The comparison relating to the solar daily variation in which the long-term changes were most significant, is shown in Figures 5.9 (a) and 5.10 (a). As to be expected, the amplitudes obtained from 6-hour running average deviates are smaller than the amplitudes of sums of harmonics of fit to the unsmoothed deviates, but the same trends are followed in each case. The sequence of diurnal amplitudes is also shown and it is clear from the SE tails that the values of the parameters, in solar and sidereal time at least, could not have been greatly affected by statistical fluctuations.

Reasons for using annual running averages were briefly mentioned earlier (Section 3.2). In the short run they are liable to give a false impression, irregular

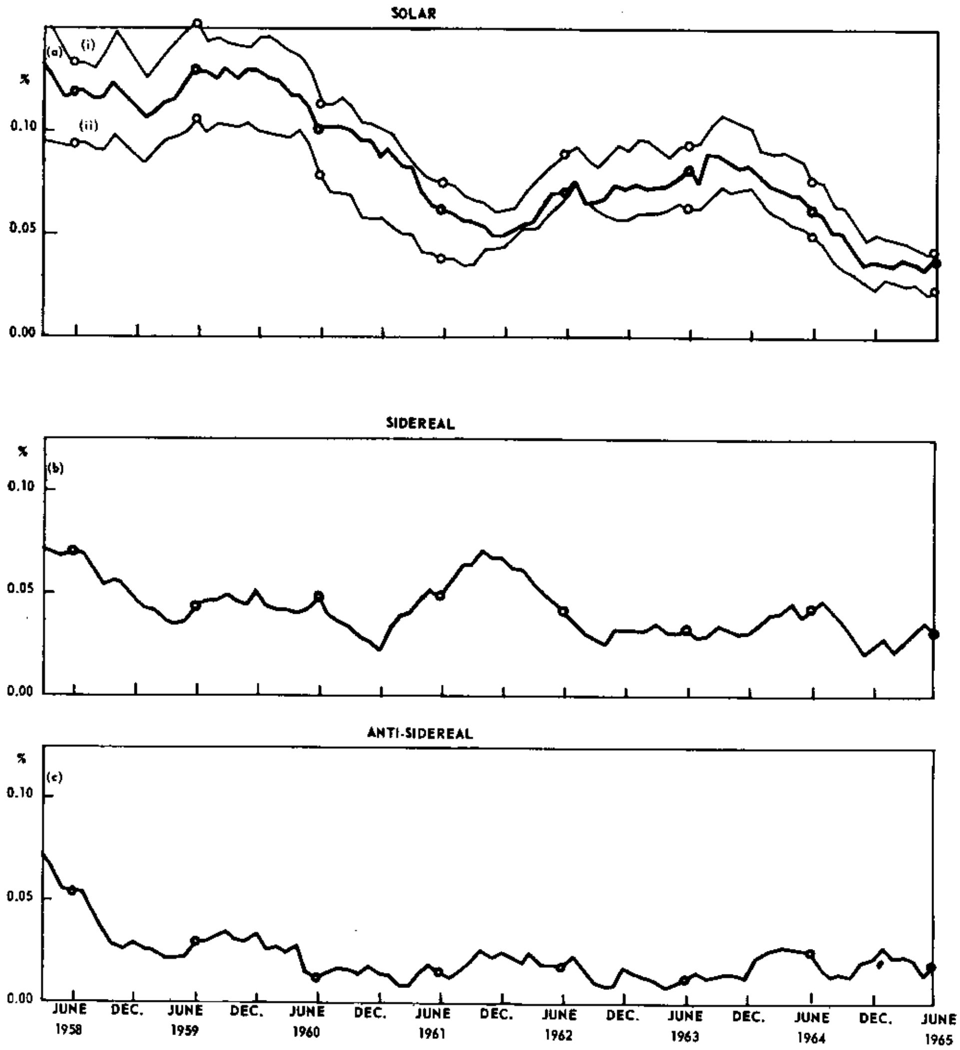



FIG. 5.9. Twelve-month running averages of amplitudes of the daily variations in solar, sidereal and anti-sidereal time observed in the vertical direction underground. The dates refer to the centre-months and the circles indicate the conventional annual mean values.

Thick lines—amplitudes derived from smoothed bi-hourly deviates (see text).

Thin lines—amplitudes in solar time (i) from the sums of first and second harmonics of best fit to unsmoothed hourly deviates and (ii) from the first harmonics.

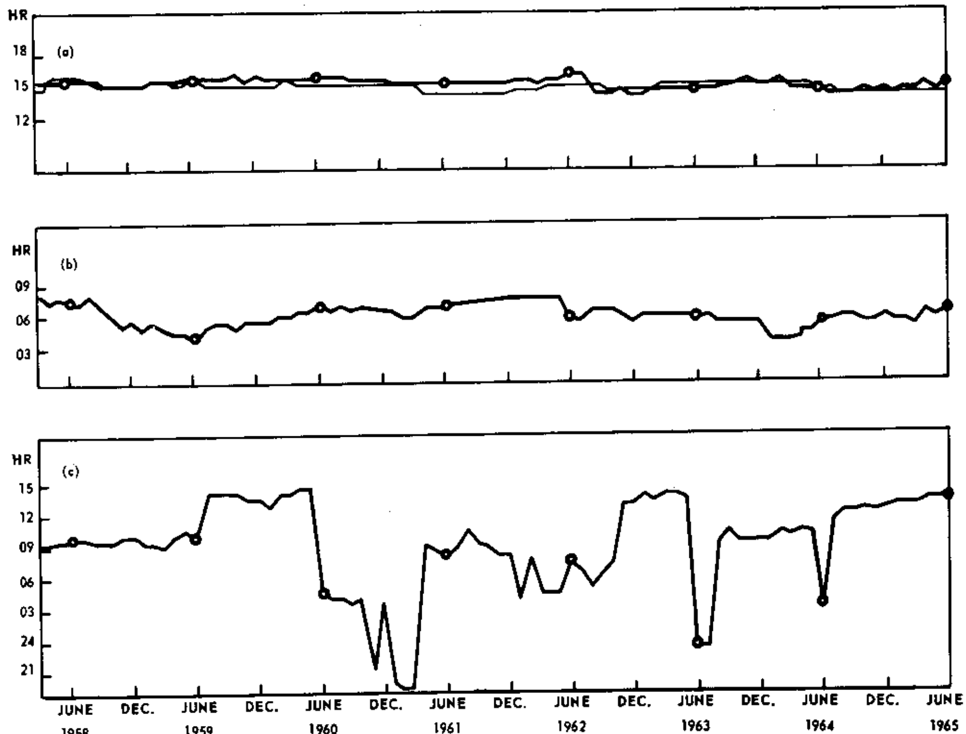



FIG. 5.10. Twelve-month running averages of times of maximum of the daily variations in solar, sidereal and anti-sidereal time observed in the vertical direction underground, corresponding to the amplitudes shown in Figure 5.9. Thin line—times of maximum of the solar daily variation from the sums of first and second harmonics of best fit to unsmoothed hourly deviates.

changes tending to appear as quasi-persistent features. However, taken over a number of years, running means portray long-term changes more effectively than do the independent annual averages. They answer the following question: would trends that might be apparent in the set of independent averages (eight in our case) have been different if the averages had been centred on another month of the year? In the case of the amplitudes of the solar daily variation it is evident that the trend indicated by the annual averages centred on June (circles) would not have differed greatly if the averages had been centred on any other month. On the other hand, independent annual averages would have represented trends in amplitude of the sidereal effect rather differently if they had been centred on October as against June. In fact, it appears from the running averages that there was a transient maximum of the sidereal effect in 1962, superimposed on the tendency for a gradual decline of amplitude.

From the point of view of seasonal modulation, the most important of the long-term changes that must be taken into account have already been discussed. We note the features of relevance in Figures 5.9 and 5.10:

(i) The great decrease in amplitude of the solar daily variation and the constancy of the time of maximum.

(ii) The less clearly defined but, by 1965, significant decrease of amplitude of the sidereal daily variation, associated with an essentially constant time of maximum, within the limits of a year to year variability.

(iii) The attenuation of the anti-sidereal effect to a level of amplitude that was generally $< 0{\cdot}02\%$ after 1960. The annual histograms (Figure 5.7) show perhaps more clearly that this effect becomes small and irregular while the sidereal daily variation, like the solar daily variation, remains well-defined and of uniform character.

### 5.4. PHASE ANOMALIES

In Figure 5.11 the graphs of annual running averages of amplitude of the solar daily variation (Figure 5.9 (a)) and of the apparent sidereal daily variation (Figure 5.9 (b)) are referred to the same origin. It will be noted there were two periods when the averaged sidereal amplitude either exceeded the solar amplitude or was only slightly less than it, namely a period of about eight months following June 1961 and a period of about one year following June 1964. The first period in particular would be expected to give important evidence as to the nature of the apparent sidereal effect. For if the latter were not spurious, then some time late in 1961 the sidereal component should have commenced to dominate the observed monthly average daily variation. It would then depend on the phase relationship between the solar and sidereal components at that time of the year as to whether an apparently enhanced solar daily variation would be observed at first (both components in phase) or whether there would be a sudden large displacement of phase as the net daily variation became orientated in sidereal time (components out of phase). The period following June 1964 might also present opportunities of observing the dominant effect of the sidereal component provided that there was assistance from relatively modest seasonal or irregular changes. If, on the other hand, in spite of the evidence already given to the contrary, the apparent sidereal effect was essentially spurious, we should expect to observe during both periods only enhanced seasonal changes in a small solar daily variation.

The sequences of amplitude and phase of the observed monthly average daily variation (arranged in solar time) are shown in Figure 5.12. The well-defined phase anomaly late in 1961, the absence of this effect at the same time of the year in 1962 and 1963 and the incomplete effects in 1964 and 1965 are clearly in accord with the trends of amplitude shown in Figure 5.11 if the annual amplitudes in sidereal time are attributed to a genuine sidereal daily variation. Confirmation is given by the equally good accord with the annual times of maximum depicted in Figures 5.10 (a) and (b). A genuine sidereal daily variation with a maximum at opproximately 0700 local sidereal time and a solar variation whose maximum occurs at approximately 1500 local solar time become 12 hours out of phase with each other in November. It is noted that on the occasions of the anomalies the large displacement of phase took place either in October or November. In fact, as the dashed line in Figure 5.12 indicates, the time of maximum of the daily variation during the phase anomaly in 1961 followed 0700 local sidereal time quite closely. Supposing the sidereal diurnal maximum to have occurred at 1900

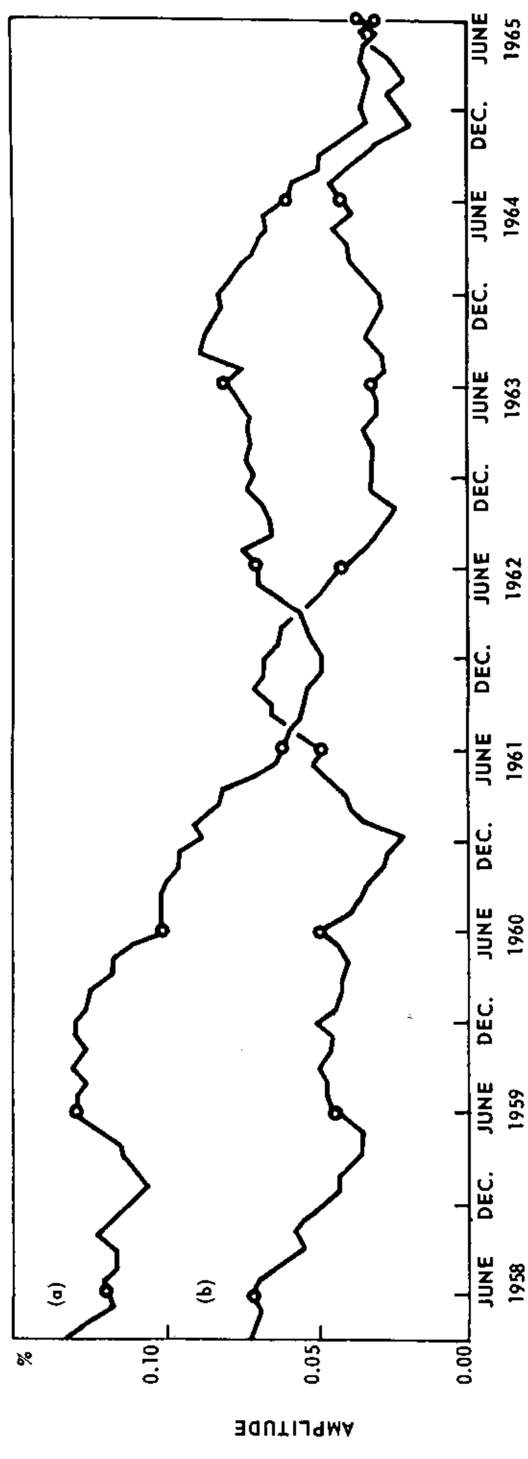



FIG. 5.11. Twelve-month running averages of amplitude of (a) the solar daily variation and (b) the apparent sidereal daily variation, reproduced from Figure 5.9 and referred to a common ordinate.

sidereal time, on the other hand, the phase anomaly would have commenced in April.

It is submitted that these phenomena not only constitute strong evidence for a genuine sidereal component in the daily variation, but indicate also that any spurious contribution to the annual apparent sidereal effect must have been very small.

An analysis of the phase anomaly in 1961 is given in Paper 5 where it is shown that, when suitable models for the solar and sidereal components are used, the significant features of the daily variation in 1961 are reproduced.

The only other known instance of a phenomenon that was apparently of this kind relates to observations at ground level in the northern hemisphere during 1954. The effect, reported by Conforto and Simpson (1957) has been discussed in Section 3.1. Monthly average daily variations of both meson and neutron intensities exhibited progressive displacements of phase towards earlier values in the manner of a sidereal component with a diurnal maximum at approximately 2000 local sidereal time. In Figure 5.13 the effect on the daily variation of meson intensity at Rome is compared with the phase anomaly observed at Hobart in the daily variation of meson intensity underground during 1961-62.

As mentioned in Section 3.1 the time of maximum of the annual average solar daily variation advanced considerably after 1950 and reached a turning point in 1954. Thereafter it began to return to later values. There seems little doubt that important changes in the solar daily variation of extra-terrestrial origin had occurred, because of the effect on the neutron component at Huancayo and Climax where the end-points of the annual running average diurnal vectors exhibited an anti-clockwise rotation between the years 1953 and 1955 (Glokova 1960). Glokova has shown that the mean-year solar diurnal variation of meson intensity did not behave in quite the same way and evidently consisted of a varying component due to the solar anisotropy and a constant component of atmospheric origin. The latter would appear to have been consistent with a diurnal variation due to atmospheric temperature as deduced by Quemby *and* Thambyahpillai (1960) from an analysis of the nucleonic and hard components at Huancayo. The estimated temperature component at Huancayo had a maximum of 0·11% at 0540 local time.

It would seem from the above considerations that greatly weakened solar modulation of the primaries was responsible for the small and complex *mean-year diurnal variation* of neutron intensity, while the corresponding solar diurnal variation of meson intensity tended to come under the control of a constant component (due to atmospheric temperature) as the extra-terrrestrial contribution weakened. On the other hand, although the *monthly average diurnal* variations were probably influenced by changing solar modulation and (in the case of the hard component) by seasonal changes in the temperature effect, they appear to have been more substantially affected by a sidereal component.

There was considerable variation of individual response in respect of the phase anomaly of 1954. This would probably depend in the main on the nature of the temperature vector at the place of observation (for mesons), on the primary cut-off rigidity of the detector, and on the spatial characteristics of the two anisotropic components, particularly their respective latitude dependences of amplitude. As we

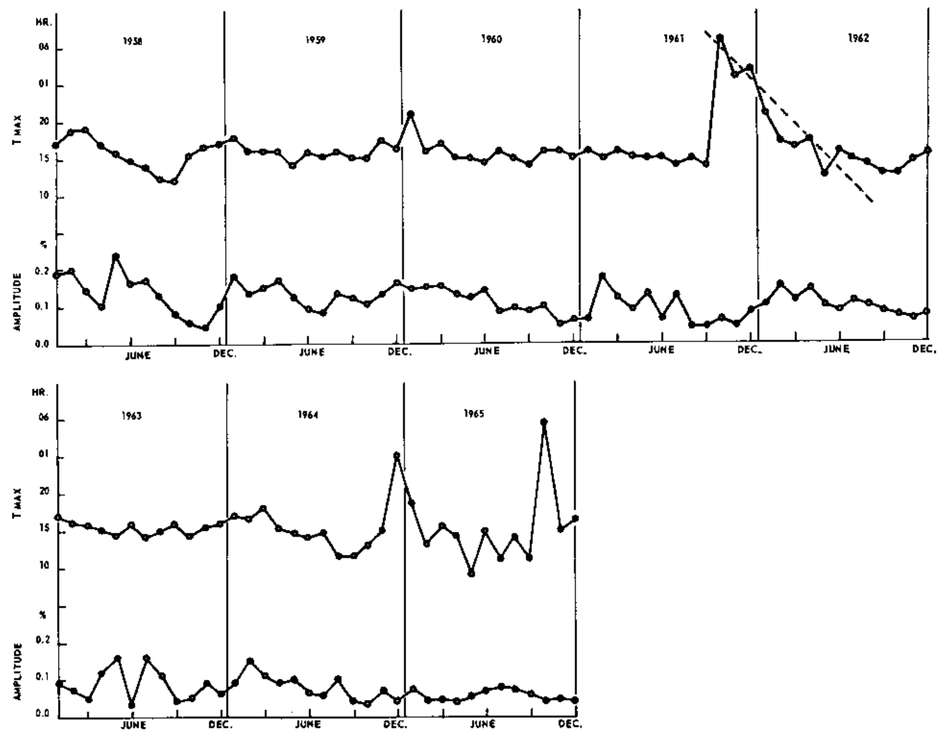



FIG. 5.12. Monthly mean values of the time of maximum (in local solar time) and amplitude of the daily variation of vertical intensity underground, Hobart. The dashed line associated with the anomalous times of maximum in 1961-62 represents 0700 local sidereal time.

have seen, the anomalous variations of phase in the hard component at Rome followed constant sidereal time quite well, perhaps in consequence of the rather high cut-off rigidity at that place and a sufficiently small diurnal variation due to atmospheric temperature. In contrast to this, the anomaly was not seen in the hard component at ground level at Hobart. The reverse situation on a later occasion was evidently connected with the fact that the average amplitude of the sidereal effect observed underground in the southern hemisphere was greater than that in the northern hemisphere by a factor of about two. The point will be discussed in Section 5.5 where annual averaged observations from the two hemispheres are compared. In later Sections the unsymmetrical nature of the latitude dependence of amplitude will be considered more closely, being one of the major features of the sidereal effect observed underground.

One cannot discuss the latitude dependence of amplitude in 1954 with confidence because of the difficulty of assessing the nature of the apparent sidereal effect in the hard component at sea level. However, it may be of interest to refer once again to Figure 3.1 which shows that over the period 1947-54 the amplitude of the apparent sidereal effect in the ion chamber data at Cheltenham was greater than it was at Christchurch, while the reverse was the case during the period 1938-1946. If a genuine sidereal component had been responsible for these events the

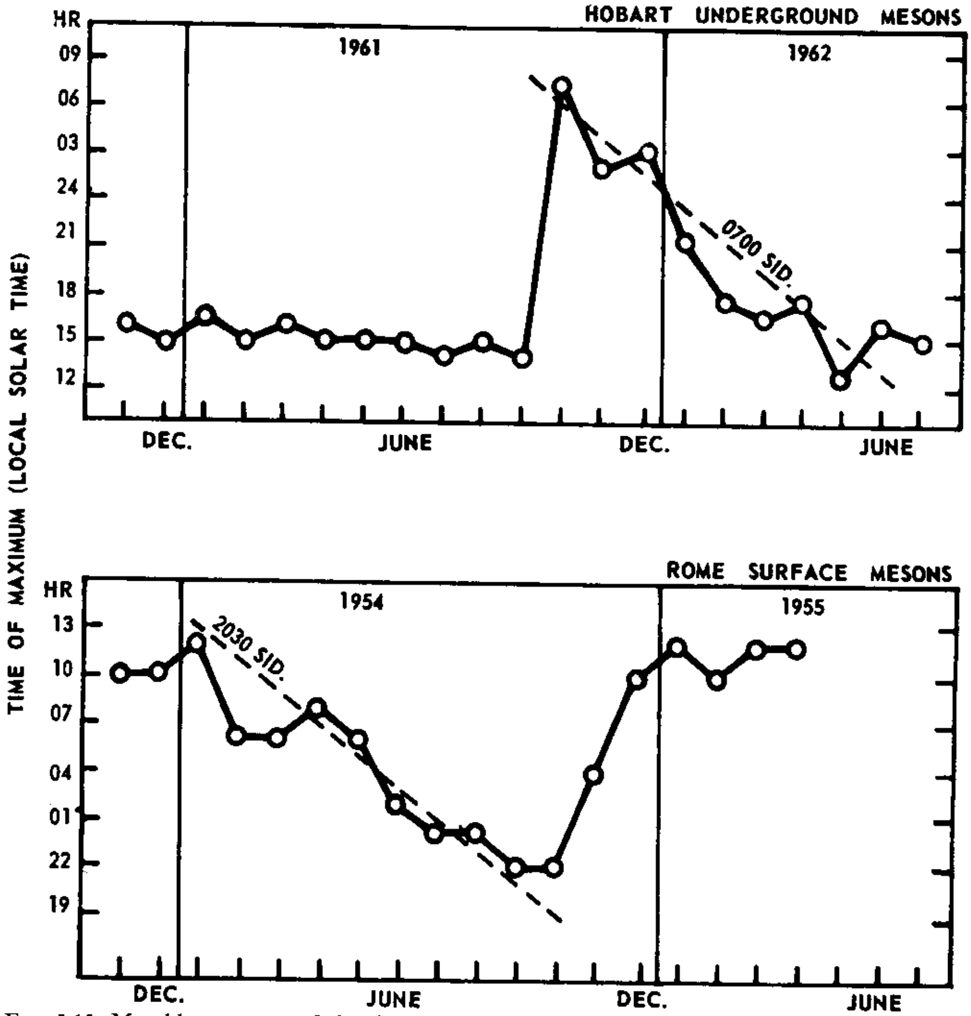



FIG. 5.13. Monthly averages of the time of maximum, in solar time, of the pressure-corrected daily variation of vertical meson intensity at the Hobart underground site (1961-1962) and at Rome (1954). The dashed lines represent the solar time equivalents of 0700 local sidereal time and 2030 local sidereal time as shown.

absence of the phase anomaly at Hobart in 1954 would appear to have a simple explanation. It is considered that there is sufficient in the evidence to suggest that the type of asymmetry in the latitude dependence of amplitude observed in the recent underground data should not necessarily be regarded as a constant feature.

It is noted that the time of maximum of the sidereal component deduced from the phase anomaly in 1954 agrees well with the time of maximum (approximately 2000 sidereal time) of the annual apparent sidereal effect averaged over a large number of years (see Figure 3.1 relating to the ionization measurements at Cheltenham). Thus, while the phase anomalies give evidence of a genuine sidereal component, acceptance of the evidence requires that there be a 12-hour phase displacement depending on whether the observations are made in the northern or the southern hemisphere. We now proceed to discuss other evidence, some of which indicates unequivocally that the 12-hour phase difference does indeed relate to a genuine sidereal daily variation.

### 5.5. THE 12-HOUR DIFFERENCE OF PHASE OBSERVED IN THE SIDEREAL MAXIMA FROM OPPOSITE HEMISPHERES

Among the many observations that have accumulated from experiments in the northern hemisphere at ground level there is an overall tendency for the maximum of the apparent sidereal effect to occur at approximately 2000 local sidereal time. The variability of these observations, and of observations at ground level generally, has been noted in Section 3. It seems likely that the spurious effects, due severally to secular, seasonal and irregular changes in the large solar component, are frequently important and would be very difficult to extract. This might raise considerable doubt as to the genuineness of even the more persistent effect at ground level. Thus it would be reasonable to suspect that the difference of roughly 12 hours between the apparent sidereal diurnal maxima at Christchurch (43·5°S) and Cheltenham (38·70°N) from ion chamber measurements (see Figure 3.1) was a sideband effect, due to seasonal modulation of the solar daily variation. Nevertheless, the average time of maximum from Christchurch was not very different from that observed underground at Hobart, while, as we noted in the previous section, the time of maximum from Cheltenham agreed quite well with that deduced from the phase anomaly in 1954. Thus there was clearly a suggestion that a genuine sidereal daily variation may have been responsible for the 12-hour phase difference.

In an examination of this aspect of the sidereal effect it is obviously desirable to have simultaneous observations from similar detectors located at approximately the same depths underground in the separate hemispheres. For the purpose of comparison with the underground observations at Hobart it was most fortunate that data were available from two (and later, three) semi-cubical vertical telescopes situated at a depth of approximately 40 m.w.e. underground at Budapest (47·5°N, 18·9°E geographic). Each telescope consisted of four trays, giving fourfold coincidences and was of dimensions 1·264 x 1·264 x 0·632 m. From February 1958 to August 1961 the duplex counting rate was approximately 100,000 pcles/hr. Thereafter three telescopes were used, with a combined counting rate of approximately 160,000 pcles/hr.

The only complete calendar years for which continuous records were available from Budapest at World Data Centre WDC2 were the years 1959 and 1961. The results of harmonic analyses of the daily variations in sidereal time observed at Hobart and Budapest over the two years are listed in Table 5.2. In Figure 5.14 the individual harmonics are shown against the histograms of the bi-hourly deviates. The 12-hour phase difference between the first harmonics is clearly indicated, the actual difference being 11 hours ± 50 minutes (SE). The sums of harmonics also exhibit this difference, the first and second harmonics being in phase.

Observations from the two underground cubical telescopes inclined 30°N and 45°S of the zenith at Hobart will be discussed in later Sections. It is noted here that significant daily variations in sidereal time were observed in the two directions

TABLE 5.2.

*Hobart and Budapest.* Harmonics of best fit to the apparent sidereal daily variations of pressure-corrected bi-hourly deviates, averaged over the two years 1959 and 1961.

| | 1st harmonic | | 2nd harmonic | | Sum of harmonics | |
|---|---|---|---|---|---|---|
| | Amplitude % | $T_{max}$ hr | Amplitude % | $T_{max}$ hr | Amplitude % | $T_{max}$ hr |
| Hobart SE | 0·036 ±0·004 | 0520 ±0020 | 0·012 ±0·004 | 0500 ±0120 | 0·048 | 0520 |
| Budapest SE | 0·023 ±0·004 | 1830 ±0040 | 0·006 ±0·004 | 1900 ±0240 | 0·028 | 1830 |

and that the times of maximum, whether of first harmonics or sums of harmonics, conformed with the result from the vertical direction. Averaged over the two years of operation, 1961 and 1962, the sidereal diurnal maximum in the north-pointing direction occurred at 0700 ± 0040. In the south-pointing direction the diurnal maximum, averaged over the four years 1961-1964, occurred at 0600 ± 0110.

A result which corroborates the observations from Budapest has recently been reported from London (Thambyahpillai *et al.* 1965) where two (and later, six) semi-cubical vertical scintillator telescopes, each of detecting area 1·44 $m^2$, had been installed at a depth of approximately 60 m.w.e. underground. The time of maximum of the observed sidereal diurnal variation, averaged over three years from 1961 to 1964, was 1800 ± 55 minutes, differing by 12 hours from the time of maximum at Hobart averaged over the eight years 1958-1965.

Table 5.3 summarizes the times of occurrence of the observed sidereal diurnal maxima that have been discussed here. From the point of view of reproducibility and statistical accuracy this is believed to be the best information available.

The 12-hour phase difference in the underground observations is unmistakable.

TABLE 5.3

Times of maximum of the first harmonics of daily variations in sidereal time, from pressure-corrected data, averaged over the years indicated.

| | Time of maximum |
|---|---|
| *Northern hemisphere* | |
| Cheltenham (38·70°N geog.) sea-level ionisation 1938-1946 (9 years) | 2120 |
| Cheltenham sea-level ionization 1947-1954 (8 years) | 2110 |
| Budapest (47·5°N geog.) vertical semi-cubical telescopes at ~ 40 m.w.e. underground, 1959, 1961 (2 years) | 1830 |
| London (50°N) vertical semi-cubical telescopes at ~ 60 m.w.e. underground, 1961-1964 (3 years) | 1800 |
| *Southern hemisphere* | |
| Christchurch (43·5°S geog.) sea-level ionization 1938-1946 (6 years) | 0630 |
| Christchurch sea-level ionization, 1947-1954 (8 years) | 0750 |
| Hobart (43°S geog.) ~ 40 m.w.e. underground | |
| vertical semi-cubical telescopes 1958-1965 (8 years) | 0610 |
| cubical telescope inclined 30°N 1961-1962 (2 years) | 0700 |
| cubical telescope inclined 45°S 1961-1964 (4 years) | 0600 |

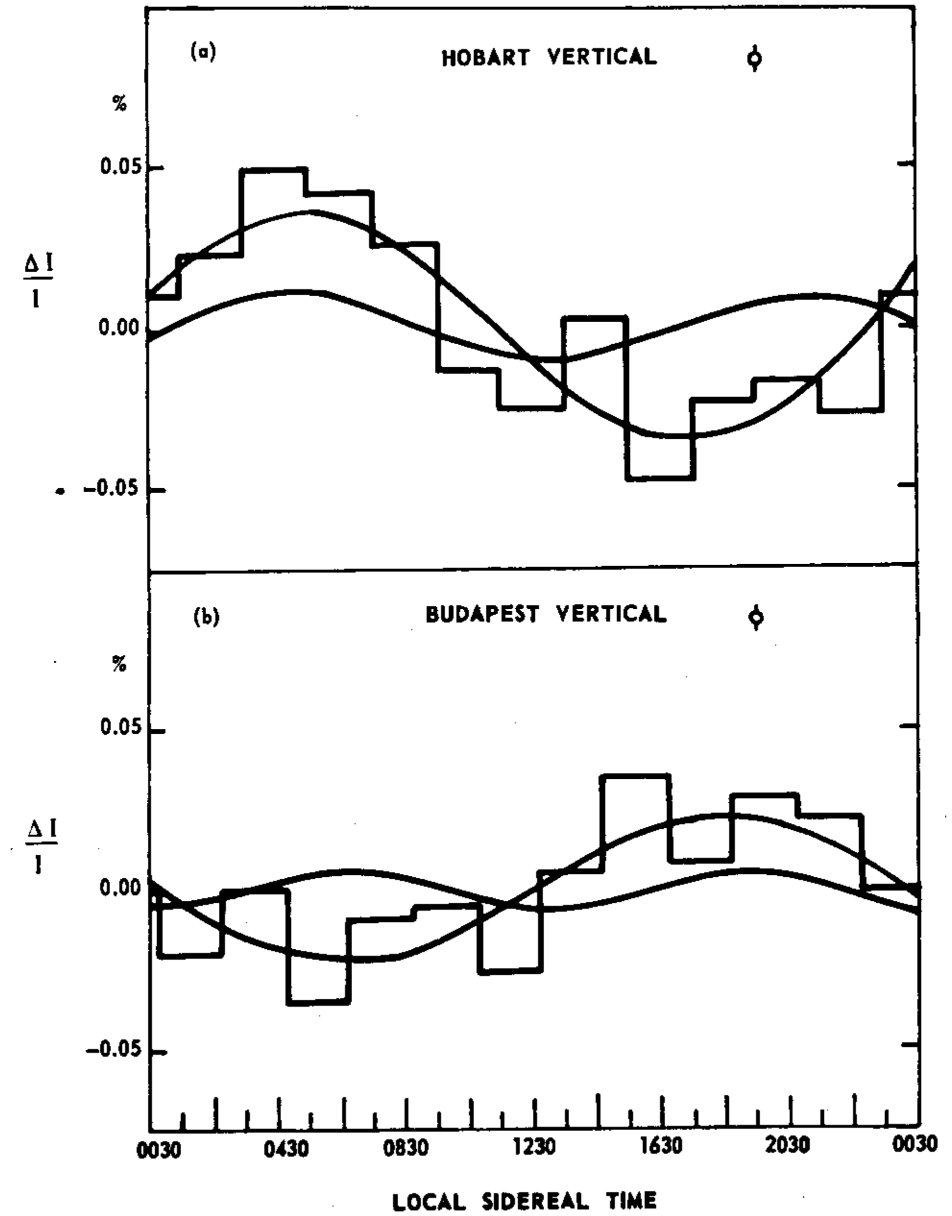



FIG. 5.14. The observed daily variation in sidereal time, averaged over the years 1959 and 1961, of pressure-corrected meson intensity from vertical semi-cubical telescopes at a depth of ≈ 40 m.w.e.

(a) at Hobart, scanning asymptotic latitudes in the vicinity of 39°S geographic

(b) at Budapest, scanning asymptotic latitudes in the vicinity of 44°N geographic.

The error tails shown are the S.E.'s of amplitude associated with the first and second harmonics of best fit.

From the Cheltenham-Christchurch comparison a similar effect at ground level is suggested, although there appears to be a tendency for the maxima to occur later and for the phase difference to be somewhat less than 12 hours.

Considered aside from any other evidence, the 12-hour phase difference underground demonstrates that the apparent sidereal daily variation is controlled either by seasonal modulation of a solar component or by a two-way sidereal anisotropy or by a combination of these two phenomena. (Note that a uni-directional sidereal anisotropy could not have been responsible for the effect.) If we now add the evidence indicating that the observed sidereal effect in the vertical direction at Hobart was essentially genuine it would appear that a two-way anisotropy was exerting the controlling influence and that the sidereal effects at Budapest and London were essentially genuine also.

The above conclusion would have been strengthened if the absence of significant spurious components in the underground observations from the northern hemisphere could have been demonstrated independently of the 12-hour phase difference and the observations from Hobart. However, some of the significant indications of a genuine sidereal effect at Hobart were not to be found in the available observations from the northern hemisphere, and it appears that this was because the sidereal amplitudes were smaller. (The amplitude at Budapest was about 60% of the amplitude at Hobart, while at London the amplitude was less by a factor of at least two, but was not directly comparable, having been obtained at an considerably greater depth.) Notably, a phase anomaly was not observed at Budapest (or at London) in 1961, although it was evident from the available data that the solar daily variation at Budapest was the same as at Hobart and exhibited the same large changes (e.g., see Table 6, Paper 7). Thus the non-appearance of the phase anomaly seems to have been directly attributable to the much smaller sidereal effect. Again, an analysis of the anti-sidereal effect would be handicapped, in the first place, because of the small observed amplitude ($\sim$ 0·015% both at Budapest and London). In the second place, there were not, as far as is known, substantial year to year changes of the kind which would either indicate the presence of the appropriate solar carrier and annual sidereal sideband to confirm suggested modulation of a solar daily variation, or indicate the absence of these if the anti-sidereal effect had been largely caused by modulation of a sidereal daily variation.

It was desirable, then, to obtain supporting evidence that the 12-hour phase difference was of sidereal origin by some quite different method. It was therefore decided to set up a telescope inclined in the plane of the meridian so as to view asymptotically entirely within the northern hemisphere from the underground laboratory at Hobart. If the phase difference that had been observed between the two hemispheres was a genuine sidereal effect there should be a displacement of approximately 12 hours between the annual apparent sidereal maxima observed in the vertical and inclined directions. As distinct from this, the maxima from these two directions should be approximately in phase, (1) if the results from Hobart and Budapest in the vertical direction had both been spurious as the result of seasonal modulation of a solar component, or (2) if the result from Hobart in the vertical direction had been genuine, but not due to a two-way anisotropy, while the result at Budapest had been spurious, or vice versa. Thus, in principle, the experiment would be able to distinguish unambiguously between a 12-hour phase difference of sidereal origin and one that was either partly or completely produced by spurious sidereal components resulting from any kind of local seasonal modulation of a solar daily variation. A narrow-angle telescope was set up underground in the direction 70°N of zenith (see 5.2 above) and was put into operation towards the end of 1963. The location of the asymptotic cone of acceptance is indicated sufficiently well from the following considerations. The total geometric aperture being 36°, both in zenith angle and azimuth, the estimated meson cut-off energy at production increases from approximately 14 GEV to 70 GEV between the lower and upper limits of zenith angle acceptance. (The values may be calculated in the manner described in Section 4.6, except that the energy loss must be determined for a curved atmosphere at high zenith angles. For near-horizontal incidence the atmospheric energy loss given by Wilson (1958) has been used.) From the coupling coefficients applicable to the production of mesons of energies $>$ 15 GEV in the vertical direction (see Section 6) it follows that the mean energy of response to the average primary spectrum in the inclined direction must be considerably in excess of 100 GEV, and we can be confident that this would also be true of the response to the variation spectrum of a sidereal anisotropy. Charged primaries of these energies arriving from the north at the latitude of Hobart experience only small deflections in the terrestrial magnetic field. It can be seen from the diagrams of Brunberg *and* Dattner (1953) that deflections would not be greater than about 3° for all directions of arrival within a cone of half angle 18° centred on the zenith angle 70°N. Consequently, the asymptotic cone is centred only a few degrees southward, in latitude, and eastward, in longitude, of the geometric cone of acceptance. Latitudes of viewing which range from 9°N to 45°N within the geometric cone are therefore estimated to range from about 7°N to 42°N in the asymptotic cone. While the boundary cannot be precisely defined because of scattering in the material absorber, particularly of those mesons which are near the end of their range on arrival at the equipment, it is clear that the asymptotic cone must be confined essentially to the northern hemisphere, being centred at about 25°N geographic latitude and not more than 3°E of the meridian through Hobart. On the other hand, when the integrated cone of acceptance for a semi-cubical vertical telescope is calculated for a sidereal anisotropy, it is found to be centred approximately 10°E of the meridian at the latitude 39°S (Section 6). Therefore, the resulting distortion due to this cause in the observation of a 12-hour phase difference of sidereal origin, is estimated to be less than half an hour.

Although only a qualitative result was to be expected from the experiment, it was hoped that it would clearly distinguish between the two possibilities. In the event of a spurious sidereal effect, due to a modulated solar daily variation, the first harmonics from the two directions should be essentially in phase. Alternatively, in the event of an effect due to a simple two-way sidereal anisotropy that was relatively undistorted beyond the earth's field region, not only the first harmonics, but the sum of first and second harmonics should be in phase. Moreover, a relatively large second harmonic would be expected from the north-pointing detector, which scans near the equator. (In advance of the quantitative treatment, it is noted that the semi-diurnal component must increase to a maximum at the equator, while the diurnal component falls to zero, either at the equator or very close to it.) Thus the best indication of the 12-hour phase difference, if it existed, would come from a comparison of sums of harmonics.

The first complete year of observation, 1964, yielded a significant daily variation in sidereal time. The time of maximum of the first harmonic, 1600 ± 0140 (SE), when compared with the time of maximum in the vertical direction, 0500 ± 0030, unmistakably favoured the 12-hour phase difference. The corresponding times of maximum of sums of harmonics were 0400 and 1340 respectively, giving a phase difference of 9 hours 40 minutes.

The sidereal maxima in the two directions were again significantly out of phase in 1965, the difference being more pronounced in the sum of harmonics (~ 10 hours) than in the first harmonic (~ 8 hours). These results were accompanied by improved evidence for a significant second harmonic in the north-pointing direction.

From the average of the two complete years 1964 and 1965, the observed phase difference between sums of harmonics was 12 hours 14 minutes and between first harmonics was 9 hours 14 minutes. The daily variations of bi-hourly deviates are shown, with the sums of harmonics of best fit, in Figure 5.15. Table 5.4 summarizes the results of harmonic analysis of the observations from the narrow-angle inclined telescope*.

An indication of the consistency of the phase difference observed between the sums of harmonics in the two directions at Hobart is given by the annual running averages listed in Table 5.5. It appears that the phase differences associated with

TABLE 5.4.

Harmonic characteristics of the apparent sidereal daily variations observed with the narrow-angle telescope inclined 70°N of zenith. The harmonics are derived from hourly deviates of pressure-corrected intensity. The errors shown are the SE's of estimate.

| | *1964* | *1965* | *1964 + 1965* |
|---|---|---|---|
| First harmonic | | | |
| Amplitude (%) | 0·104 ± ·044 | 0·095 ± ·044 | 0·092 ± ·031 |
| $T_{max}$ (Hr) | 1600 ± 0130 | 1310 ± 0140 | 1430 ± 0120 |
| Second harmonic | | | |
| Amplitude (%) | 0·029 ± ·044 | 0·087 ± ·044 | 0·055 ± ·031 |
| $T_{max}$ (Hr) | 1700 | 1830 ± 0140 | 1810 ± 0200 |
| Sum of harmonics | | | |
| Amplitude (%) | 0·091 | 0·112 | 0·110 |
| $T_{max}$ (Hr) | 1340 | 1740 | 1730 |

* Following the installation at the University of the Elliott 503 computer, it has become the general practice to analyse hourly, rather than bi-hourly, count totals. Thus, some of the more recent groups of harmonics presented in this report have been computed from hourly deviates. Care is taken to ensure that all harmonics employed in a given comparison are derived in the same manner.

TABLE 5.5.

Comparison of the apparent sidereal daily variations observed with the vertical semi-cubical telescope and the narrow-angle telescope inclined 70°N of zenith. Times of maximum of sums of first and second harmonics of best fit to the hourly deviates are tabulated in the form of annual running averages.

| Year Ending | | Vertical<br>$T_{max}$<br>Hr | Inclined 70°N<br>$T_{max}$<br>Hr | Difference (inclined–vertical)<br>Hr |
|---|---|---|---|---|
| 1964 | December | 0400 | 1340 | 0940 |
| 1965 | January | 0450 | 1810 | 1320 |
| | February | 0540 | 0900 | 0320 |
| | March | 0520 | 1840 | 1320 |
| | April | 0610 | 1830 | 1220 |
| | May | 0800 | 1820 | 1020 |
| | June | 0820 | 1820 | 1000 |
| | July | 0710 | 1740 | 1030 |
| | August | 0720 | 1820 | 1100 |
| | September | 0720 | 1830 | 1110 |
| | October | 0740 | 1840 | 1100 |
| | November | 0650 | 1830 | 1140 |
| | December | 0710 | 1730 | 1020 |

the year ending in December 1964 and December 1965 were representative of a persistent effect. The only important divergence from a large phase displacement occurred in the year ending in February 1965. It came about through a somewhat early maximum (~ 1400) of the first harmonic in the inclined direction, favouring the first maximum of the second harmonic.

It is clear that greatly improved counting statistics are needed in the inclined direction if useful information concerning the relative amplitudes of the harmonics is to be obtained. It will be shown later that a two-way anisotropy which conforms with observations from other latitudes of viewing would require the amplitude of the second harmonic, appropriate to the cone of viewing of the inclined telescope, to be somewhat greater than the amplitude of the first harmonic. The only suggestion of this in the present data is that on eight of the thirteen occasions to which Table 5.5 refers, the amplitude of the second harmonic exceeded that of the first.

## 5.6. THE OBSERVED FIRST AND SECOND HARMONICS

In his considerations of acceleration mechanisms associated with the galactic magnetic field, Davis (1954) has described the two-way anisotropy that results when excess fluxes of particles with steep helices are being propagated in both directions along the field. He shows that the sidereal daily variation observed terrestrially should consist of first and second harmonics that are in phase (see Table 1.1). In Section 6 of this report, the harmonics that Davis had obtained in his more generalized treatment of anisotropies will be derived independently, employing an empirical expression for the two-way anisotropy. On the assumption that the intensity maxima are equal in the opposite directions and that the anisotropy is being observed directly (i.e., ignoring the influence of local magnetic fields) the amplitude of the observed sidereal diurnal variation would reach its

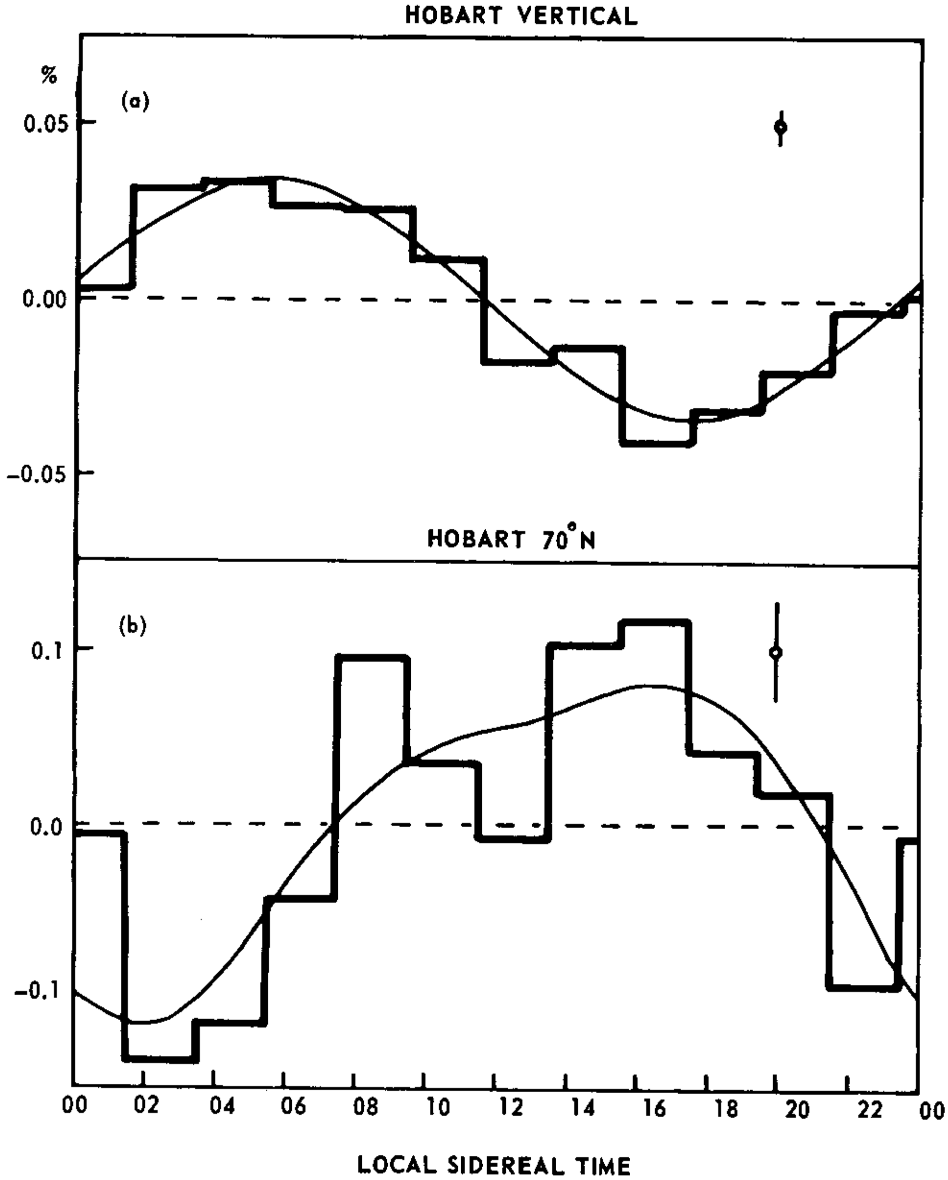



FIG. 5.15. The observed sidereal daily variation of underground meson intensity at Hobart, averaged over the years 1964 and 1965

(a) from the vertical semi-cubical telescopes, scanning asymptotic latitudes in the vicinity of 39°S geographic and

(b) from a narrow angle telescope inclined 70° to the north of zenith, scanning asymptotic latitudes in the vicinity of 20°N geographic.

The error tails shown are the S.E.'s of amplitude of the individual harmonics.

maximum value at the latitudes of viewing 45°S and 45°N and would be zero at the equator and the poles. On the other hand the amplitude of the second harmonic would reach its maximum value at the equator and would be zero at the poles. The maximum amplitudes of the two harmonics would be equal.

In reality, because local magnetic fields intervene, the above characteristics more properly relate to the free-space harmonics as functions of asymptotic latitude of viewing. Moreover, the evidence will suggest that the intensity maxima in the opposite directions along the axis of anisotropy are not equal. It will be shown that the sidereal diurnal variation may in fact consist of two parts, one part being identifiable with the two-way anisotropy and hence associated with a semi-diurnal variation to which it bears a fixed relation at the given latitude of viewing. It will be proposed that there is an additional contribution to the diurnal variation from a single intensity maximum along the axis of the anisotropy, due perhaps to a balance of streaming. Thus, if the two components of the diurnal variation changed with respect to each other, the ratio of amplitudes of the observed first and second harmonics would vary accordingly. It follows that a survey of the latitude dependence of the observed harmonics should relate to observations that are as nearly as possible simultaneous with each other.

The factors that have just been mentioned will be given full consideration in the later Sections. Our main concern in this Section is to demonstrate that the observed sidereal daily variation does comprise first and second harmonics, which, when significant, are in phase, both from observations in the vertical direction at Hobart relating to individual years and from simultaneous observations at different latitudes of viewing.

*(a) Observations in the vertical direction at Hobart, 1958-1965*

Figure 5.16 shows the results of harmonic analysis of the observations averaged over each of the four biennial periods and their sum. It can be seen that, when the small second harmonic was detectable, it was in phase with the first harmonic and that, in three of the four biennial periods and in the eight-year average, the amplitude was just significant at the 5% level of probability. From annual running averages derived from hourly deviates (Figure 5.17) there is some indication that the amplitude decreased over the eight years, with a tendency for large excursions in the time of maximum to occur when the amplitude was persistently less than about 0·01%. This is about what would be expected for an annual SE of amplitude of $\pm$ 0·006%.

*(b) Observations at other latitudes of viewing*

Under this heading the most useful information to date has come from the vertical semi-cubical telescope at Budapest (asymptotic latitude $\sim$ 43°N) and from the cubical telescope inclined 30°N of zenith (asymptotic latitude $\sim$ 17°S) at Hobart. In Figure 5.18 the harmonics relating to the two complete years of operation of the north-pointing cubical telescope are compared with simultaneous observations in the vertical direction at Hobart and with the available observations from Budapest. Not only are the observed harmonics in phase at each of the three different latitudes of viewing but, as will be shown, the corresponding free-space

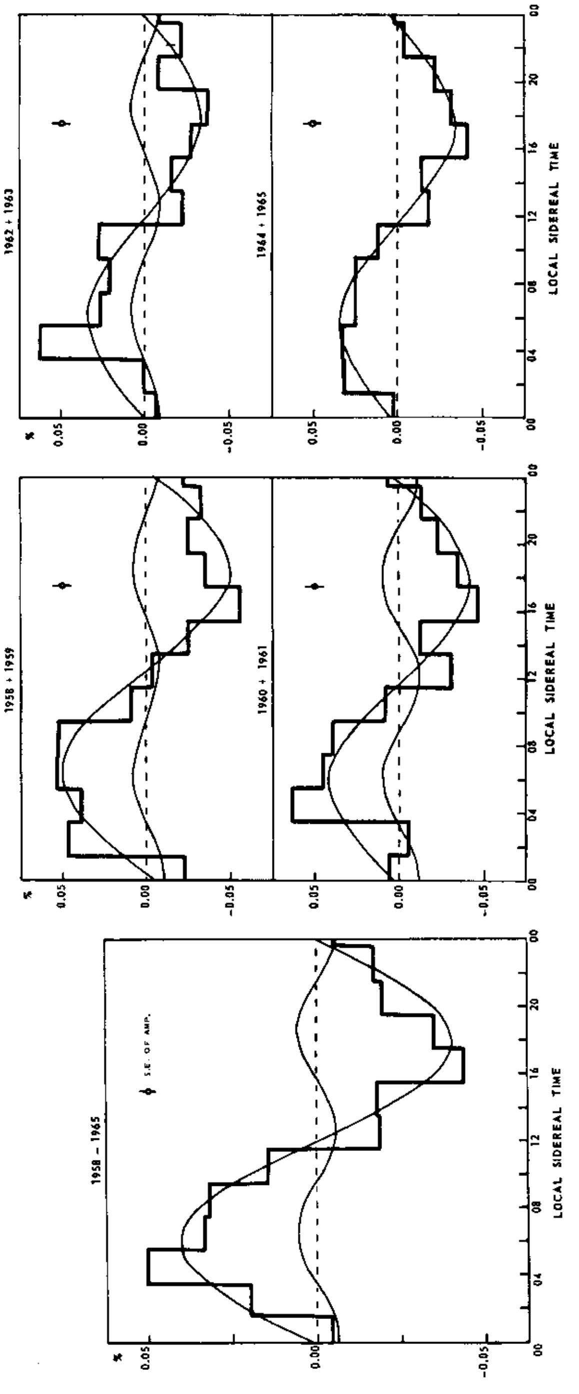



FIG. 5.16. The apparent sidereal daily variation of vertical intensity underground, Hobart. The histograms, relating to the eight-year average and the four biennial periods, are shown with first and second harmonics of best fit.

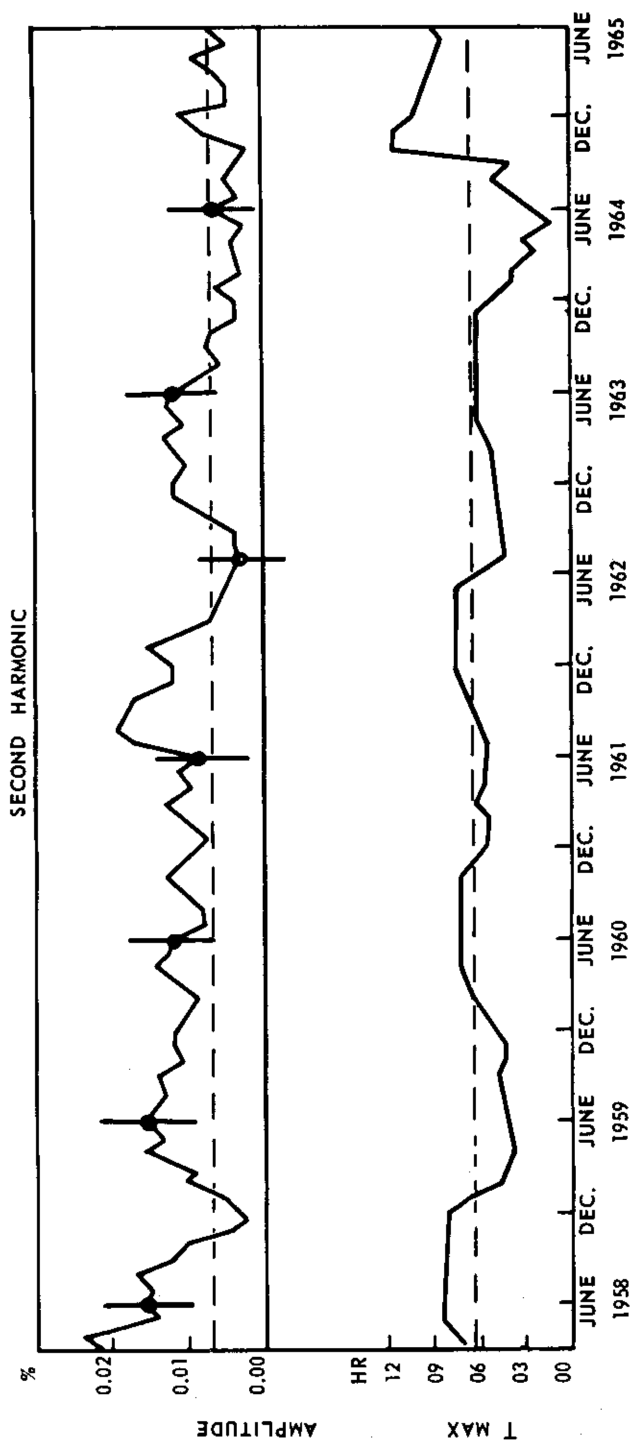



FIG. 5.17. The apparent sidereal daily variation of vertical intensity underground. Twelve-month running averages of amplitude and time of maximum of the second harmonic, derived from hourly deviates. The dashed lines represent the long-term averages. S.E. tails are attached to the calendar-year values of amplitude.

harmonics exhibit a latitude dependence of amplitude that conforms completely with the predictions of the model for the anisotropy. This set of observations is the one that will be employed in the estimation of the direction of the anisotropy.

Thambyahpillai *et al.* (1965) have found that at the depth of 60 m.w.e. at London there was no significant second harmonic in the observed sidereal daily variation averaged over the years 1961-1965. Thus, if it was present, it appears that the amplitude of the second harmonic must have been less than about 0·008%.

The other result that should be mentioned was obtained from the cubical telescope inclined in the direction 45°S of zenith (mean asymptotic latitude ~ 60°S) at Hobart. As shown in Table 5.1, the counting rate was less than that in the direction 30°N of zenith by a factor of about two, because of the higher zenith angle and the greater amount of material absorber. The counting rate was insufficient, in fact, for the effective observation, on an annual basis, of the small daily variations (solar and sidereal) that would be expected at the high latitude of viewing. Over the two years of operation of the experiment in the north-pointing direction, the harmonics of the sidereal effect observed with the south-pointing telescope were not statistically significant. The south-pointing run was continued for a further two years in the hope of obtaining a more definite result. The final values of the harmonics and of their sum, averaged in sidereal time over the four complete years 1961-1964, are as follows:

| | Amplitude (%) | $T_{max}$ (Hr) |
|---|---|---|
| First Harmonic | 0·025 ± 0·008 | 0610 ± 0110 |
| Second Harmonic | 0·020 ± 0·008 | 0550 ± 0050 |
| Sum of Harmonics | 0·045 | 0600 |

While the observations from the south-pointing telescope give useful supporting evidence for the 0600 maximum of the sidereal daily variation in the southern hemisphere (albeit an earlier value would be expected—see below) and for harmonics that are in phase, the amplitude of the observed second harmonic is clearly anomalous. At the high asymptotic latitude of viewing the amplitude of any second harmonic of extra-terrestrial origin should be very small indeed, independently of the nature of the anisotropy. With improved counting statistics a more realistic value should be observed.

In principle, a south-pointing experiment at Hobart is an important one. High energy primaries which arrive from the south approach the earth transverse to the terrestrial magnetic field and the longitude co-ordinates of the velocity vectors are displaced appreciably eastwards. Accordingly, there should be a phase-shift in the time of maximum of the sidereal daily variation observed in the south-pointing direction relative to observations in the vertical direction. This would constitute valuable evidence that the sidereal anisotropy was due to charged primaries and would give information as to the variation spectrum. From a high counting rate experiment that is being planned with this object in view, it is hoped that relatively accurate estimates of the amplitude of the diurnal and semi-diurnal sidereal components will be obtained. In the following Sections the asymptotic response in the south-pointing direction will be closely examined.

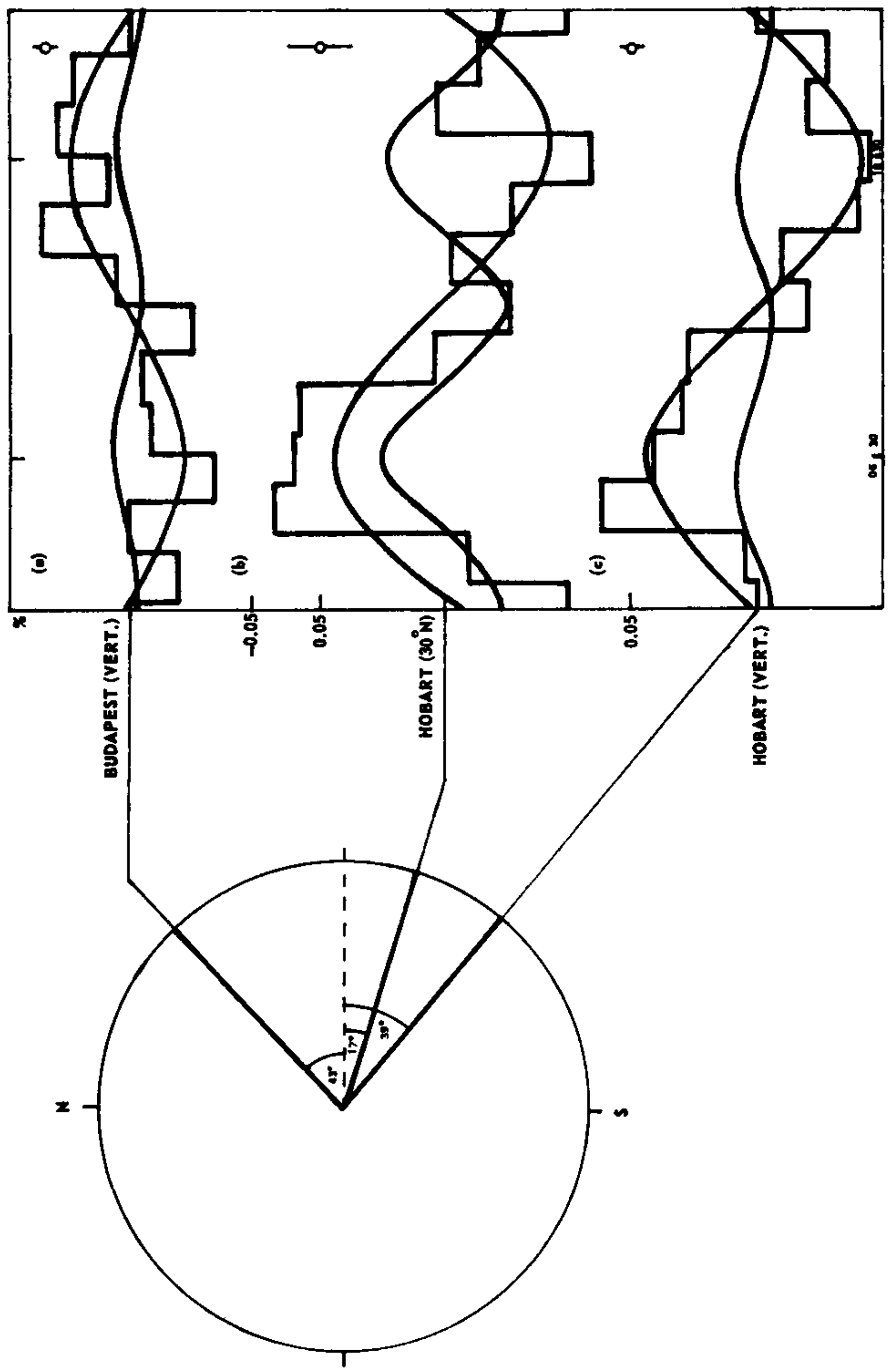



FIG. 5.18. The sidereal daily variation observed under-ground at three different asymptotic latitudes:
(a) at Budapest (1959 + 1961) with semi-cubical vertical telescopes
(b) at Hobart (1961 + 1962) with a cubical tele-scope inclined 30° north of zenith
(c) at Hobart (1961 + 1962) with semi-cubical vertical telescopes.
The first and second harmonics of best fit to the histograms are shown.
The error tails are the S.E.'s of amplitude.

### 5.7 (a). *Summary of evidence*

Table 5.6 summarizes the annual averaged observations from underground that have been discussed in the Section and that, for one reason or another, are of relevance. For the sake of completeness, all known harmonics are shown although not all of the experiments have given, or were intended to give, useful information concerning individual harmonics, particularly the second.

From the analyses of the underground data there is evidence in the first place for an annual apparent sidereal effect that is both statistically significant and reproducible, i.e., neither statistical no quasi-statistical fluctuations obscure the main effect. Thus a phenomenon is being observed which, like the solar daily variation, is continuously present and requires an explanation in terms of some physical process.

The observations indicate that there is in fact a genuine sidereal daily variation of intensity, associated with a two-way anisotropy, and that if there are any spurious contributions to the annual average they are relatively unimportant. In particular, if there is a sideband of atmospheric origin it must be small in comparison with the genuine sidereal component.

Three of the most important lines of evidence—from the annual averages relating to the vertical intensity at Hobart, from the phase anomalies and from the phase difference of 12 hours in the sidereal diurnal variations as viewed in the different hemispheres—were of widely differing character. Each would seem to have required the presence of a sidereal component in the daily variation. Of particular significance, however, were two other features of this evidence:

(i) It was found that the phase anomalies at Hobart were synchronized with the trends of the annual average amplitudes in solar and sidereal time. This was mandatory if the annual sidereal effect was to be essentially genuine. On the other hand, if there had been a substantial spurious sidereal component in the annual average the degree of co-ordination of events could scarcely have occurred.

(ii) The evidence from Hobart relating to asymptotic scans in the southern hemisphere (in the axial directions 30°N, 0° and 45°S underground), when compared with the observations from Budapest, indicated the existence of a two-way sidereal anisotropy. The vertical—70°N experiment underground constituted an acid test of this hypothesis. Both from individual years and from the biennial average the result from this experiment was significant and unequivocally in favour of the hypothesis. Moreover, of all the evidence, this most directly indicated that the apparent sidereal effect could not have been due to local seasonal modulation of a solar component.

The evidence for a two-way anisotropy led to a consideration of the two harmonics and the latitude dependence of their amplitudes. For a proper study of the relationship between the harmonics, a simultaneous latitude survey with much higher counting rates than we have at present is needed. Nevertheless, the present experiments have given significant evidence from the observed first and second harmonics for the existence of the two-way anisotropy. It has been shown in this Section that there is a persistent tendency for the two harmonics to be in phase. In the final Sections it will be shown how the amplitudes of the free-space

TABLE 5.6.

Harmonics of the apparent sidereal daily variation observed underground in the two hemispheres, estimated from bi-hourly deviates of pressure-corrected intensity. SE tails are shown.

| Location and detector | Geog. latitude of viewing* | Absorber† (m.w.e.) | Period of obs. | First Harmonic Amplitude % | First Harmonic $T_{max}$ Hr | Second Harmonic Amplitude % | Second Harmonic $T_{max}$ Hr | Sum of Harmonics Amplitude % | Sum of Harmonics $T_{max}$ Hr |
|---|---|---|---|---|---|---|---|---|---|
| | | | *NORTHERN ASYMPTOTIC LATITUDES* | | | | | | |
| Budapest vertical | 43°N | 50 | 1959, 1961 (2 yrs) | 0·023 ± 0·004 | 1830 ± 0040 | 0·006 ± 0·004 | 1900 ± 0240 | 0·028 | 1830 |
| London vertical | 46°N | 70 | 1961–1964 (3 yrs) | 0·015 ± 0·004 | 1800 ± 0055 | | | 0·015 | 1800 |
| Hobart 70°N of zenith | 20°N | 60 | 1964–1965 (2 yrs) | 0·092 ± 0·031 | 1430 ± 0120 | 0·025 ± 0·031 | 1930 | 0·080 | 1710 |
| | | | *SOUTHERN ASYMPTOTIC LATITUDES* | | | | | | |
| Hobart vertical | 39°S | 46 | 1958–1965 (8 yrs) | 0·040 ± 0·002 | 0554 ± 0012 | 0·006 ± 0·002 | 0640 ± 0120 | 0·046 | 0600 |
| Hobart 30°N of zenith | 17°S | 42 | 1961–1962 (2 yrs) | 0·043 ± 0·008 | 0650 ± 0040 | 0·024 ± 0·008 | 0630 ± 0050 | 0·067 | 0650 |
| Hobart 45°S of zenith | 60°S | 56 | 1961–1964 (4 yrs) | 0·025 ± 0·008 | 0610 ± 0110 | 0·020 ± 0·008 | 0550 ± 0050 | 0·045 | 0600 |

* Estimated mean asymptotic latitude of viewing. † Approximate mass to the top of the atmosphere.

harmonics deduced from the observations in the vertical direction at Hobart and at Budapest and in the axial direction 30°N at Hobart conform with the requirements of the anisotropy. The mean asymptotic latitudes of viewing of the three telescopes are particularly well located in the latitude range from approximately 40°N to approximately 40°S; over this range the greatest variation in the ratio of amplitudes of harmonics is to be expected, the amplitudes of one or other of the harmonics should be relatively large and maximum resolution of amplitude is achieved from the response characteristics of the telescopes. A value for the declination will be calculated from the harmonics, leading finally to an estimate of the direction of the anisotropy in free space.

There seems to be no doubt, from equivalent observations at the same depths underground, that at present the amplitude of the sidereal diurnal variation is much greater in the southern than in the northern hemisphere. In the long term it will be most desirable to monitor the ratio of amplitudes in the two hemispheres to determine the constancy or otherwise of the amplitude asymmetry. In the quantitative treatment it will be assumed that there are in effect two parts to the anisotropy—one part being the two-way anisotropy with equal amplitudes in opposite directions along an axis and the other part being a superimposed intensity maximum (e.g., due to net streaming) in one of these two directions.

There has been evidence that the sidereal daily variation exhibits two important time-variations. In the first place there was an overall tendency for the amplitude in the vertical direction at Hobart to decrease by a factor of perhaps two between 1958 and 1965. This must be of great significance for the interpretation of the anisotropy, but for a true appreciation of this effect it would be essential to have simultaneous observations of the long-term trends of amplitude from both hemispheres. A brief discussion of the phenomenon will be given later.

Again, it has been necessary to rely on the observations from Hobart for the evidence of seasonal modulation of the sidereal component. Although the explanation must be regarded as tentative, it is suggested that pronounced semi-annual modulation of amplitude and, to a lesser extent, of phase, occurred at sunspot maximum, that there was a rather rapid attenuation of modulation thereafter and that the phenomenon possibly survived in a greatly weakened form over the years of minimum activity. This hypothesis appears to be the only one which is compatible with all of the changes that have occurred in the components of the daily variation of vertical intensity underground and is not compromised by the evidence from other measurements (e.g., the narrow-angle north-pointing experiment) that the sidereal effect at Hobart is essentially genuine.

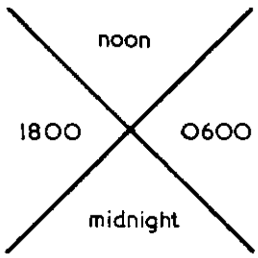
The interplanetary field of the sun as a possible source of semi-annual variation of the sidereal effect will be discussed below, in an attempt to visualize what the influence of the field might be. However, it seems that a correct assessment will have to wait until substantial evidence for the type of seasonal modulation observed at Hobart is to be had from both hemispheres simultaneously, perhaps at the next sunspot maximum, and until we have a more complete description of the interplanetary field.

### *5.7 (b). Semi-annual modulation of the sidereal effect in relation to the interplanetary magnetic field*

It will be recalled that the large anti-sidereal effect observed in 1958 was compatible with semi-annual modulation of the sidereal component (hypothesis IIIa). On this interpretation the amplitude of the sidereal component decreased almost to zero in June and December, as the difference ($D_r$) variations show (Figure 5.6). The sidereal maximum at these times would have occurred at approximately 0600 local sidereal time (Figure 5.5). It is noted that 0600 is also approximately the sidereal time of midday in June and of midnight in December. In other words, when the direction of maximum intensity approximately coincided with the direction of the earth-sun line, towards the sun in June and away from the sun in December, the intensity maximum was not observable. In this connection mirroring in the interplanetary field may be important and some of the factors involved should be mentioned.

If a sidereal anisotropy is to be observed at the earth's orbit, it is to be inferred that the charged primaries responsible for it could not have approached with guiding-centre motions along lines of the interplanetary magnetic field. Moreover, the threshold rigidity for observation should be not less than the upper limiting rigidity for observation of the solar anisotropy. A method of estimating annual values, to be described in Section 6, gives approximately 100 GV as the upper limiting rigidity in 1958. For the present, we assume that this also approximated the threshold rigidity, $R_c$, for observation of sidereal anisotropy during that year. It will be shown later that approximately 75% of the underground response relates to primaries in the rigidity range 100-500 GV, for $R_c = 100$ GV and a flat spectrum of variation.

It appears, then, that at the earth's orbit where the interplanetary field strength is $\sim 5 \times 10^{-5}$ gauss the gyroradii of the majority of primaries of interest would be between 0·4 A.U. and 2·0 A.U., provided that there was no substantial spatial variation of the field, normal to its direction in the plane of the ecliptic, over these distances. Although it seems that great variation would be likely, the nature of the field over distances of astronomical units above and below the plane of the ecliptic is not known. On the other hand, the Imp-I data give evidence that in the ecliptic plane there are four sectors in heliographic longitude, the field in neighbouring sectors being oppositely directed (Ness *and* Wilcox 1965). At the earth's orbit, the width of a sector must on the average be approximately 1·5 AU, although one sector was observed to be much narrower than this and the others somewhat wider. Therefore, the high energy primaries with relatively large pitch angles would usually not complete one turn of a helix near the earth's orbit without crossing from one field sector to another, and this would prevent mirroring along the direction of the spiral field from the earth ($\sim$ 45°W of the earth-sun line). However, there would be a greater tendency for trapping and mirroring in a field sector as the particles approached the sun. That is to say, for a field strength $B_r$ proportional to $r^{-2}$, where $r$ is distance from the sun, the gyroradius $\rho$ is proportional to $r^2$ and thus $\rho/r \propto r$. This means that as $r$ decreases the gyroradius at mirroring becomes progressively smaller in relation to the width of a field sector, thus materially increasing the probability of reflection in addition to scattering. It appears, then, that high energy primaries which approach the earth from the anti-sun direction in the plane of the ecliptic will have a tendency to be subsequently reflected as well as scattered from the general direction of the sun. It follows that, when the asymptotic cone of the detector scans in the direction of the sun, a proportion of the flux to which it responds will have originated from the anti-sun direction. We now divide the solar day into four zones of time, arbitrarily shown to be equal in the sketch. When the asymptotic cone of the detector is in the noon zone and in the midnight zone, the primary response tends to be the same, there being a tendency to view in the anti-sun direction in the noon zone.




Let us now suppose that there is a sidereal anisotropy at the earth's orbit and that the detector scanning asymptotically in the plane of the ecliptic records a sidereal diurnal variation $V_1 = A \cos \phi$ at the times of the year when there should be least interference due to mirroring, i.e., when the sidereal maximum occurs at 0600 and at 1800 solar time respectively. Now, when the sidereal maximum occurs in the midnight zone, it can easily be seen that mirroring tends to cause rectification of the diurnal wave, giving rise to a semi-diurnal component $V_2 = \frac{A}{2} \cos 2\phi$ instead. Six months later, the intensity minimum occurs in the midnight zone and, because of rectification of the negative half of the diurnal wave, it tends to be replaced by a semi-diurnal component, $V_2 = \frac{A}{2} \cos 2 \ (\phi - \frac{\pi}{2})$. Over these two particular periods of the year six months apart, then, the sidereal diurnal variation would tend to vanish and be replaced by a smaller semi-diurnal component, exhibiting a phase displacement of six hours as between the two periods. It will be noted that, in the difference ($D_r$) type of observation, which gives the sidereal effect averaged over pairs of months six months apart, the semi-diurnal sidereal components produced by mirroring must vanish. Therefore, depending on the efficiency of the process, the sidereal daily variation observed in this form should tend to vanish twice a year, when the sidereal maximum occurs at about noon and at midnight respectively.

The times of occurrence of minimum amplitude predicted above agree very well with the underground observations in 1958 if, as it seems, the difference ($D_r$) variations were essentially due to a genuine sidereal component. As described, however, the reflection process relates to near-equatorial observations of a unidirectional anisotropy. In reality, we are concerned with mid-latitude observations of an apparent two-way anisotropy whose axis is steeply inclined to the plane of the ecliptic, as will be shown later. The main difference here is that, if the process of reflection and scattering near the sun is to be of relevance in this situation, it seems that the smoothing out of intensity differences between the noon and midnight zones must be effective for directions of arrival inclined as much as approximately 60° to the plane of the ecliptic.

Although the sidereal diurnal maximum occurs approximately 12 hours later in the northern hemisphere, on the above hypothesis June and December should have been the months of minimum amplitude in that hemisphere also. Because of the small annual amplitudes in the northern hemisphere and the lack of a complete year's data from Budapest in 1958, the use of difference daily variations to obtain evidence of a semi-annual modulation of amplitude was not practicable. However, another indication is given by the anti-sidereal effect, which should exhibit 12 hours difference in time of maximum between the two hemispheres, although it should be remembered that there could be other reasons for this. We can simplify the argument, without serious error, by neglecting the relatively minor semi-annual variations of phase that appear to have been associated with amplitude modulation in 1958. The time of maximum $\tau_m''$ of the anti-sidereal sideband due to semi-annual modulation of amplitude of a sidereal diurnal variation (time of maximum $\tau_m'$) is given by

$$\tau_m'' = \tau_m' - 2T_o$$

where $T_o$ is the time of the year when maximum amplitude occurs. The derivation is given in Paper 2, Section 10. Clearly, if $T_o$ is the same in both hemispheres of viewing, but $\tau_m'$ is 12 hours different, the phase of the anti-sidereal effect must also be 12 hours different. For what it is worth, it is noted that during years of pronounced seasonal modulation the maximum of the anti-sidereal effect observed underground at Hobart occurred at approximately 1000, while (a) at London, averaged over the three years 1961-1964, the time of maximum was approximately 2200 and (b) at Budapest, averaged over the two available years 1959 and 1961, the maximum occurred at approximately 2100. While the result is consistent with the type of semi-annual modulation we are considering, it is recognized that there may have been other contributions to the anti-sidereal effects at London and Budapest. To assess the true nature of these effects it would be most desirable to have evidence of significant long-term changes of seasonal modulation at the two places.

On the evidence, the mechanism for seasonal modulation of the sidereal component must have operated with great efficiency in 1958, but with rapidly decreasing effect thereafter (Figure 5.9 (c)). From the overall constancy of the solar diurnal variation of neutron intensity over the years 1958-65, it is clear that solar modulation of lower energy primaries in the plane of the ecliptic had maintained its level. Therefore it is suggested that the lessening of seasonal modulation of the sidereal component may have been connected with considerable changes in the field in regions away from the plane of the ecliptic, perhaps a lateral contraction of the volume of the field. There is an indication of this, independently, from the great decrease in amplitude of the solar daily variation observed underground, as will be discussed later.

## 6. TRANSFORMATION OF CO-ORDINATES AND THE ASYMPTOTIC CONE OF ACCEPTANCE

It is the purpose of this Section to describe the daily variation that is expected to be observed underground, in response to an intensity anisotropy of the primaries as specified in the frame of reference in which it is fixed. There are three parts to the treatment:

(1) an empirical description of the intensity anisotropy outside the earth's field in the fixed frame of reference;

(2) transformation to the rotating celestial co-ordinate system, to give the free-space harmonics and an average intensity term;

(3) estimation of the harmonics that would be observed underground in response to the free-space harmonics.

We consider first a model for the sidereal anisotropy (subdivision A of the Section) and then go on to consider the relevant features of the solar anisotropy (subdivision B).

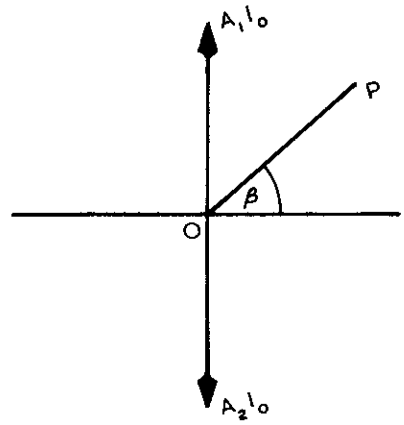
## 6A. THE SIDEREAL ANISOTROPY

### 6A.1. DESCRIPTION OF THE ANISOTROPY

It will be assumed from the underground evidence that there is (a) a two-way sidereal anisotropy such as might be associated with a galactic magnetic field when there are equal excess fluxes of particles with steep helices in both directions along the field and (b) an additional intensity maximum in one of the two directions, such as might be associated with net streaming of the cosmic ray gas.

For simplicity of the present treatment it will be assumed that both contributions to the total anisotropy exhibit the same rigidity dependence of amplitude. It is also assumed for the present that the anisotropy results from modulation of the isotropic flux and that the omnidirectional intensity is unaffected.

Suppose that observations of the directional intensity of high energy primaries is made from some position O on the earth's orbit, remote from the earth and its magnetic field. The point O is the origin of a frame of reference determined by the axis of the anisotropy and the plane normal to it. The frame of reference moves like the earth on an orbital path round the sun, but does not rotate. We proceed to an empirical model which describes the intensity anisotropy that is expected to be observed on the annual average at O. In this model the differential intensity for primaries of rigidity R in the direction of viewing OP, which makes an angle $\beta$ with the plane normal to the axis of the anisotropy (see sketch), is given by




$$I_{R,\beta} = I_{oR}[1 + R^{\gamma}\{m \sin \beta + n(\sin^2 \beta - \tfrac{1}{3})\}] \tag{6.1}$$

where $I_{oR}$ is the isotropic flux, $\gamma$ is the index of the variation spectrum of the anisotropy, and $m$ and $n$ are amplitude constants. The term $-\frac{1}{3} I_{oR} R^{\gamma} n$ allows for the fact that $I_{oR}$ is also the omnidirectional flux, obtained by averaging $I_{R,\beta}$ over all directions of viewing, $OP$, within a sphere centred on $O$.

The unequal and oppositely directed differential intensity maxima will therefore be

$$A_{1R}I_{oR} = \left\{1 + \frac{R^{\gamma}}{3}(2n + 3m)\right\} I_{oR} \quad \text{when} \quad \beta = +\frac{\pi}{2}$$

and

$$A_{2R}I_{oR} = \left\{1 + \frac{R^{\gamma}}{3}(2n - 3m)\right\} I_{oR} \quad \text{when} \quad \beta = -\frac{\pi}{2}.$$

6A.2. TRANSFORMATION TO THE ROTATING CELESTIAL FRAME OF REFERENCE

The well-known celestial system of spherical co-ordinates is coincident with the rotating terrestrial system, but is fixed with respect to neighbouring stars, A point P on the celestial sphere is specified by declination (dec.) synonymous with geographic latitude, and Right Ascension (RA), measured eastward of the First Point of Aries, ♈.

We now wish to describe the anisotropic intensity in terms of celestial co-ordinates, in the direction OP when it partakes of terrestrial rotation. We then have a description of the anisotropy as observed at any time in the vertical direction at some point P on the earth, when all environmental factors such as the terrestrial magnetic field, the atmosphere and the material absorber underground, are absent. That is to say, we have a *free-space* description in celestial co-ordinates, as a function of local sidereal time.

In Figure 6.1 the frame of reference of the anisotropy and the celestial frame of reference are referred to the common origin O. In the celestial system the co-ordinates of the direction of the intensity maximum $A_1I_0$ at $\beta = \pi/2$ are specified by Right Ascension $\alpha_s$ and declination $d$. It can be seen that the equatorial plane of the anisotropy intersects the celestial equator along the line whose RA is $\alpha_s + 90$ in the direction coming out of the page from O, and that the transformation from one frame of reference to the other is effected by simple rotation of axes about the $\alpha_s + 90$ direction through the angle $\varepsilon = 90 - d$. Using the appropriate transformation formula, we now relate the co-ordinate $\beta$ of the direction OP, in the anisotropic frame of reference, to the co-ordinates of OP in the celestial system. Since the anisotropy is described entirely in terms of $\beta$ we then immediately have the means of describing it in terms of celestial co-ordinates.

In the celestial system the direction OP is specified by declination $\delta$ and Right Ascension $\phi'$, $\phi'$ being the local sidereal time at P. However, to transform from the one co-ordinate system to the other, it is necessary to refer bearings in RA to the common axis at $\alpha_s + 90$. Referred to this direction, the RA of the direction OP is $(\phi' - \alpha_s - 90)$.

The appropriate co-ordinate conversion formula for expressing $\beta$ in terms of celestial co-ordinates (e.g., see Explanatory Supplement to the Ephemeris, 1961) is then

$$\sin\beta = \cos d \cos\delta \cos(\phi' - \alpha_s) + \cos^2 d \cos^2\delta \cos 2(\phi' - \alpha_s) \qquad (6.2)$$

It will be noted that, once we have the corresponding expression for $\sin^2\beta$, the anisotropic intensity $I_{R,\beta}$, as observed in the rotating terrestrial system, can be expressed in terms of celestial co-ordinates and local sidereal time using equation (6.1).

Squaring 6.2:
$$\sin^2\beta = \frac{\sin 2d \sin 2\delta}{2}\cos(\phi' - \alpha_s) + \frac{\cos^2 d \cos^2\delta}{2}\cos 2(\phi' - \alpha_s) + \left(\frac{\cos^2\delta}{2}(3\cos^2 d - 2) - \cos^2 d + 1\right) \qquad (6.3)$$

Therefore the intensity anisotropy specified in the terrestrial framework is, from 6.1, 6.2 and 6.3,

$$I_R = I_{oR}\left\{1 + R^\gamma(m + 2n\sin d\sin\delta)\cos d\cos\delta\cos(\phi' - \alpha_s) + R^\gamma\frac{n}{2}\cos^2 d\cos^2\delta\cos 2(\phi' - \alpha_s) + m\sin d\sin\delta + n\left(\frac{\cos^2\delta}{2}(3\cos^2 d - 2) - \cos^2 d + 1 - \frac{R^\gamma}{3}\right)\right\} \qquad (6.4)$$
$$= I_{oR}(1 + T(\phi') + T(\delta)),$$

where $T(\phi')$ is the daily variation term and $T(\delta)$ is the latitude-dependent daily average term. In the present context $T(\delta)$ must be small compared with $I_{oR}$ and may be disregarded in the expression for

$$T(\phi') = \frac{\Delta I_R(\phi')}{I_{oR}},$$

where $\Delta I_R(\phi') = I_R - I_{oR}(1 + T(\delta))$.

The daily variation term consists of first and second harmonic compents $V_{1R}$ and $V_{2R}$:

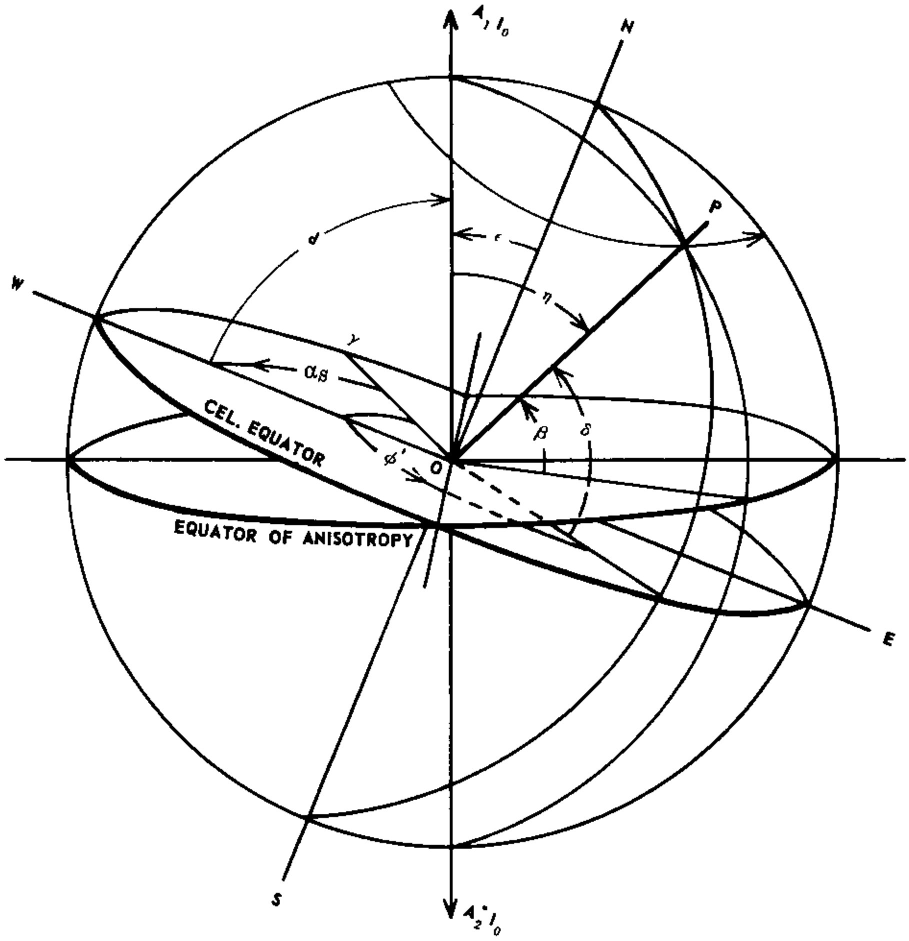



FIG. 6.1. The frame of reference of the sidereal intensity anisotropy, in relation to the frame of reference of the rotating direction of observation OP.

$$V_{1R} = \frac{\Delta I_{1R}}{I_{oR}} = R^\gamma(m + 2n \sin d \sin \delta) \cos d \cos \delta \cos (\phi - \alpha_s) \tag{6.5}$$

$$V_{2R} = \frac{\Delta I_{2R}}{I_{oR}} = R^\gamma \frac{n}{2} \cos^2 d \cos^2 \delta \cos 2(\phi' - \alpha_s), \tag{6.6}$$

These are the *differential free-space harmonics* that we will be concerned with. They are identical with the composite of terms due to diffusion and acceleration derived by L. Davis (1954) in his generalized treatment of high energy anisotropies associated with the galactic magnetic field.

We may represent $V_{1R}$ by

$$V_{1R} = R^\gamma M \cos (\phi' - \alpha_s),$$

where $M$ is the rigidity-independent constant $(m + 2n \sin d \sin \delta) \cos d \cos \delta$. Then the *total free-space first harmonic* will be

$$V_1 = \frac{\int_{Rc}^{Rm} \Delta I_{1R}\, dR}{\int_{Rc}^{Rm} I_{oR} dR} = \frac{\int_{Rc}^{Rm} I_{oR} R^{\gamma} dR}{\int_{Rc}^{Rm} I_{oR} dR} \cdot M \cos(\phi' - \alpha_s)$$

$$= \frac{\int_{Rc}^{Rm} R^{\varepsilon} R^{\gamma} dR}{\int_{Rc}^{Rm} R^{\varepsilon} dR} M \cos(\phi' - \alpha_s)$$

$$= \kappa_c M \cos(\phi' - \alpha_s),$$

where $k_c$ is an amplitude constant depending on $R_c$ and $R_m$, these being the upper and lower cut-off rigidities for observation of the anisotropy at the earth's orbit, and $\varepsilon$ is the index of the average primary differential spectrum. Since only negative values of $\gamma$ will be considered here, it suffices to put $R_m = \infty$, and integration gives

$$\kappa_c = \frac{\varepsilon + 1}{\varepsilon + \gamma + 1} R_c^{\gamma}.$$

Over the primary rigidity range of significance (approximately $10^{11}$ to $10^{13}$ev) $\varepsilon$ is thought to be approximately $-2{\cdot}5$ (e.g., see Parker, 1965, Figure 1) and thus

$$\kappa_c = \frac{1{\cdot}5}{1{\cdot}5 - \gamma} R_c^{\gamma}. \tag{6.6a}$$

For the case $\gamma = 0$ it is noted that $k_c = 1$ and therefore

$$V_1 = V_{12} = M \cos(\phi' - \alpha_s).$$

The *total free-space second harmonic* is similarly

$$V_2 = \kappa_c N \cos 2(\phi' - \alpha_s),$$

where $N$ is the amplitude constant $\frac{n}{2} \cos^2 d \cos^2 \delta$.

We now have to calculate the corresponding daily variation to be expected at the telescope. The following factors, some of which depend considerably on primary rigidity, must be taken into account: the deflections of the primaries in the terrestrial magnetic field, the coupling between the primaries and the observed meson component, the influence of the atmosphere and the underground absorber on the counting rate as a function of direction of arrival of the mesons and the physical geometry of the telescope. These are all taken care of, to the best of our knowledge, by the asymptotic constants of response.

The first and second harmonics of the sidereal daily variation expected to be observed will be

$$\nu_1 = K_1 M \cos(\phi' - \alpha_s + \phi_1)$$

and

$$\nu_2 = K_2 N \cos 2(\phi' - \alpha_s + \phi_2).$$

It can be seen that the asymptotic constants $K_1$, $K_2$, $\phi_1$ and $\phi_2$ and the free-space amplitude constant $k_c$ connect the observed daily variation with the daily variation of primaries in free space. Thus, denoting the observed diurnal amplitude by $\nu_{1max}$, the corresponding free-space amplitude is $V_{1max} = \frac{k_c}{K_1} v_{1max}$ and the phase difference is $\phi_1$. Similarly, for the second harmonic, $V_{2max} = \frac{k_c}{K_2} v_{2max}$ and the phase difference is $\phi_2$.

It may of interest to note that, when $\gamma = 0$, the constants $K_1$ and $\phi_1$ are analagous to $B_1$ and $\rho_1$ that have been calculated for the first harmonic of the solar daily variation as functions of the upper limiting rigidity $R_u$, for $\beta = 0$ (where $\beta$ is the index of the variation spectrum of the solar anisotropy). Values of $B_1$ *and* $\rho_1$ versus $R_u$ are shown in Figures 2 and 3 of Paper 7.

### 6A.3. THE ANISOTROPIC CONSTANTS OF RESPONSE

The given detector responds to a free-space first harmonic

$$V_1 = \kappa_c \left(m \cos d \cos \bar{\delta} + \frac{n}{2} \sin 2d \sin 2\bar{\delta}\right) \cos (\phi' - \alpha_s) \tag{6.7}$$

at the mean asymptotic latitude of response, $\bar{\delta}$. The observed harmonic will be

$$v_1 = K_1 \left(m \cos d \cos \bar{\delta} + \frac{n}{2} \sin 2d \sin 2\bar{\delta}\right) \cos (\phi' - \alpha_s + \phi_1) \tag{6.8}$$

where $K_1$, $\phi_1$ and $\bar{\delta}$ are to be determined.

Similarly, the observed second harmonic will be

$$v_2 = K_2 \frac{n}{2} \cos^2 d \cos^2 \bar{\delta} \cos 2(\phi' - \alpha_s + \phi_2) \tag{6.9}$$

A simplification of the present treatment is that the mean asymptotic latitude of viewing for the second harmonic is defined to be the same as that for the first harmonic. The constants of response are therefore $k_c$, $K_1$, $K_2$, $\phi_1$, $\phi_2$ and $\bar{\delta}$.

Alternatively, the observed harmonics of response may be written as

$$v_1 = \left(C_1 m \cos d + C_2 \frac{n}{2} \sin 2d\right) \cos (\phi' - \alpha_s + \phi_1) \tag{6.10}$$

and

$$v_2 = C_3 \frac{n}{2} \cos^2 d \cos 2(\phi' - \alpha_s + \phi_2) \tag{6.11}$$

where
$$C_1 = K_1 \cos \bar{\delta} \tag{6.12}$$
$$C_2 = K_1 \sin 2\bar{\delta} \tag{6.13}$$
$$C_3 = K_2 \cos^2 \bar{\delta} \tag{6.14}$$

Thus
$$\sin \bar{\delta} = \frac{C_2}{2C_1} \tag{6.15}$$

The method used here is to estimate approximate values of the amplitude constants $C_1$, $C_2$ and $C_3$ from which $\bar{\delta}$, $K_1$ and $K_2$ may be determined.

### 6A.4. CALCULATION OF THE CONSTANTS OF RESPONSE: OUTLINE OF METHOD

The calculation of the underground reponse to the solar anisotropy has been described in Paper 7. Since the technique was designed to have a more general application, the following is essentially a recapitulation of the treatment given in the paper, suitably modified so as to apply to the sidereal anisotropy.

The fraction of the observed first harmonic underground that is due to anisotropic primaries of rigidity $R$ will differ considerably from $V_{IR}$ (equation (6.5)), partly because at any instant the primaries involved do not approach the earth's magnetic field from a single direction $(\delta, \phi' - \alpha_s)$ but from a cone of directions that will allow them to arrive at the meson production level within the solid angle of viewing of the recorder, after penetrating the field. The cone is the asymptotic cone of acceptance of which the concept and general method of application to an anisotropy have been fully outlined by Rao, McCracken *and* Venkatasan (1963). The treatment given here differs somewhat in detail from theirs and is perhaps only appropriate when dealing with high energy primaries. The cone of acceptance is divided into $N$ segments specified by intervals of asymptotic latitude. These intervals can be conveniently defined so that each segment of the cone contributes about the same amount to the fraction of the counting rate that is due to primaries of the given rigidity. The contribution $v_{1R}$ to the first harmonic of the sidereal daily variation from primaries of rigidity $R$ is then approximated by the summation:

$$v_{1R} = \frac{Y_R R^{\gamma}}{N} \sum_{n=1}^{N} \left\{ m \cos d\, A_{Rn} \cos \delta_{Rn} + \frac{n}{2} \sin 2d\, A_{Rn} \sin 2\delta_{Rn} \right\} \cos (\phi' - \alpha_s + \phi_{1Rn}) \quad (6.16)$$

where the summation symbol $n$ refers to subscripts only.

In this expression, $Y_R$ is the differential coupling coefficient and $A_{Rn}$ is an amplitude reduction factor determined by the manner in which the contributions to $Y_R$ from the $n^{\text{th}}$ segment of the cone are distributed in asymptotic longitude. An individual contribution will de defined more precisely in Section 6A.5 as the "differential radiation sensitivity", $I(R, \omega)$. The distribution also determines an effective longitude of viewing $\phi_{1Rn}$ (east of the observer's meridian) with respect to a first harmonic of the anisotropy. The mean asymptotic latitude $\delta_{Rn}$ is obtained from the distribution in asymptotic latitude of the contributions to $Y_R$ from the $n^{\text{th}}$ segment of the cone.

The total first harmonic $v_1$ is estimated as

$$v_1 = \sum_{R=Rc}^{Rm} v_{1R}.$$

In the calculations of response, the upper cut-off $R_m$ is put equal to infinity, since the telescope response becomes negligible at very high rigidities.

A useful form of $v_{1R}$ is the vector

$$\begin{aligned} \hat{v}_{1R} &= |v_{1R}|\, \hat{\alpha}_{1R} \\ &= m \cos d \frac{Y_R}{N} R^{\gamma} \sum_{n=1}^{N} A_{Rn} \cos \delta_{Rn}\, \hat{\alpha}_{1Rn} \\ &\quad + \frac{n}{2} \sin 2d \frac{Y_R}{N} R^{\gamma} \sum_{n=1}^{N} A_{Rn} \sin 2\delta_{Rn} \hat{\alpha}_{Rn} \end{aligned} \quad (6.17)$$

where the summation symbol $n$ refers to subscripts only and where $\alpha_{1Rn} = \alpha_s - \phi_{1Rn}$ is the phase angle on the harmonic dial. In turn, the total first harmonic vector is

$$\hat{v}_1 = | v_1 | \hat{\alpha}_1 = \sum_{R=Rc}^{\infty} \hat{V}_{1R} = C_1' \, m \cos d \, \hat{\alpha}_{1a} + C_2' \frac{n}{2} \sin 2d \, \hat{\alpha}_{1b} \qquad (6.18)$$

where the constants $C_1'$ and $C_2'$ are determined as

$$C_1' \hat{\alpha}_{1a} = \sum_{R=Rc}^{\infty} \frac{Y_R}{N} R^{\gamma} \sum_{n=1}^{N} A_{Rn} \cos \delta_{Rn} \hat{\alpha}_{1Rn} \qquad (6.19)$$

and

$$C_2' \hat{\alpha}_{1b} = \sum_{R=Rc}^{\infty} \frac{Y_R}{N} R^{\gamma} \sum_{n=1}^{N} A_{Rn} \sin 2\delta_{Rn} \hat{\alpha}_{1Rn} \qquad (6.20)$$

It will be noted that $C_1' m \cos d\hat{\alpha}_{1a}$ and $C_2' \frac{n}{2} \sin 2d\hat{\alpha}_{1b}$ form a vector triangle with $| v_1 | \hat{\alpha}_1$ and that $C_1' = C_1$ and $C_2' = C_2$ only if the unit vectors $\hat{\alpha}_{1a}$ and $\hat{\alpha}_{1b}$ are the same. In the various determinations of the underground response it has been found that $\phi_{1a}$ and $\phi_{1b}$, the deflection angles corresponding to $\alpha_{1a}$ and $\alpha_{1b}$, rarely differ by more than two degrees and that therefore it suffices to approximate $C_1$ by $C_1'$, $C_2$ by $C_2'$ and to put

$$\hat{\alpha}_{1a} = \hat{\alpha}_{1b} = \hat{\alpha}_1.$$

The response to the free-space second harmonic is calculated in a like manner, the differential contribution $v_{2R}$ being given by

$$v_{2R} = \frac{Y_R}{N} R^{\gamma} \frac{n}{2} \cos^2 d \sum_{n=1}^{N} B_{Rn} \cos^2 \delta_{Rn} \cos 2(\phi' - \alpha_s + \phi_{2Rn}) \qquad (6.21)$$

where $B_{Rn}$ is analagous to $A_{Rn}$, being the amplitude reduction factor for a second harmonic in relation to the $n^{\text{th}}$ segment of the asymptotic cone.

The vector form of $v_{2R}$ is

$$\hat{v}_{2R} = | v_{2R} | \hat{\alpha}_{2R}$$

and we have finally the total second harmonic vector

$$\hat{v}_2 = | v_2' | \hat{\alpha}_2 = \sum_{R=Rc}^{\infty} \hat{v}_{2R}$$

$$= C_3 \frac{n}{2} \cos^2 d \hat{\alpha}_2, \qquad (6.22)$$

where

$$C_3 \hat{\alpha}_2 = \sum_{R=Rc}^{\infty} \frac{Y_R}{N} R^{\gamma} \sum_{n=1}^{N} B_{Rn} \cos^2 \delta_{Rn} \hat{\alpha}_{2Rn} \qquad (6.23)$$

In summary, the integrated asymptotic amplitude and phase constants of the anisotropy are derived in sequence as follows:

$$\left.\begin{aligned} C_1 &= C_1' \text{ (eqn. 6.19)} \\ C_2 &= C_2' \text{ (eqn. 6.20)} \end{aligned}\right\} \rightarrow \bar{\delta}, \mathrm{K}_1$$

$$C_3 = C_3' \text{ (eqn. 6.23)} \rightarrow \mathrm{K}_2$$

$$\hat{\alpha}_{1a} = \hat{\alpha}_{1b} = \hat{\alpha}_1 \rightarrow \phi_1$$

$$\hat{\alpha}_2 \rightarrow \phi_2.$$

The constants $\tilde{\delta}$, $K_1$, $K_2$, $\phi_1$ and $\phi_2$, appropriate to the given detector, may now be estimated for selected values of $\gamma$ and $R_c$. Together with $k_c$ (see equation (6.6a)), these constants relate the observed harmonics $v_1$ and $v_2$ to the corresponding integrated free-space harmonics $V_1$ and $V_2$.

It will be shown in Section 7 that, if sets of estimated free-space harmonics are available from two, or preferably three, well-separated latitudes of viewing, these suffice for the determination of the *declination* $d$ of the anisotropy. That is to say, $d$ can be estimated without having to estimate $m$ and $n$, the amplitude constants of the anisotropy.

### 6A.5. THE CALCULATIONS IN DETAIL: EVALUATION OF $\delta_{Rn}$, $A_{Rn}$, $\phi_{Rn}$, $B_{Rn}$, $\phi_{2Rn}$

*(a) Method of determination*

In calculating the response underground to a first harmonic of the anisotropy we first determine sets of values of $A_{Rn}$, $\delta_{Rn}$, and $\phi_{Rn}$ for selected rigidities so that by curve-fitting they may be read off as functions of $R$. It should be noted that $A_{Rn}$, $\delta_{Rn}$ and $\phi_{Rn}$ are *constants of the detector* at the given location and may be used to determine the response to any free-space first harmonic, either solar or sidereal. Corresponding quantities $A_{2Rn}$, $\delta_{2Rn}$ and $\phi_{2Rn}$, relating to a second harmonic, may be obtained by the same procedures.

Consider the $\mu$-mesons, due to isotropic primaries of rigidity $R$, which arrive at the detector within a small solid angle $\omega_r$ specified by zenith angle $Z_r$ and azimuth $a_r$. They constitute a fraction of the total counting rate that is known as the differential radiation sensitivity $I(R, \omega_r)$ given by

$$I(R, \omega_r) = Y_R F(\omega_r) \cos^n Z_r \qquad (6.24)$$

The factor $F(\omega_r)$, the geometric sensitivity (viz., Parsons, 1957), is the fraction of the total counting rate apportioned to $\omega_r$ by virtue of the geometry of the telescope, while $\cos^n Z_r$ expresses the well-known zenith angle dependence of intensity. The primaries of rigidity $R$ that are responsible for $I(R, \omega_r)$ will have approached the earth's magnetic field from directions within some asymptotic volume element $(\Omega_r)_R$ specified by latitude $(\delta_r)_R$ and longitude $(\phi_r)_R$. Such asymptotic co-ordinates are usually found either by computing the trajectories of the primaries outward from the direction $(Z_r, a_r)$ at the geomagnetic location of the recorder (viz., McCracken *et al.* 1962) or else from observations of the deflections of charged particles in a physical simulation of the geomagnetic field (viz., the terrella experiments of Brunberg *and* Dattner 1953). When $I(R, \omega_r)$ and the associated asymptotic co-ordinates $(\delta_r)_R$ and $(\phi_r)_R$ have been calculated for each of the volume elements $\omega_r$ which make up the solid angle of the recorder, the distribution of the fractional $\mu$-meson intensity due to primaries of rigidity $R$ can be determined with respect both to asymptotic latitude and asymptotic longitude of the primaries. The distribution with latitude can be simply divided up amongst $N$ latitude intervals giving equal contributions to the differential counting rate, thereby defining the $N$ segments of the total asymptotic cone. The mean latitudes $\delta_{Rn}$ are obtained from the latitude distribution within the individual segments. The distribution with respect to longitude within each segment, when applied to a first harmonic cos $\phi$, provides the amplitude reduction factor $A_{Rn}$ and the longitude displacement angle $\phi_{Rn}$. However, it should be noted that the distributions themselves are derived on the assumption of isotropic primaries and an average cosmic ray spectrum.

When the constants $A_{Rn}$, $\delta_{Rn}$ and $\phi_{Rn}$ are applied to a particular model for the anisotropy, the important simplification in the present treatment is that the mean value $\delta_{Rn}$ replaces a latitude distribution. This suggests that the latitude intervals specifying the segments of the asymptotic cones should be small. Several factors are involved in the choice of intervals, one of them being the manner of variation of the longitude distribution with respect to asymptotic latitude. As far as the underground vertical semi-cube is concerned, the final result does not seem to depend at all markedly on the number of latitude intervals used.

*(b) Application to detectors : (i) the vertical semi-cube underground*

The solid angle of the semi-cube was divided up into 576 elements ($\omega_r$) of dimensions 5° in aximuth and 7·5° in zenith angles. In calculating the values of $I(R, \omega_r)$, Fenton's coupling coefficients and the geometric sensitivity characteristics worked out by Parsons were used. Both $n = 2{\cdot}0$ and $n = 2{\cdot}2$ were tried for the $\cos^n Z$ zenith angle dependence of intensity, but for all practical purposes they each led to the same final asymptotic distribution.

Since about 90% of the counting rate underground appears to be due to primaries of energy > 50 GEV, it was considered that a centred dipole was a sufficient approximation to the real geomagnetic field for calculations of the asymptotic co-ordinates $(\delta_r)_R$. and $(\phi_r)_R$. [Calculations using a more complex representation of the field tended to confirm this view and are described in Paper 7, Section IV (a).] Consequently, the particle deflections were estimated from the diagrams of Brunberg *and* Dattner for the geomagnetic southern latitude of 50°. Considerable interpolation was necessary and for this reason it was found more convenient to express the asymptotic latitude and longitude data in the form of deflections in zenith angle and azimuth. A technique for converting co-ordinates by means of a terrestrial globe is described in Appendix I, where a general method of obtaining radiation sensitivity $I(R. \omega_r)$, as a function of asymptotic direction of viewing, is described.

At the higher rigidities, the differential cones of acceptance change rather slowly with rigidity and therefore it was decided to work them out only for the three most important rigidities, namely 50 GV (a practical lower limiting rigidity of response), 150 GV (near the mean rigidity of response) and infinite rigidity (providing an asymptote for the response constants). Figures I.2 to I.9 (Appendix I) give the deflection data for particles of rigidity 50 GV and 150 GV at the geomagnetic latitude of observation $\lambda = -50°$.

Each cone of acceptance was initially divided up into three segments defined by intervals of asymptotic geographic latitude. For example, it was found that of the 50 GV primaries contributing to the counting rate, one third came from the latitude range 90°S to 46°S, one third from the range 45°S to 28°S and the rest from the range 27°S to 20°N. These high, middle and low latitude intervals were found to be almost exactly the same at the higher rigidities.

The mean latitude $\delta_{Rn}$ was worked out for each segment from the distribution $I(R, \omega_r)$ versus $(\delta_r)_R$, after the data had been grouped at 3° intervals of latitude. The distribution $I(R, \omega_r)$ *versus* $(\phi_r)_R$ was based on longitude intervals of 20° and provided the amplitude constant $A_{Rn}$ and phase constant $\phi_{Rn}$ when a first harmonic cos $\phi$ was impressed on it. In addition, constants $A_R$, $\delta_R$ and $\phi_R$ were obtained for the total (undivided) cone. Values of the asymptotic constants for individual segments and for the total cone are given in Table 6.1 for each of the three rigidities. It can be seen that (a) above 150 GV the cone of acceptance does not change with increasing rigidity except for a gradual displacement towards the observer's meridian, and that (b) the distribution $I(R, \omega_r)$ versus $(\phi_r)_R$ changes only slightly with latitude, as would be expected.

TABLE 6.1.

Asymptotic constants for the vertical semi-cube underground at 50° geomagnetic latitude. The constants specify the differential response to a free-space first harmonic at the primary rigidities 50 GV, 150 GV and infinity.

Primary Rigidity (GV)

| | 50 | | | 150 | | | ∞ | | |
|---|---|---|---|---|---|---|---|---|---|
| | $A_{Rn}$ | $\delta_{Rn}$ | $\phi_{Rn}$ | $A_{Rn}$ | $\delta_{Rn}$ | $\phi_{Rn}$ | $A_{Rn}$ | $\delta_{Rn}$ | $\phi_{Rn}$ |
| High Latitude Range (H) | 0·84 | −56° | 25° | 0·75 | −60° | 12° | 0·74 | −62° | 0° |
| Mid Latitude Range (M) | 0·86 | −36° | 35° | 0·86 | −38° | 10° | 0·86 | −41° | 0° |
| Low Latitude Range (L) | 0·91 | −13° | 33° | 0·90 | −15° | 12° | 0·89 | −18° | 0° |
| Total Cone | 0·87 | −34° | 30° | 0·83 | −39° | 11° | 0·83 | −39° | 0° |

Curves of fit were drawn through values of $A_R$, $\delta_R$ and $\phi_R$ given at the bottom of the Table and are shown in Figure 6.2. These curves were used in all subsequent calculations requiring values of the asymptotic constants as functions of $R$. In a similar manner, curves of $B_R$ and $\phi_{2R}$ may be drawn as functions of $R$, characterizing the differential response to a generalized second harmonic.

*(ii) The cube inclined 30°N underground*

In the axial direction, the telescope happens to view upwards along the magnetic field at Hobart and at the same time this represents a low latitude of viewing. It is for this reason that the asymptotic acceptance cones are relatively compact, centred only slightly east of the observer's meridian and are not strongly dependent on rigidity, while the distribution of $I(R, \omega_r)$ with asymptotic longitude is virtually independent of asymptotic latitude. Consequently the constants $A_R$, $\delta_R$ and $\phi_R$ can be computed from the undivided cones of acceptance with rather less loss of accuracy than in the case of the vertical semi-cube.

*(iii) The south-pointing cube underground*

The axial direction of the telescope is 45° south of the zenith and this corresponds to a geographic latitude of 88°S. The situation is the reverse of that in the N-pointing direction. Directions of arrival tend to be transverse to the magnetic

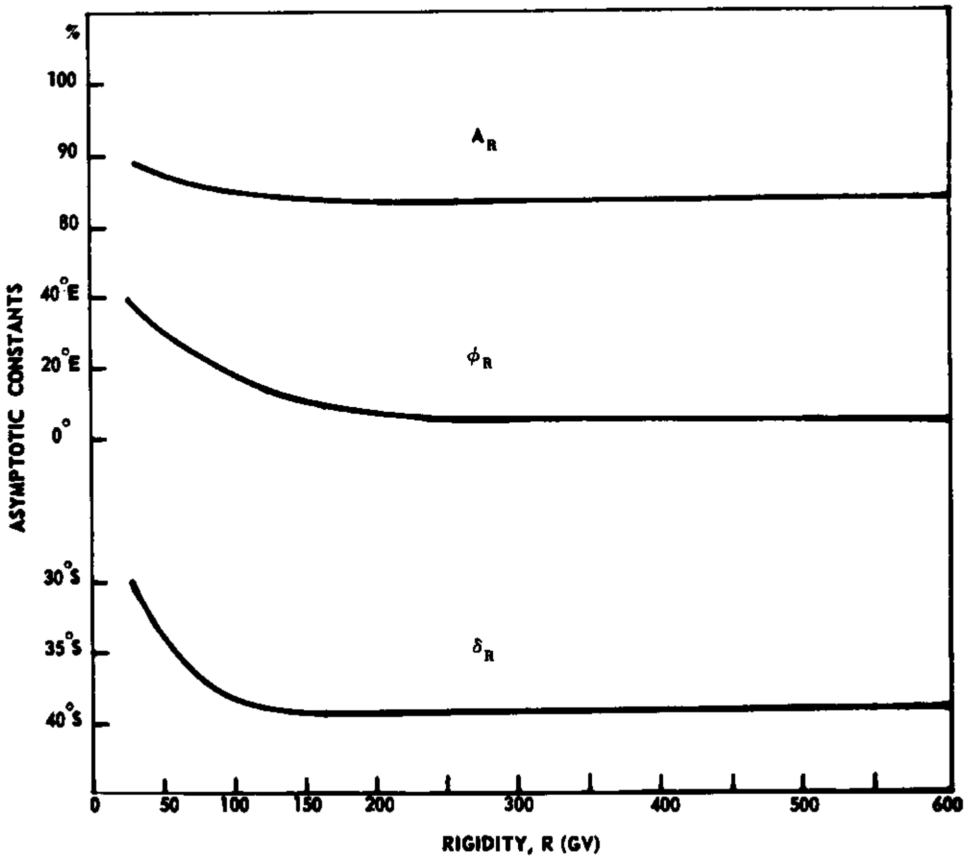



FIG. 6.2. The differential asymptotic constants of response to a diurnal variation of the primaries in free space, for a vertical semi-cubical telescope at a depth of 40 m.w.e. at $\lambda = 50°S$.

field and at higher zenith angles are accessible from all 360° of asymptotic longitude. The cones of acceptance, divided up into high, middle and low latitude segments (Figure 6.3), are centred far to the east of the observer's meridian at rigidities near the cut-off and change markedly with increasing rigidity. Figure 6.4 shows how the spread in asymptotic latitude varies with rigidity.

It is clearly necessary to use divided cones of acceptance for the calculation of response in the south-pointing direction. It would also be desirable in an improved treatment to determine the response at other values of rigidity between 50 GV and 150 GV.

In Figure 6.5 are shown curves of the estimated differential response constants as functions of primary rigidity.

### 6A.6. THE CALCULATIONS IN DETAIL: DETERMINATIONS OF $\vec{\delta}$, $K_1$, $K_2$, $\phi_1$ AND $\phi_2$

In order to proceed from the differential response constants of the detector to the integrated constants that relate to the anisotropy, it is necessary to obtain

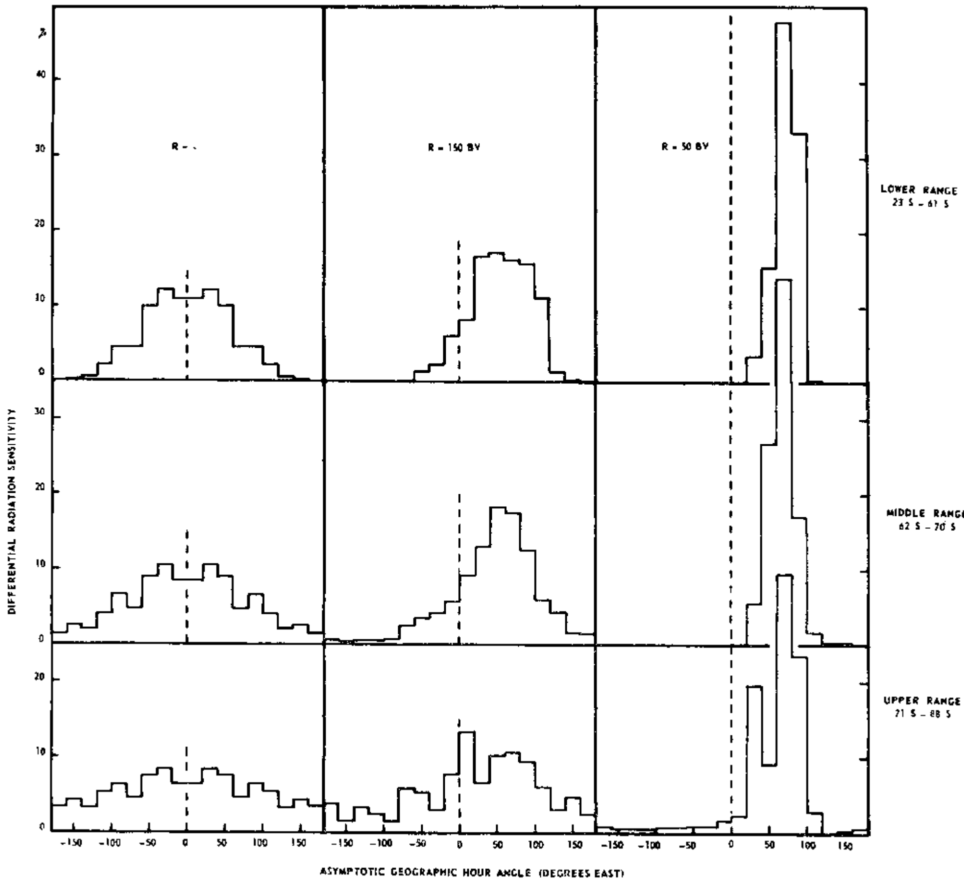



FIG. 6.3. Differential asymptotic cones of acceptance for sectional latitude ranges of viewing, relating to a cubical telescope inclined 45° south of zenith at a depth of about 40 m.w.e. at Hobart. The asymptotic latitude ranges have been chosen to give approximately equal contributions to the differential counting rate that derives from primaries of the given rigidity.

the differential coupling coefficients, $Y_R$, and to select values for $\gamma$, the index of the variation spectrum, and for $R_c$, the primary threshold rigidity for observation of the anisotropy.

*(a) The coupling coefficients*

Fenton's coupling coefficients relate the vertical intensity of $\mu$-mesons at a depth of 40 m.w.e. to the primary proton spectrum. In Figure 6.6 the relationship is presented in the form of an integrated response curve. The corresponding curves for mesons at sea level (Quenby *and* Webber 1959) and neutrons at sea level (Dorman 1957) are shown for comparison. It would appear that only 8% of the counting rate underground is due to protons of energy $<$ 50 GEV as against 50% in respect of the meson intensity at sea level and 74% in respect of the neutron intensity.

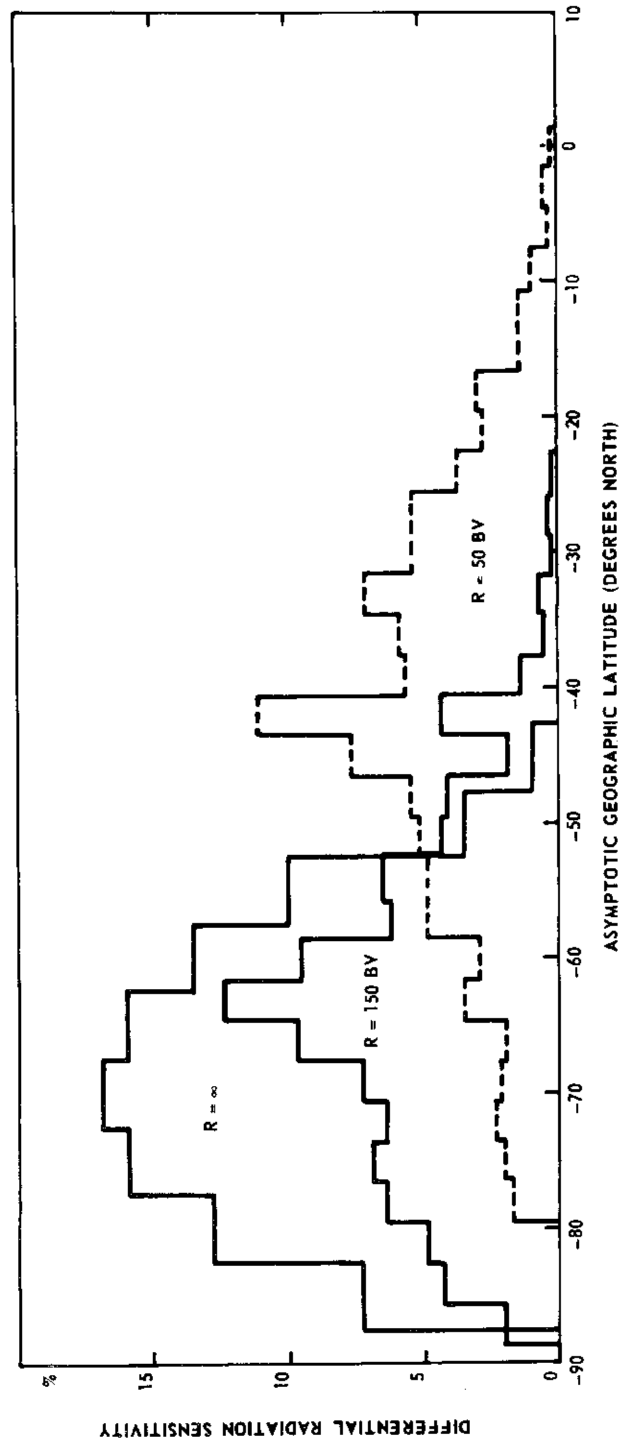



FIG. 6.4. Response characteristics of a cubical telescope inclined 45° south of zenith at a depth of about 40 m.w.e. at Hobart. The estimated contributions to the differential counting rate are shown as functions of asymptotic latitude of viewing, for primaries of low, average and infinite rigidity respectively.

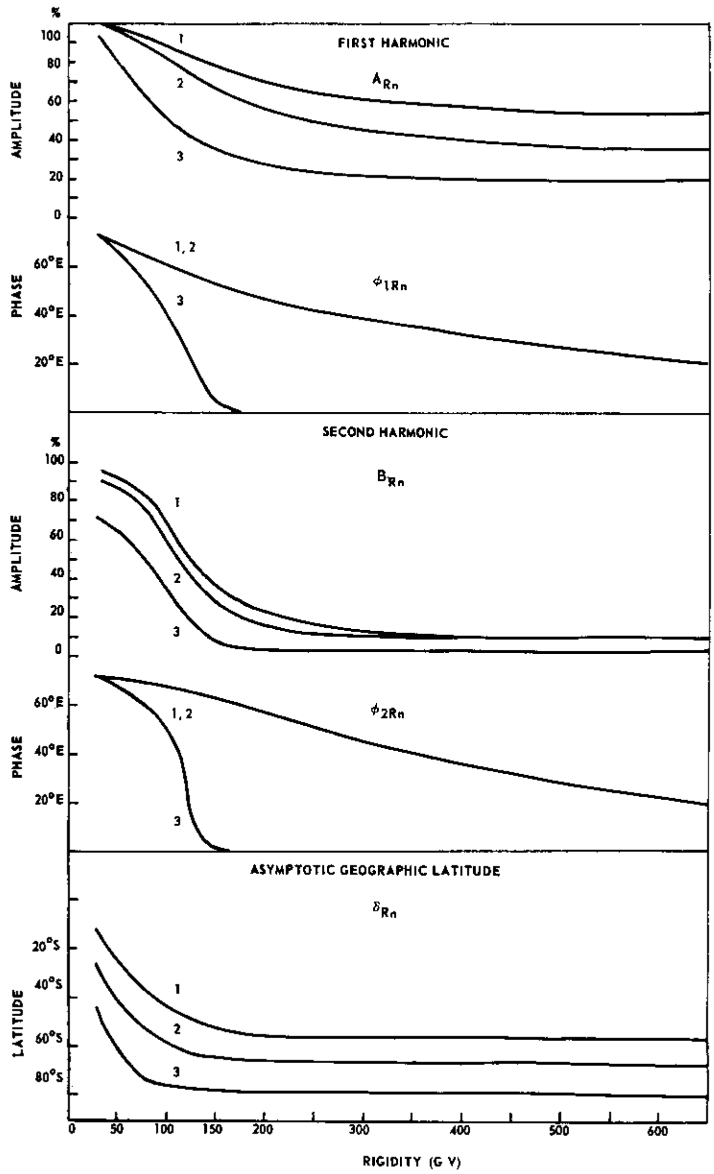



FIG. 6.5. The differential response constants of the south-pointing cube underground, relating to cones of acceptance divided up into low, middle and high latitude segments according as $n$ takes the values 1, 2 and 3 respectively.

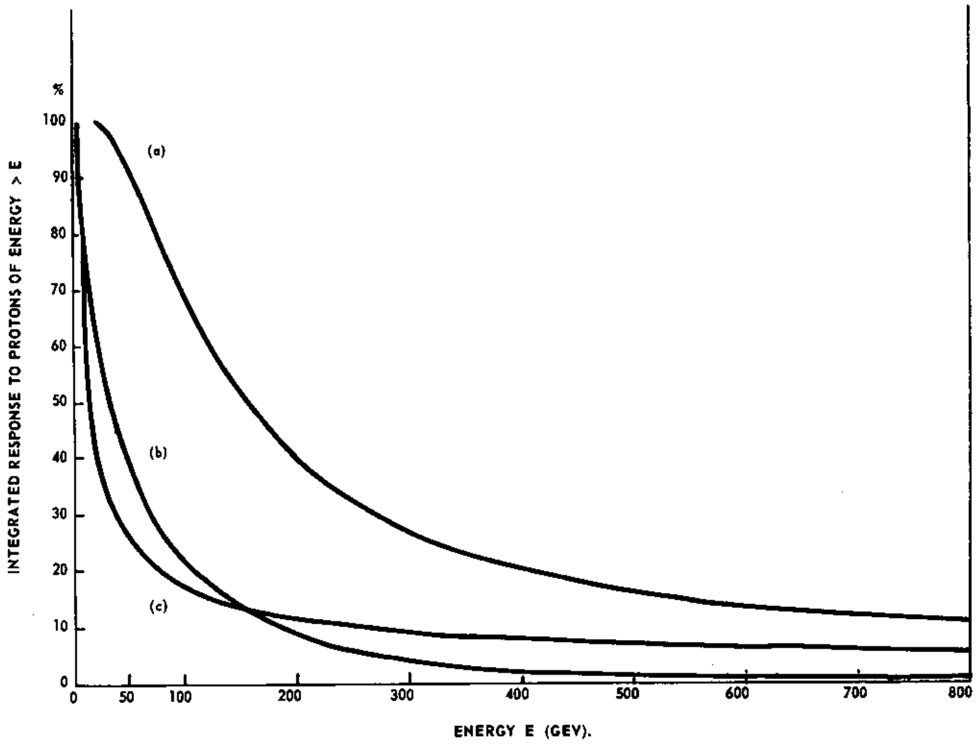



FIG. 6.6 Estimated proportion of the counting rate due to primary protons of energy > E, in respect of:

(a) meson intensity at 40 m.w.e. underground (from Fenton 1963)
(b) meson intensity at sea level (from Webber *and* Quenby 1959)
(c) neutron intensity at sea level (from Dorman 1957).

Fenton's coefficients derive from considerations of the theoretical yield of pions of energy $\geqslant$ 15 GEV (it being assumed that 15 GEV is the minimum energy required by a $\mu$-meson at production to penetrate the atmosphere plus 40 m.w.e. of material absorber). A similar result was obtained by Mathews (1963) from an empirical response curve extrapolated from latitude-intensity data.

The provisional nature of the coefficients is an important factor inhibiting accurate estimations of response to anisotropies, in particular the solar anisotropy. In respect of the sidereal anisotropy the existing coefficients should suffice for the preliminary calculations of response. However, revised estimates of the underground absorber already suggest that the coupling coefficients for the Hobart detectors should be appropriately modified. In Section 4.7 it was shown that the mean cut-off at production for the vertical semi-cube must be approximately 11 GEV. From this point of view the existing coefficients tend to underestimate the contributions to the counting rate from the lower energy primaries. As against this, Fenton (1963) has pointed out that his calculations of the yield of pions may have led to an overestimation of the response to the low energy primaries.

*(b) The index, $\gamma$, of the variation spectrum*

The little evidence that is available, from the comparison of (i) observed sidereal diurnal amplitudes at sea level and underground (amplitudes comparable), and (ii) the amplitude observed at approximately 60 m.w.e., London, and that observed at approximately 40 m.w.e., Budapest (amplitudes equal, within statistical errors of $\sim$ 25%), suggests that the amplitudes of the free-space harmonics of the sidereal anisotropy are not steeply rigidity-dependent.

Again, if the anisotropy is connected with a Fermi type of acceleration process —although as noted in Section 1, there are other possibilities to consider as well— the amplitude would be expected to decrease with increasing rigidity.

However, it is only of concern in this report to find out whether estimates of the direction of the anisotropy depend weakly or strongly on the values assigned to $\gamma$. Consequently, it has been decided to compare estimations based on $\gamma = 0$ and $\gamma = -2$, on the supposition that the true value of $\gamma$ might lie somewhere between these two.

*(c) The primary threshold rigidity, $R_c$, for observation of the anisotropy*

We are guided here by the estimations of the upper limiting rigidity, $R_u$, for observation of the solar anisotropy. Provisionally, $R_c$ is equated with $R_u$, although it is recognized (see discussion in Paper 7, Section 9) that they are not necessarily the same. For this reason, and from the indications that $R_u$ itself changes considerably during the course of the cycle of solar activity, the asymptotic constants are calculated over a range of values of $R_c$.

*(d) Determination of the constants*

Approximate values of the constants, referring to the vertical semi-cubical telescope and the north-pointing and south-pointing cubical telescopes, were obtained by vector summations. Table 6.2 and 6.3 list the results of the determinations for $\gamma = 0$ and $\gamma = -2$ respectively, and for the following values of $R_c$: 20, 50, 70, 100, 150 and 200 GV. In Figure 6.7, the relative (observed/free-space) diurnal and semi-diurnal amplitudes of response that apply to the vertical telescopes are shown versus $R_c$ for the two values of $\gamma$. They are in fact the tabulated values of $K_1/k_c$ ($= K_1$ when $\gamma = 0$).

Since the constants have been determined for the geomagnetic latitude $\lambda = -50°$, it has been considered sufficient in this treatment to use the asymptotic constants for the Hobart ($\lambda = -52°$) and Budapest ($\lambda = +45°$) vertical semi-cubical telescopes. It will be noted that the appropriate values of the mean asymptotic latitude of viewing at Budapest, $\bar{\delta}_{Bud}$, are listed under the Hobart values.

## 6B. THE SOLAR ANISOTROPY

Brief reviews are given here of two features of the solar anisotropy: the seasonal characteristics of the *free-space* diurnal variation and the upper limiting rigidity for observation of the anisotropy, as described in Papers 1 and 7 respectively.

TABLE 6.2.

Integrated constants of response relating to underground observations of the sidereal anisotropy, for $\gamma = 0$.

| | | *Cut-off Rigidity* $R_c(GV)$ | | | | | |
|---|---|---|---|---|---|---|---|
| | | *20* | *50* | *70* | *100* | *150* | *200* |
| Semi-cube vertical (Hobart and Budapest) | $K_1$ | 0·828 | 0·764 | 0·676 | 0·566 | 0·424 | 0·327 |
| | $\phi_1$ | 15° | 12° | 10° | 8° | 7° | 6° |
| | $\delta_{HOB}$ | −37° | −38° | −39° | −39° | −39° | −39° |
| | $\delta_{BUD}$ | +42° | +43° | +43° | +43° | +43° | +43° |
| | $K_2$ | 0·509 | 0·460 | 0·405 | 0·343 | 0·257 | 0·199 |
| | $\phi_2$ | 16° | 12° | 10° | 8° | 7° | 6° |
| Cube 30°N of zenith (Hobart) | $K_1$ | 0·942 | 0·864 | 0·756 | 0·646 | 0·485 | 0·380 |
| | $\phi_1$ | 4° | 3° | 2° | 1° | 1° | 1° |
| | $\delta$ | −18° | −18° | −18° | −18° | −17° | −17° |
| | $K_2$ | 0·812 | 0·754 | 0·656 | 0·567 | 0·415 | 0·326 |
| | $\phi_2$ | 4° | 3° | 2° | 1° | 1° | 1° |
| Cube 45°S of zenith (Hobart) | $K_1$ | 0·465 | 0·415 | 0·341 | 0·269 | 0·189 | 0·146 |
| | $\phi_1$ | 53° | 49° | 45° | 39° | 31° | 25° |
| | $\delta$ | −45° | −52° | −55° | −59° | −61° | −61° |
| | $K_2$ | 0·320 | 0·265 | 0·173 | 0·128 | 0·063 | 0·029 |
| | $\phi_2$ | 67° | 65° | 62° | 56° | 41° | 34° |

TABLE 6.3.

Integrated constants of response relating to underground observations of the sidereal anisotropy, for $\gamma = -2$.

| | | *Cut-off Rigidity* $R_c(GV)$ | | | | | |
|---|---|---|---|---|---|---|---|
| | | *20* | *50* | *70* | *100* | *150* | *200* |
| | $\kappa_c \times 10^3$ | 1·07 | 0·172 | 0·087 | 0·043 | 0·019 | 0·011 |
| Semi-cube vertical (Hobart and Budapest) | $K_1/\kappa_c$ | 0·125 | 0·296 | 0·339 | 0·334 | 0·277 | 0·210 |
| | $\phi_1$ | 33° | 20° | 17° | 11° | 7° | 6° |
| | $\delta_{HOB}$ | −32° | −37° | −40° | −40° | −40° | −40° |
| | $\delta_{BUD}$ | +37° | +42° | +44° | +44° | +44° | +44° |
| | $K_2/\kappa_c$ | 0·084 | 0·180 | 0·215 | 0·214 | 0·179 | 0·139 |
| | $\phi_2$ | 35° | 21° | 17° | 11° | 7° | 6° |
| Cube 30°N of zenith (Hobart) | $K_1/\kappa_c$ | 0·136 | 0·325 | 0·376 | 0·386 | 0·332 | 0·255 |
| | $\phi_1$ | 10° | 5° | 4° | 1° | 1° | 1° |
| | $\delta$ | −19° | −17° | −17° | −17° | −17° | −17° |
| | $K_2/\kappa_c$ | 0·111 | 0·273 | 0·319 | 0·330 | 0·281 | 0·217 |
| | $\phi_2$ | 10° | 5° | 4° | 1° | 1° | 1° |
| Cube 45°S of zenith (Hobart) | $K_1/\kappa_c$ | 0·121 | 0·249 | 0·217 | 0·186 | 0·134 | 0·098 |
| | $\phi_1$ | 69° | 63° | 56° | 48° | 47° | 40° |
| | $\delta$ | −29° | −43° | −48° | −47° | −60° | −63° |
| | $K_2/\kappa_c$ | 0·102 | 0·197 | 0·137 | 0·065 | 0·036 | 0·014 |
| | $\phi_2$ | 71° | 69° | 67° | 65° | 49° | 42° |

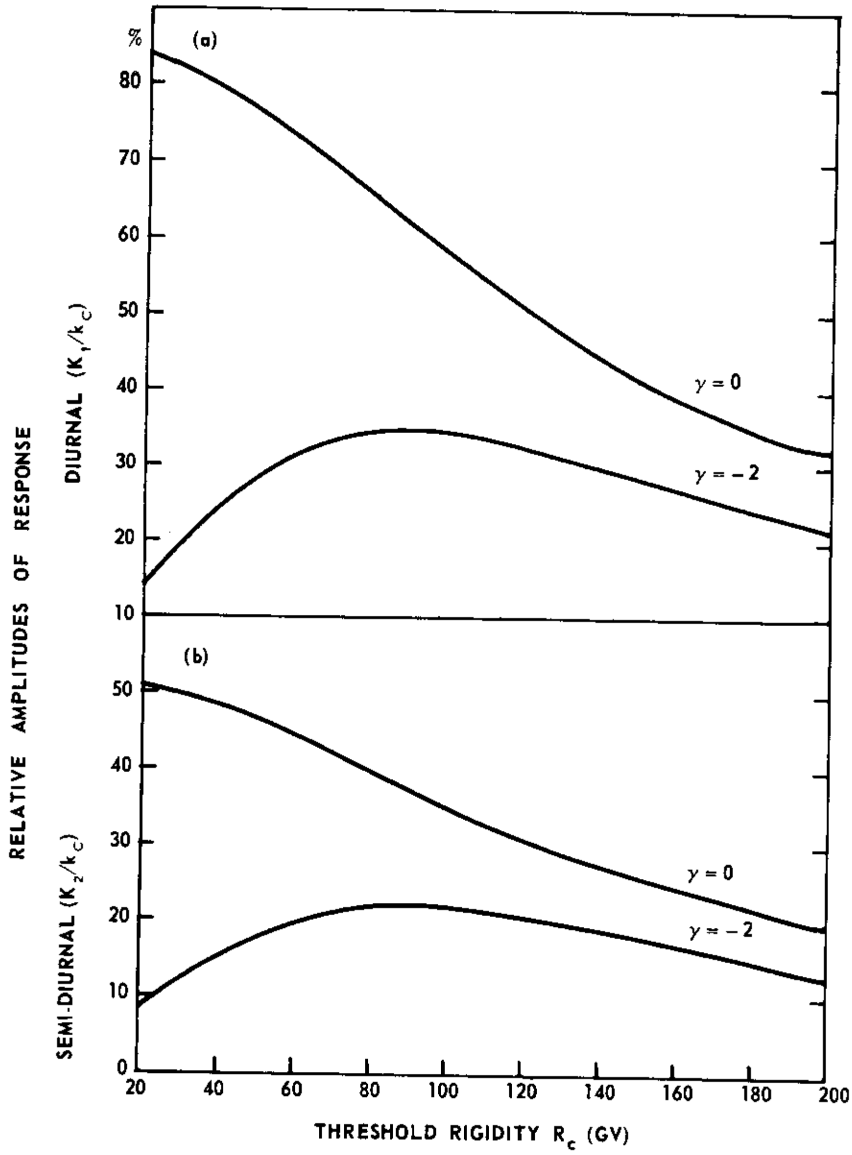



FIG. 6.7. Calculated amplitudes of response to the two-way anisotropy from a vertical semi-cubical telescope at a depth of ~ 40 m.w.e., Hobart, relative to the corresponding free-space amplitudes.

### 6B.1. THE SEASONAL CHARACTERISTICS OF THE FREE-SPACE DIURNAL VARIATION

The solar anisotropy as originally described by Rao *et al.* (1963) was specified in the system of spherical co-ordinates whose reference planes were the ecliptic and the plane normal to it through the earth-sun line, this being the frame of reference in which the anisotropy was assumed to be fixed. The anisotropic component of intensity for primaries of rigidity $R$ was specified as

$$\Delta I(R, \Lambda, \psi) = AI_o(R) \cos \Lambda \cos (\psi - \psi_o) \qquad (6.25)$$

where $A$ = the amplitude constant;
$I_o(R)$ = the average differential rigidity spectrum;
$\Lambda$ = the ecliptic latitude of the detector;
$\psi$ = the direction of viewing in the plane of the ecliptic, east of the earth-sun line;
$\psi_o$ = the direction of maximum intensity.

After transforming to the conventional ecliptic system, specified by the plane of the ecliptic and the equinox, it is necessary to transform to the frame of reference of the observer, the terrestrial equatorial system. This is achieved by rotation of axes through the angle $\varepsilon$, the obliquity of the ecliptic, as shown in Figure 6.8. In terrestrial co-ordinates the expression for the anisotropy outside the earth's field region then becomes (see equation (4) of Paper 1):

$$\Delta I(R_1\alpha_s, \delta, \phi) = AI_o(R)\{[1 - 0{\cdot}0826 \sin^2 (\alpha_s + \psi_o)] \cos \delta \cos (\phi - \psi_o)$$
$$-0{\cdot}0413 \cos \delta \sin 2(\alpha_s + \psi_o) \sin (\phi - \psi_o) + 0{\cdot}3979 \sin \delta \sin (\alpha_s + \psi_o)\} \qquad (6.26)$$

(for $\varepsilon = 23° 27'$),

where $\alpha_s = RA$ of the sun at transit at Greenwich,
$\phi$ = time of day relative to local noon,
$\delta$ = geographic latitude of observation.

Bearings of interest in the ecliptic and terrestrial equatorial planes are shown in Figure 6.9.

The presence of the annually varying factor $\alpha_s$ leads to an annual variation of the secular component $0{\cdot}3979 \sin \delta \sin(\alpha_s + \psi_o)$ and to a semi-annual variation of the diurnal component. If the diurnal component is the same on any given day as it was six months previously, it then must vanish when arranged in sidereal time and averaged over a complete year, since all days can be grouped in pairs six months apart. Therefore semi-annual modulation of this kind cannot possibly be responsible for a spurious sidereal effect.

By similar methods, some preliminary calculations have been made for a solar anisotropy whose latitude dependence of amplitude differs from $\cos \Lambda$. The possibility of latitude-independence of amplitude has been considered, and of latitude dependence specified by $\cos^2 \Lambda$, $\cos^3 \Lambda$ and $\cos^5 \Lambda$ respectively. In all of these cases it appears that the seasonal variation of the free-space first harmonic must be very small and that spurious sidereal effects are negligible.

Much the same considerations apply if the solar anisotropy is assumed to be located in the solar equatorial plane, and as McCracken *and* Rao (1965) have indicated in their review of the diurnal anisotropy, the free-space diurnal variation must exhibit a semi-annual variation, which they have specified as a semi-annual variation of amplitude. As we have seen, this cannot give rise to an annual sidereal sideband component.

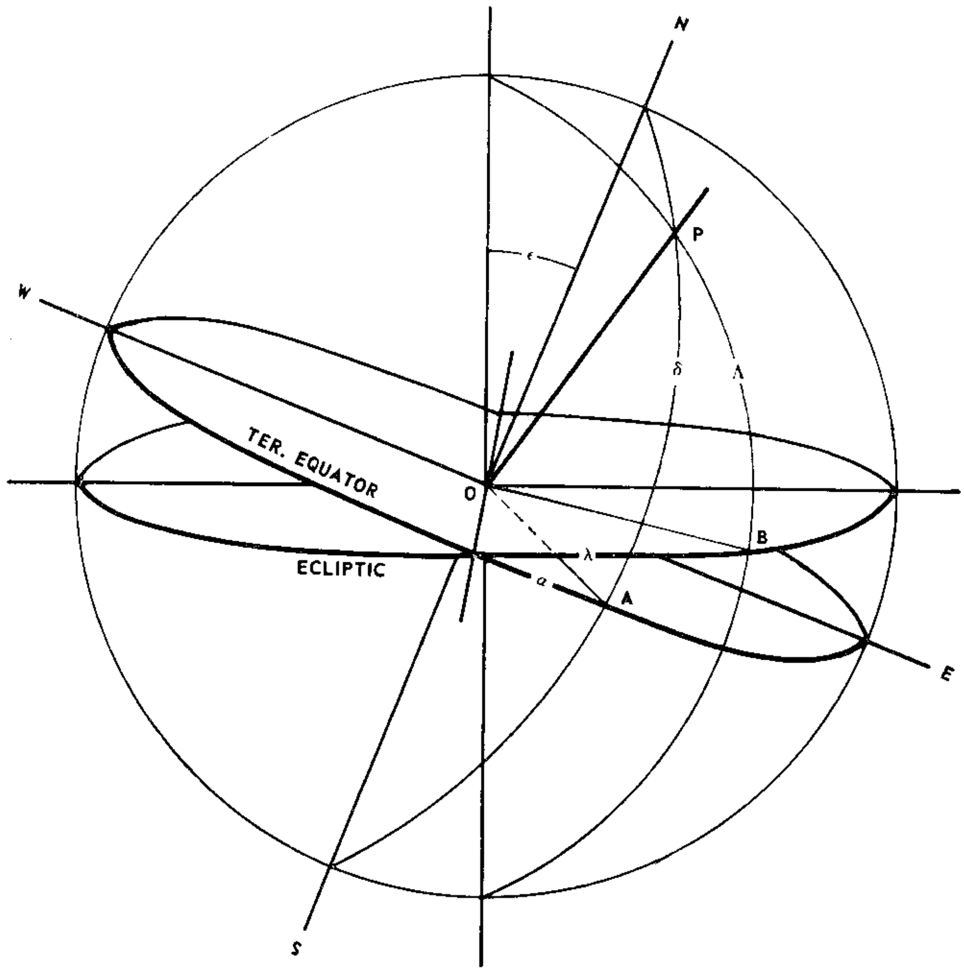



FIG. 6.8. The spherical co-ordinates of the direction of observation, OP, in the terrestrial and ecliptic frames of reference.

It is concluded that, on our present understanding of the solar anisotropy, the transformation from the frame of reference in which it is orientated to the rotating terrestrial system of co-ordinates does not in itself bring into being any significant spurious sidereal effect.

### 6B.2. THE UPPER LIMITING RIGIDITY OF THE SOLAR ANISOTROPY

A method of determining $R_u$ is fully described in Paper 7. It depends on the fact that changes in $R_u$ would produce a much greater response in the solar diurnal variation as observed underground than in the diurnal variation as observed with neutron monitors at high latitude sea-level stations. Only a brief summary of the steps in the determination is given here.

The amplitude constant of response $B_1$, analogous to $K_1$, is calculated for the given detector as a function of $R_u$ (Figure 6.10). Now $B_1$ is $|v|/\alpha_1$, where $|v|$ is

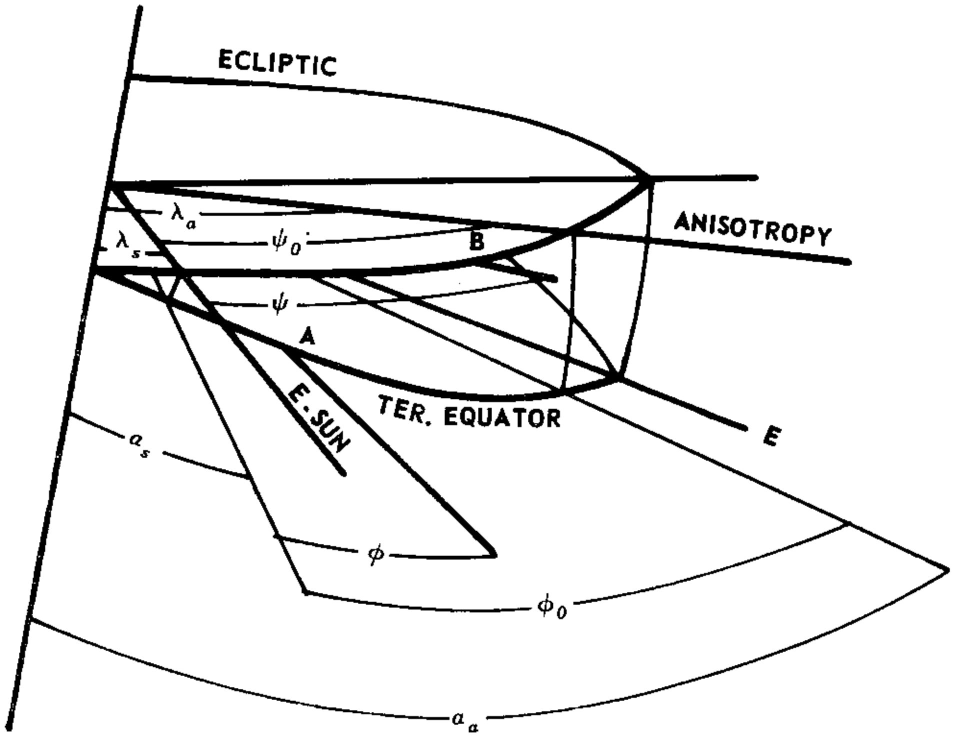



FIG. 6.9. The ecliptic and terrestrial longitude bearings of the observational direction OP, in relation to the Earth-Sun line and to the direction of an anisotropy located in the plane of the ecliptic. The points A and B are the same as those shown in Figure 6.8.

the amplitude of that part of the observed first harmonic that is caused by the primary anistropy, and $\alpha_1$ is the amplitude constant of the free-space first harmonic. Corresponding to a given value of $|v|$ (e.g., an observed annual amplitude, after the removal of estimated extraneous contributions) a curve can be obtained giving estimates of $\alpha_1$ over a range of values of $R_u$.

Provided $R_u$ takes values in excess of, say, 50 GV, the curve relating to a high latitude neutron monitor should be relatively flat, since the observed amplitude will vary only slowly with $R_u$ at these high rigidities. On the other hand, the curve relating to an underground telescope, for which the primary spectrum of response virtually commences at about 50 GV, should be very steep. Consequently, the intersection of the two curves should be clearly defined and should give the actual values of $\alpha_1$ and $R_u$ appropriate to the period of observation.

Figure 6.12 shows the result for the year 1958. Curve B, relating to high latitude neutron monitors, was estimated on the basis of the analysis by Rao *et al.* of the solar diurnal variation of neutron intensity observed that year at the network of sea-level stations. The intersections of curves A and B gave $R_u = 95$ GV with an S.E. of estimate of $\pm$ 5 GV due to count rate statistics. Naturally, other uncertainties connected with the calculations of response would be expected to

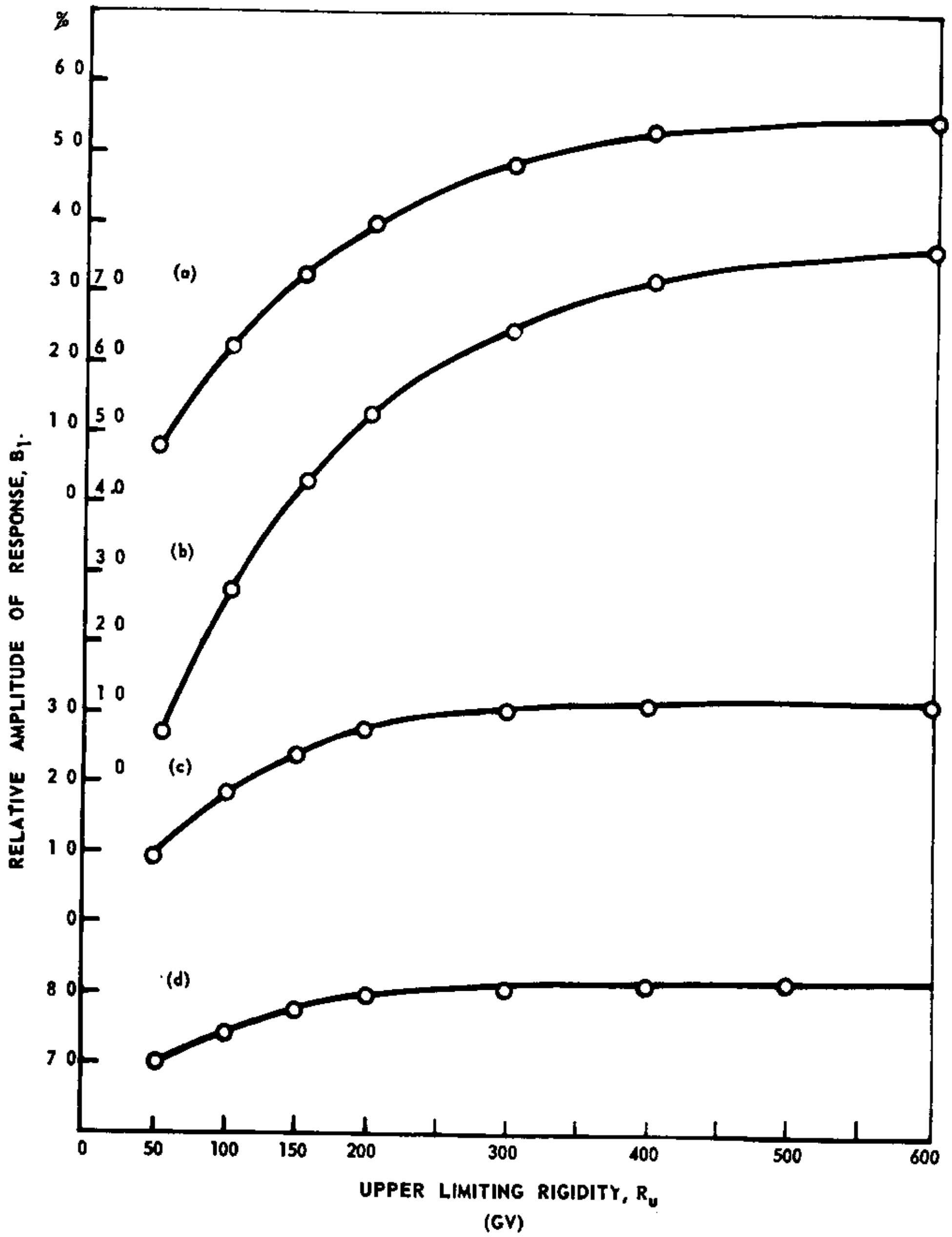



FIG. 6.10. The estimated amplitude of the solar diurnal variation, relative to the amplitude of the average free-space first harmonic, versus upper limiting rigidity, relating to

(a) a vertical semi-cube at a depth of ~ 40 m.w.e. at Hobart
(b) a cube inclined 30° north of the zenith at a depth of ~ 40 m.w.e. at Hobart
(c) a cube inclined 45° south of the zenith at a depth of ~ 40 m.w.e. at Hobart
(d) a high-latitude neutron monitor at sea level.

Circled points indicate calculated values. The model for the free-space first harmonic is that proposed by Rao, McCracken *and* Venkatesen.

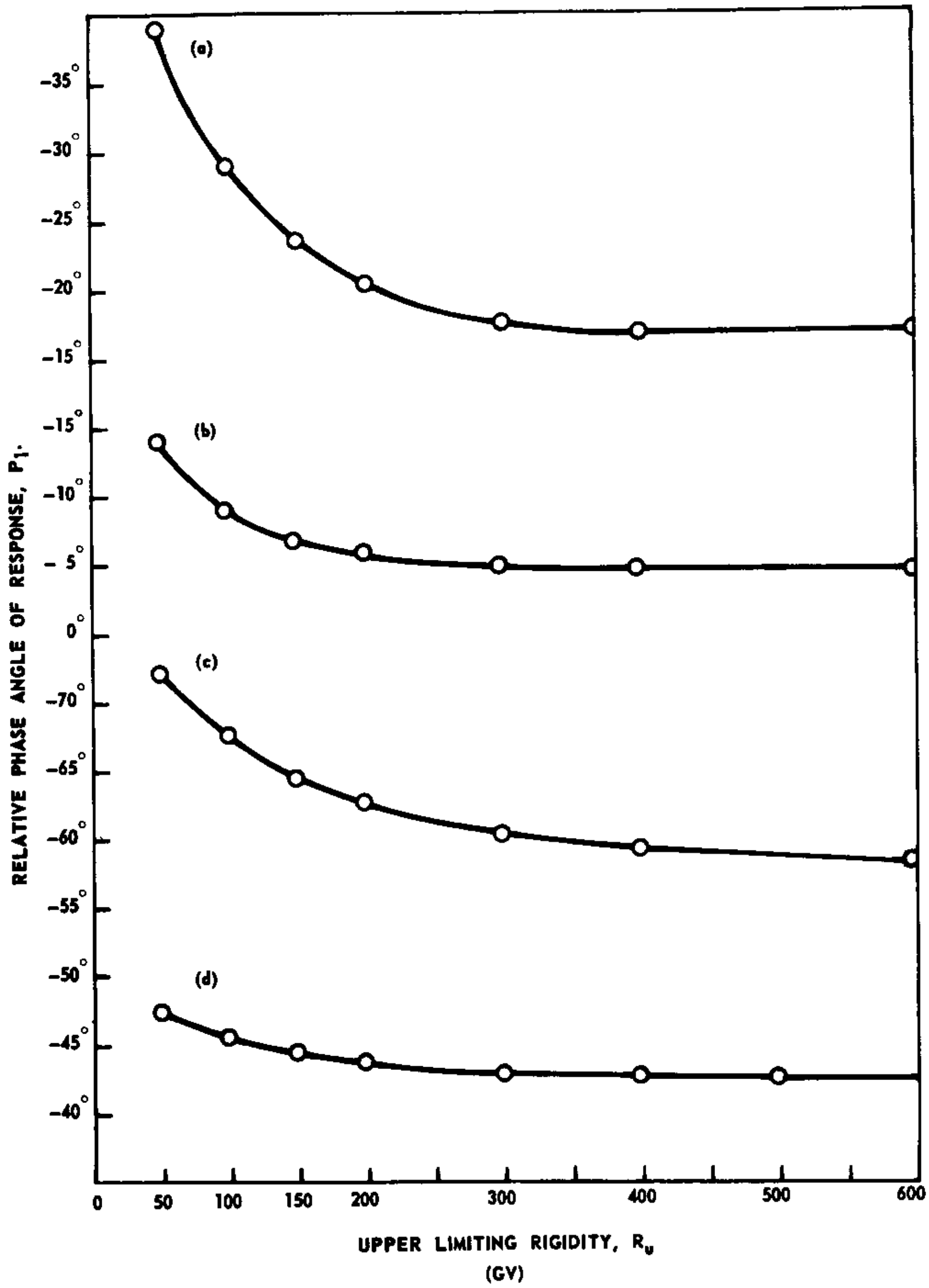



FIG. 6.11. The estimated direction of the maximum of the solar diurnal variation, measured east of the direction of maximum intensity of the free space first harmonic, versus upper limiting rigidity, relating to

(a) a vertical semi-cube at a depth of ~ 40 m.w.e. at Hobart
(b) a cube inclined 30° north of the zenith at a depth of ~ 40 m.w.e. at Hobart
(c) a cube inclined 45° south of the zenith at a depth of ~ 40 m.w.e. at Hobart
(d) a high-latitude neutron monitor at sea level.

Circled points indicate calculated values. The model for the anisotropy is that proposed by Rao, McCracken *and* Venkatesen.

predominate and it was thought that, overall, the true value of $R_u$ should lie within about 20 GV of 95 GV.

The other period examined was that of the underground latitude survey, 1961-1962. At the time there were no means of adequately estimating the new curve B, relating to the neutron monitors, but the displacement of the new curve A relative to that of 1958 indicated a significant decrease in the value of $R_u$ (Figure 6.13). It was noted that the curves deduced from the observations in the inclined directions

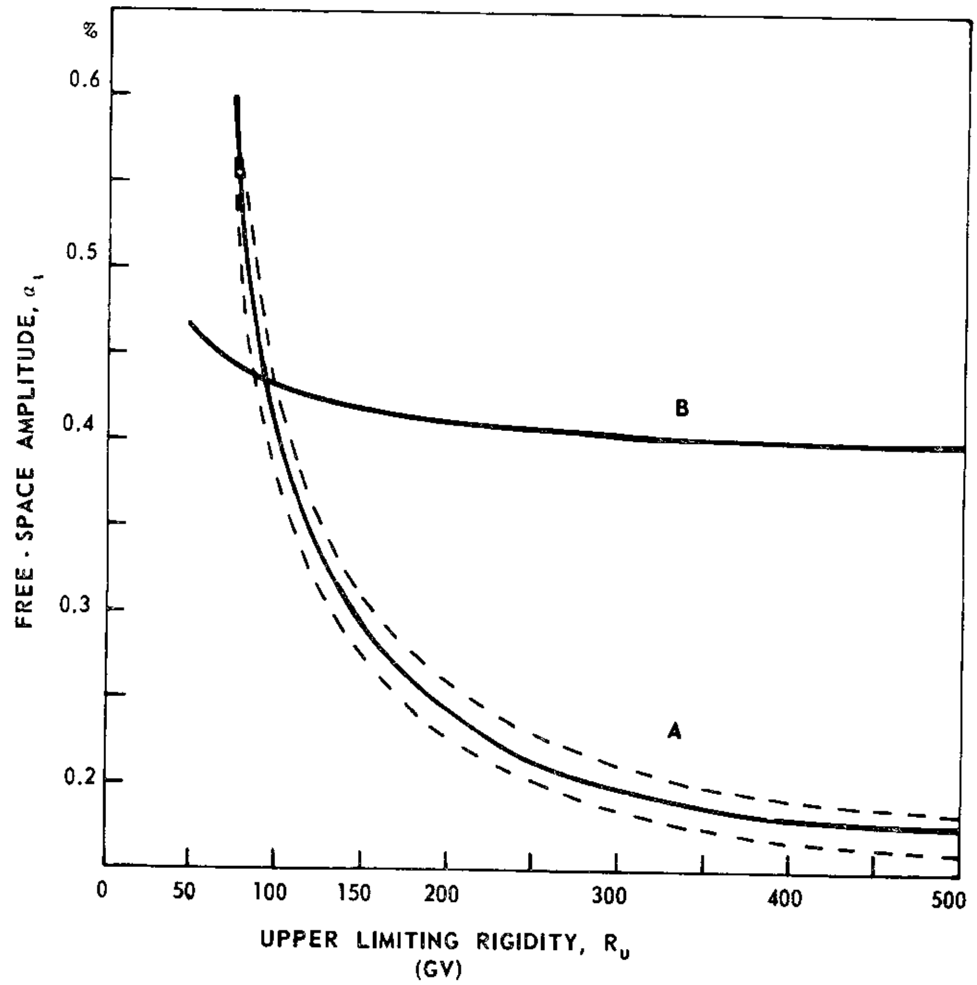



FIG. 6.12. Estimates of the amplitude ($a_1$) of the average free-space solar diurnal variation of the primaries in 1958 for different values of the upper limiting rigidity ($R_u$) based on

(a) observations with a vertical semi-cubical telescope at a depth of 40 m.w.e. at Hobart: curve A;

(b) the estimated solar diurnal variation of neutron intensity at a high-latitude sea-level station, deduced from world-wide observations of neutron intensity: curve B.

The average values of $R_u$ and $a_1$ for 1958 are given by the intersections of the two curves. All the estimates derive from the model for the free-space first harmonic proposed by Rao *et al.* The dashed lines represent the S.E.'s of error of individual points on curve A.

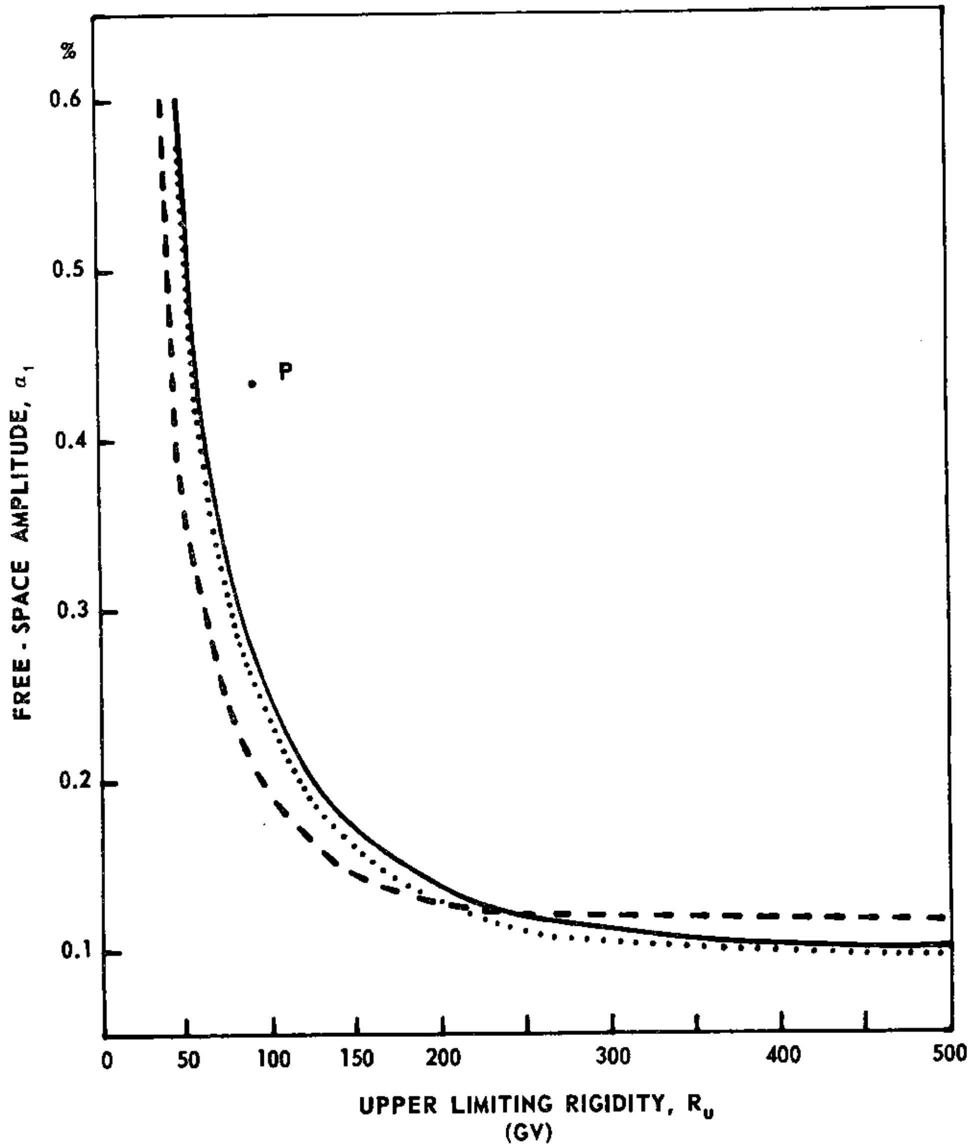



FIG. 6.13. Estimates of the amplitude ($a_1$) of the free-space solar diurnal variation of the primaries, averaged over 1961 and 1962, for different values of $R_u$ and for $\beta = 0$, based on observations underground at Hobart with

(a) the vertical semi-cubes: full line;
(b) the north-pointing cube: dotted line;
(c) the south-pointing cube: dashed line.

The point P represents the intersection of curves A and B for 1958 (Fig. 6.12).

(particularly from the more accurate observations in the axial direction 30°N of zenith) agreed quite closely with the curve obtained from the vertical observations. This tended to confirm the validity of the method of calculation of response and also demonstrated that the observed amplitudes in the three directions must have been essentially due to the solar anisotropy. Figure 6.14 depicts the compatability in respect of amplitude and phase, the observed first harmonic vectors in the three directions being compared with curves of estimates (dashed lines) calculated as functions of $R_u$. The estimates were derived from amplitude and phase constants of response appropriate to each detector (Figures 6.10 and 6.11), on the assump-

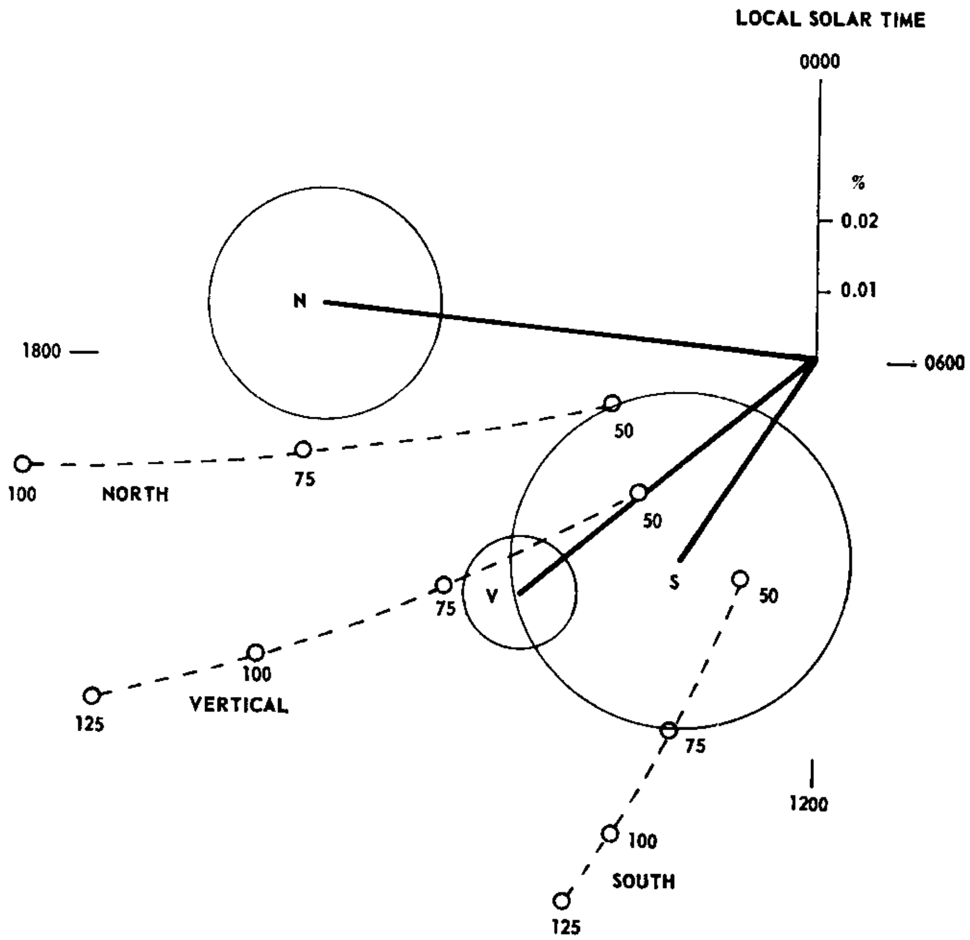



FIG. 6.14. The first harmonics of the pressure-corrected daily variations observed in the north (N), vertical (V) and south-pointing (S) directions underground at Hobart, averaged over the years 1961 and 1962. The radius of each error circle is $2\sigma$.

The dashed lines give estimates of the first harmonics to be expected in each of the three directions, for different values of upper limiting rigidity ($R_u$). Some values of interest that are shown are in GV. The estimates are based on the model for the free-space solar diurnal variation of the primaries proposed by Rao *et al.*

tion that the free-space constants $\beta$, $\alpha$, and $\psi_o$ had not changed appreciably between 1958 and 1961-1962.

On the above assumptions it was concluded that $R_u$ had decreased by approximately 30 GV over the period from 1958 to 1962. There was supporting evidence for this from the behaviour of the solar diurnal variation observed underground at Budapest (see Table 6, Paper 7) and from the differences in behaviour of the solar daily variations of neutron intensity at Huancayo and Churchill, as reported by Sarabhai *and* Subramanian (1963).

It now seems (McCracken *and* Rao 1965; J. E. Humble, private communication) that the amplitude constant $\alpha$, and the direction $\psi_o$ of the diurnal part of the solar anisotropy, did not change significantly at any time, on the annual average, between 1958 and 1965 and that, as far as could be judged, the differential free-space amplitude was independent of rigidity ($\beta = 0$). It is submitted that these findings confirm the view that the large decreases of amplitude observed at high mean rigidities of response over the same period were due essentially to decrease in the upper limiting rigidity $R_u$.

It seems that even existing counting rates underground would warrant more accurate calculations of $R_u$. It is suggested that the following points in particular should be taken into account in an improved treatment:

(1) It would be necessary to revise the calculations of the coupling coefficients, basing them on a more sophisticated model for the yield of mesons and using more accurate estimates of the cut-off energy for mesons at production, as these relate to given detectors. The coupling coefficients used in the present investigation have probably led to overestimation of $R_u$.

(2) As Thambyahpillai *et al.* (1965) have pointed out, the streaming of cosmic radiation, due to the orbital motion of the earth around the sun with a speed of 30 km/sec, should generate a solar diurnal variation, with maximum intensity at 0600 local solar time in free-space. They have estimated the amplitude to be 0·02%. Compensation for this effect increases the underground amplitude of response to the solar anisotropy by a little less than 0·02% and has the effect of increasing the estimated value of $R_u$ by approximately 20 GV.

(3) No calculations have yet been made of the effect of the scattering of $\mu$-mesons in the material absorber. It has been assumed that the integrated amplitude constants of response for wide-angle detectors would require little modification because of scattering. However, by not taking into account the smoothing of amplitudes produced by scattering, existing values of $R_u$ have been underestimated.

# 7. THE ESTIMATED DIRECTION OF THE ANISOTROPY

The direction of the sidereal anisotropy outside the earth's magnetic field will now be estimated as a function of the cut-off for observation ($R_c$) and the index ($\gamma$) of the variation spectrum. In the process, the latitude dependences of the free-space diurnal and semi-diurnal amplitudes are determined. These latitude effects give important indications as to the validity of the model for the anisotropy and as to the presence or otherwise of spurious components in the sidereal daily variation.

The estimated direction is compared with recent optical and radio determinations of the direction of the local spiral arm magnetic field.

## 7.1. THE ESTIMATED DECLINATION

### (a) *Outline of procedure*

Let us suppose that simultaneous annual averaged observations of the two-way anisotropy are available from two detectors A and B, which are located at approximately the same depth underground, but which scan widely differing asymptotic strips of latitude. Now, provided that there are (a) no important spurious contributions to the observations and (b) no additional genuine contributions from other directions, the observations from A and B suffice for the determination of the *declination d*, as a function of threshold rigidity $R_c$ and index of the variation spectrum $\gamma$. While it would appear from the evidence given in Section 5 that, if there are spurious effects in the underground observations they are not significant, it is nevertheless desirable to test against possibilities (a) and (b) with observations from a third detector C. This detector should scan a latitude strip which differs considerably from the strips scanned by A and B. In effect, having estimated d with the data from detectors A and B one notes whether such a value of $d$ is compatible with the observations from detector C. If there is a significant discrepancy, various possible reasons for this, including possibilities (a) and (b) above, must be examined. If there is no significant discrepancy it is concluded that, within the limits of error of observation, the anisotropy being observed is the one described by the model.

Evidently, then, the procedure not only gives an estimate of the declination, but provides a test of the genuineness of the observed sidereal effect.

### (b) *The expression for the declination*

It is proposed to evaluate the declination of the *southward* direction of the axis of the two-way anisotropy. Thus, following the usual convention, the value of $d$ will be negative. If we go back to equation 6.7 and put $\phi' = \alpha_s$, this gives a diurnal maximum for detector A scanning asymptotically in the southern hemisphere ($\delta$ negative), depending on the influence of the additional component $m \cos d \cos \delta \cos (\phi' - \alpha_s)$. It should be explained (see equation (6.1)) that the amplitude constant $n$ is positive, while $m$ may take a positive or a negative value depending on whether the intensity maximum from this streaming type of component is from the southward or northward direction of the axis of the two-way anisotropy.

The respective observed diurnal and semi-diurnal amplitudes of response from detectors A and B are denoted by $(v_{1A\max}, v_{2A\max})$ and $(v_{1B\max}, v_{2B\max})$ and relate to the mean asymptotic latitudes of viewing $\delta_A$ and $\delta_B$. The corresponding free-space amplitudes are $(V_{1A\max}, V_{2A\max})$ and $(V_{1B\max}, V_{2B\max})$.

Putting $\phi' = \alpha_s$ in equation (6.7) we may write in respect of detector A,

$$V_{1A\ \max} = \kappa_c\left(m \cos d \cos \delta_A + \frac{n}{2} \sin 2d \sin 2\delta_A\right) \tag{7.1}$$

Similarly

$$V_{2A\ \max} = \kappa_c \frac{n}{2} \cos^2 d \cos^2 \delta_A \tag{7.2}$$

Substituting for $n$ from equation (7.2) in equation (7.1), we get

$$V_{1A\ \max} = \kappa_c\left(m \cos d \cos \delta_A + \frac{V_{2A\ \max} \sin 2d \sin 2\delta_A}{\kappa_c \cos^2 d \cos^2 \delta_A}\right) \tag{7.3}$$

Likewise, if detector B scans asymptotically in the same hemisphere as A and the observed diurnal components from the two detectors are *in phase*, putting $\phi' = \alpha_s$ gives

$$V_{1B\ \max} = \kappa_c\left(m \cos d \cos \delta_B + \frac{V_{2B\ \max} \sin 2d \sin 2\delta_B}{\kappa_c \cos^2 d \cos^2 \delta_B}\right) \tag{7.4}$$

However, it is important to note that, if B scans in the opposite hemisphere from A and the diurnal maximum is 12 hours out of phase with that from A, then $\phi' = \alpha_s$ is the condition for $V_{1B\min} = -V_{1B\max}$ and it is necessary to use minus the observed amplitude in the subsequent calculations. Thus, in what follows, negative values of amplitude denote intensity minima that may occur in the northern hemisphere when $\phi' = \alpha_s$.

We now substitute for $m \cos d$ in equation (7.3) from equation (7.4). Thereby, the declination $d$ may be expressed solely in terms of the free-space amplitudes and associated asymptotic latitudes of viewing that are deducible from the harmonics obtained from detectors A and B. After substitution for $m$ and rearrangement of terms we finally get, in terms of *free-space harmonics,*

$$\tan d = \frac{V_{1B\ \max} \dfrac{\cos^2 \delta_A}{\cos \delta_B} - V_{1A\ \max} \cos \delta_A}{4V_{2A\ \max}(\sin \delta_B - \sin \delta_A)} \tag{7.5}$$

Now

$$V_{1B\ \max} = \frac{\kappa_c}{\kappa_{1B}} v_{1B\ \max} \tag{7.6}$$

$$V_{1A\ \max} = \frac{\kappa_c}{\kappa_{1A}} v_{1A\ \max} \tag{7.7}$$

and

$$V_{2A\ \max} = \frac{\kappa_c}{\kappa_{2A}} v_{2A\ \max}. \tag{7.8}$$

Therefore, in terms of the *observed harmonics*,

$$\tan d = \frac{\dfrac{K_{2A}\cos^2\delta_A}{K_{1B}\cos^2\delta_B}\,v_{1B\,\max} - \dfrac{K_{2A}}{K_{1A}}\cos\delta_A v_{1A\,\max}}{4v_{2A\,\max}(\sin\delta_B - \sin\delta_A)} \tag{7.9}$$

It is important to note that only the declination $d$ of the *symmetrical bi-directional component* is obtained from equations (7.5) and (7.9). If the *uni-directional component* $k_c\, m \cos d \cos \delta_A$ should happen to have a declination $d'$ different from the value $d$ gives to it here, then $m \cos d$ would become $m \cos d'$. However, this term is eliminated in the derivation of $\tan d$.

*(c) S.E. of estimate of tan d*

We may write $\tan d$ as

$$\tan d = ax + by,$$

where $a$ and $b$ are asymptotic constants, and

$$x = v_{1B\max}/v_{2A\max}$$

and

$$y = v_{1A\max}/v_{2A\max}.$$

In respect of random fluctuations of intensity, $v_{1B\max}$ and $v_{1A\max}$ may be regarded as independent Poisson variables and the variance of $x$ may be calculated accordingly. It is not quite clear that $y$ can be treated in the same way, being the quotient of the diurnal and semi-diurnal amplitudes from detector A. That is to say, it might be asked if there is any association between the amplitudes of the first and second harmonics of best fit to random values of the bi-hourly deviates. [A similar question arises earlier, in the analysis of variance of an individual harmonic $R_m = a_m \cos m\theta + b_m \sin m\theta$, where the harmonic coefficients $a_m$ and $b_m$ are regarded as being statistically independent (e.g., see Section 4.10 (c))]. Some indication should be given by Fourier analysing sets of random numbers. Ninety-nine sets of such numbers were formed by adding a constant large base value to random numbers (0 to 9) listed in Table 8 of "Cambridge Elementary Statistical Tables" (Lindley *and* Miller 1953) and arranging them in groups of thirteen. Harmonic analyses were then carried out in the same way as for normal groups of bi-hourly intensity values. A scatter diagram of pairs of first and second harmonic amplitudes is shown in Figure 7.1. The result suggests that, if there is an association between $R_1$ and $R_2$, it must be very weak. Therefore it is assumed for the present that, if $v_{1A\max}$ and $v_{2A\max}$ are treated as independent chance variables, in respect of random fluctuations of intensity, this will not lead to serious error in calculating the variance of their quotient, $y$.

Accordingly

$$\sigma(\tan d) = \pm\sqrt{a^2\sigma_x^2 + b^2\sigma_y^2}, \tag{7.10}$$

where

$$\sigma_x^2 = \frac{1}{(v_{2A\,\max})^4}\left[(v_{2A\,\max})^2\sigma^2(v_{1B\,\max}) + (v_{1B\,\max})^2\sigma^2(v_{2A\,\max})\right]$$

and

$$\sigma_y^2 = \frac{1}{(v_{2A\,\max})^4}\left[(v_{2A\,\max})^2\sigma^2(v_{1A\,\max}) + (v_{1A\,\max})^2\sigma^2(v_{2A\,\max})\right]:$$

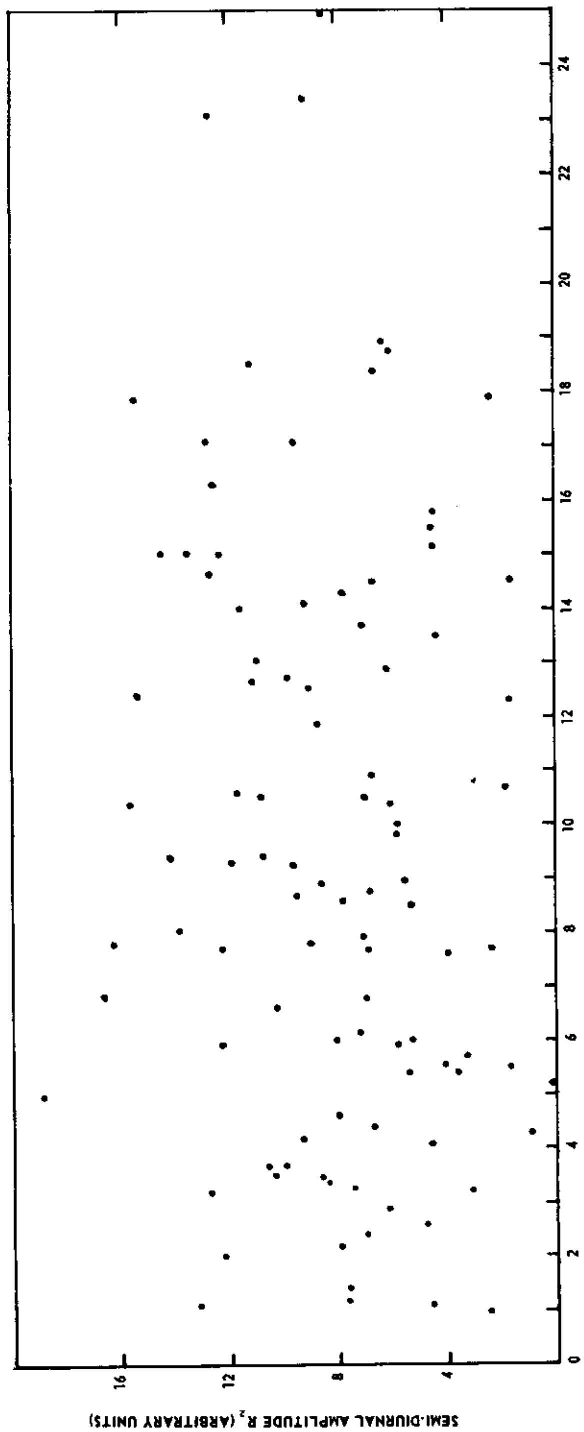



FIG. 7.1. Scatter diagram of pairs of values of $R_1$ and $R_2$ from Fourier analysis of 99 independent groups of 12 random numbers, representing bi-hourly deviates.

The estimated variances of the observed amplitudes $v_{1A\,max}$, $v_{1B\,max}$ and $v_{2A\,max}$ are found by the method described in 4.10.

*(d) Evaluation of d, using data from detectors A and B*

The vertical semi-cubical telescopes at Hobart ($\delta = -39°$) and the vertical semi-cubical telescopes at Budapest ($\delta = +43°$) were chosen as detectors A and B respectively and the cubical telescope directed 30°N of the zenith at Hobart ($\delta = -18°$) was chosen as detector C.

Data from detector A were available from 1958 onwards, from detector B they were available for the years 1959 and 1961, and from detector C they were available for the years 1961 and 1962. Since there did not appear to be a marked trend in the amplitude observed with the vertical telescopes at Hobart between 1958 and 1962, it was decided that the greatly improved statistical accuracy gained by using all data that were available from 1958 to 1962 outweighed the advantage of using only the data from 1961, the one year when simultaneous observations could be obtained from all three sets of detectors.

Allowance was made for an apparent decrease in the cut-off rigidity $R_c$ over the period, as deduced from observations of the solar diurnal variation (e.g., see Section 6 B.2). However, as will be shown below, the value estimated for $d$ depends only weakly on the value assigned to $R_c$.

The observational data and asymptotic constants that were required are listed in Table 7.1. The constants have been extracted from Table 6.2 and 6.3, the values for $R_c = 85$ GV having been obtained by graphing and interpolating.

TABLE 7.1.

The observed amplitudes and asymptotic constants used in the determination of $d$, in respect of the detectors $A$ (Hobart semi-cube, vertical), $B$ (Budapest semi-cube, vertical) and $C$ (Hobart cube, 30°N of zenith).

| Detector | *A* | *B* | C |
|---|---|---|---|
| Period | 1958–1962 | 1959; 1961 | 1961–1962 |
| Assumed $R_c(GV)$ | 85 | 85 | 70 |
| $v_{1\,max}$% | 0·0450 | 0·0233 | 0·0430 |
| $v_{2\,max}$% | 0·0080 | 0·0055 | 0·0240 |
| SE% | ±0·0025 | ±0·0027 | ±0·0080 |
| $\gamma = 0$ $K_1$ | 0·620 | 0·620 | 0·756 |
| $\gamma = 0$ $K_2$ | 0·370 | 0·370 | 0·656 |
| $\gamma = 0$ $\delta$ | −39° | +43° | −18° |
| $\gamma = 2$ $K_1 \times 10^6$ | 20 | 13 | 28 |
| $\gamma = 2$ $K_2 \times 10^6$ | 13 | 13 | 28 |
| $\gamma = 2$ $\delta$ | −40° | +44° | −17° |

*The estimate for $\gamma = 0$*

Equation (7.9) yields the following value for tan $d$, based on the constants derived from Table 6.2 and the observed amplitudes $v_{1A\max}$, $v_{1B\max}$ and $v_{2A\max}$:

$$\tan d = -0{\cdot}7710$$

From equation (7.10), the S.E. of estimate is

$$6\ (\tan d) = \pm 0{\cdot}1822.$$

Therefore, the estimate of $d$ from the vertical observations at Hobart and Budapest, for $\gamma = 0$, is

$$d = -37{\cdot}6 \pm 7^\circ.$$

*The estimate for* $\gamma = -2$

Using the observed amplitudes and the appropriate constants derived from Table 6.3, it is found that

$$d = -38{\cdot}7 \pm 7^\circ.$$

Table 7.2 shows that determinations of $d$ are not seriously affected by the choice of $\gamma$ and $R_c$, for values of $\gamma$ in the range 0 to —2, except for the combination of negative $\gamma$ and a value of $R_c$ close to the absolute primary cut-off for detection underground. On present evidence of the influence of the interplanetary magnetic field, such a low value for $R_c$, must be regarded as being unlikely.

TABLE 7.2.

Estimates of the declination of the anisotropy, in the southward direction, versus threshold rigidity $R_c$, for $\gamma = 0$ and $\gamma = -2$. The determinations are based on the amplitudes of the harmonics of the sidereal daily variations observed in the vertical direction at Hobart (1958–1962) and at Budapest (1959, 1961), and on the constants of response given in Tables 6.2 and 6.3.

| | *THRESHOLD RIGIDITY,* $R_c(GV)$ | | | | | |
|---|---|---|---|---|---|---|
| | 20 | 50 | 70 | 85 | 100 | 200 |
| Estimated $d$ ($\gamma = 0$) | −39° | −38° | −38° | −38° | −38° | −38° |
| Estimated $d$ ($\gamma = -2$) | −48° | −40° | −38° | −39° | −39° | −39° |

Aside from errors of measurement, the estimated value of $d$ is clearly defined by the relationship between the amplitudes observed with detectors A and B. This may be shown by calculating $v_{1B\max}$ as a function of $d$ for an observed pair of values of $v_{1A\max}$ and $v_{2A\max}$. By re-arranging equation (7.9), $v_{1B\max}$ (specified for $\phi' = \alpha_s$) is expressed as

$$v_{1B\,\max} = \frac{\cos\delta_B}{\cos\delta_A}\left\{\frac{K_{1B}}{K_{1A}}v_{1A\,\max} + \frac{4K_{1B\,\max}}{K_{2A\,\max}}\frac{\sin\delta_B - \sin\delta_A}{\cos\delta_A}\tan d\, v_{2A\,\max}\right\} \quad (7.11)$$

In terms of the vertical observations at Hobart and for ($\gamma = 0$, $R_c = 85$ GV) equation (7.11) gives

$$v_{1B\max}(\%) = 0{\cdot}042 + 0{\cdot}085 \tan d \quad (7.12)$$

The curve $v_{1B\max}$ versus $d$ shown in Figure 7.2 is specified by equation (7.12). Its intersection with the dashed line representing the observed value of $v_{1B\max}$ in the vertical direction at Budapest gives the estimated declination $d = -38°$. Because of the presence of the term $K_{1B}\, m \cos d \cos \delta_A \cos (\phi - \alpha_s)$ representing streaming, it can be seen from the figure, that if $d$ had been smaller than approximately —27°, the observed diurnal variation at Budapest would have been in phase with the diurnal variation at Hobart.

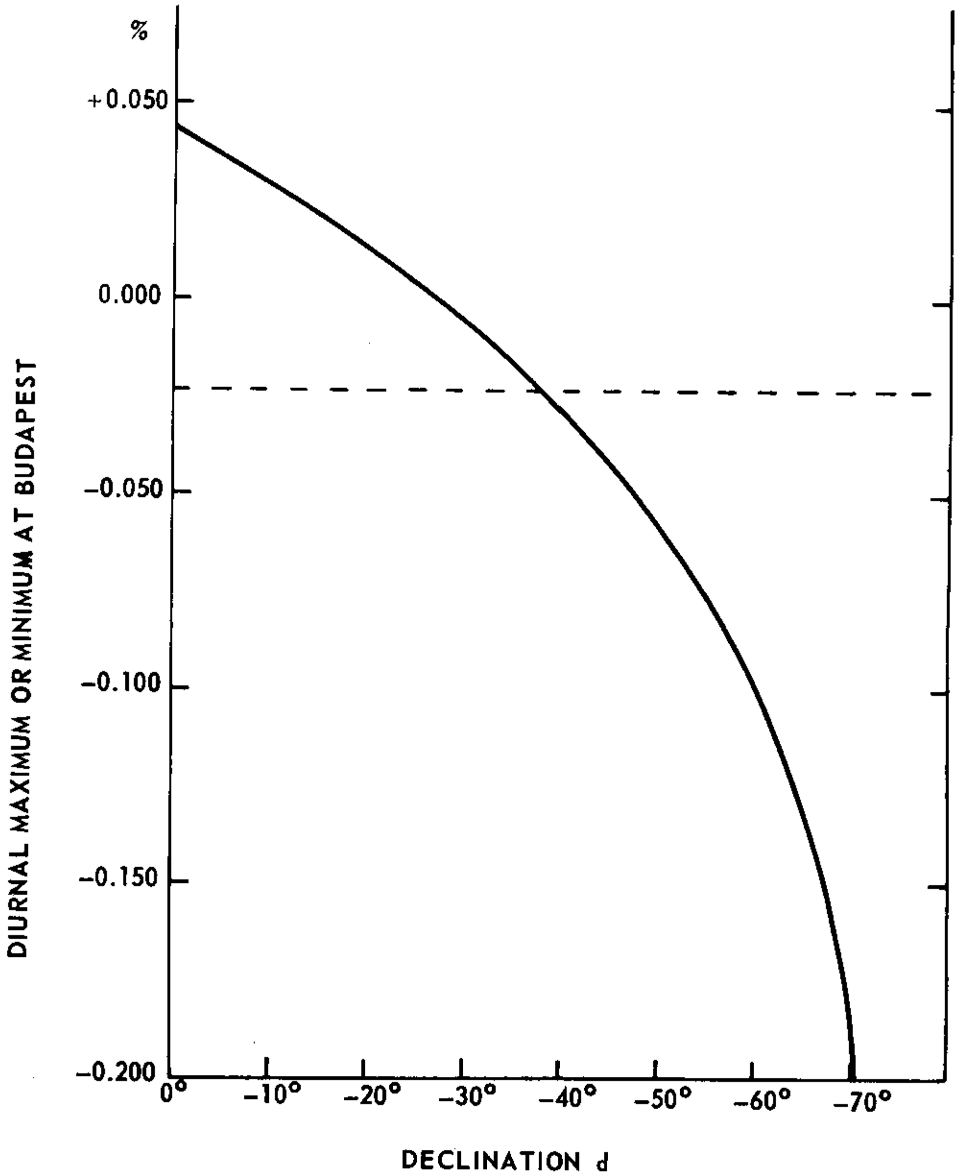



FIG. 7.2. The sidereal diurnal maximum or minimum that would be observed in the vertical direction at Budapest simultaneously with the sidereal diurnal maximum in the vertical direction at Hobart, calculated from the observed amplitude at Hobart, 1958-1962, as a function of the (southward) declination of the two-way anisotropy, for $\gamma = 0$. The dashed line represents the diurnal minimum which occurred at Budapest at the same sidereal time as the maximum observed at Hobart.

*(e) Test of the validity of the estimate of d, using detector C*

Having obtained an estimate of $d$ from detectors A and B, one may calculate the diurnal and semi-diurnal amplitudes that should be observed with detector C, scanning a different asymptotic strip of latitude. Agreement with the observations from C would indicate that, within limits of error of measurement, the model for the anisotropy was basically the correct one and would consitute supporting evidence that there was not an important spurious contribution to the observed sidereal effect.

The underground cubical telescope inclined 30°N of zenith at Hobart was well suited for use as detector C. As Table 7.1 shows, the telescope exhibits a very good response to first and second harmonics of the anisotropy, while the free-space diurnal and semi-diurnal amplitudes would be expected to be relatively large at the asymptotic latitude of viewing $\delta = -18°$.

The amplitudes $v_{1C\max}$ and $v_{2C\max}$, relating to detector C, have to be expressed in terms of the estimated declination $d$, the amplitudes observed with one of the detectors A and B (we choose A) and the appropriate asymptotic constants. Equation 7.11 is therefore used for the diurnal amplitude and we have

$$v_{1C\max} = \frac{\cos\delta_C}{\cos\delta_A}\left\{\frac{K_{1C}}{K_{1A}}v_{1A\max} + \frac{4K_{1C}}{K_{2A}}\frac{\sin\delta_C - \sin\delta_A}{\cos\delta_A}\tan d\, v_{2A\max}\right\} \quad (7.13)$$

The expression for the semi-diurnal amplitude $v_{2C\max}$ is obtained by substituting $\frac{\kappa_c}{K_2}v_{2\max}$ for the free-space amplitude $V_{2\max}$ in equation (7.2), so that

$$v_{2A\max} = K_{2A}\frac{n}{2}\cos^2 d\cos^2\delta_A$$

and

$$v_{2C\max} = K_{2C}\frac{n}{2}\cos^2 d\cos^2\delta_C;$$

therefore,

$$v_{2C\max} = \frac{K_{2C}}{K_{2A}}\frac{\cos^2\delta_C}{\cos^2\delta_A}v_{2A\max} \quad (7.14)$$

It is noted that the relationship between the semi-diurnal amplitudes observed at two different latitudes is independent of $d$.

The expressions for $v_{1C\max}$ and $v_{2C\max}$ given by equations (7.13) and (7.14) were evaluated for $\gamma = 0$ and $\gamma = -2$, using the appropriate constants from Table 7.1. Table 7.3 shows that the observed and calculated amplitudes were in close agreement, particularly for the case $\gamma = 0$. The discrepancy when $\gamma = -2$ is not significant and, from considerations of counting rate statistics alone, it could not be said that one index was to be preferred to the other.

*(f) The latitude dependence of the free-space diurnal amplitude*

A more comprehensive variant of method (e) of testing the validity of the estimate of $d$ comes from the determination of the free-space amplitude $V_{1l\max}$ as a function of asymptotic latitude of viewing $\delta_l$, based on the estimate of $d$ which satisfies observations from detectors A and B, for given values of $\gamma$ and $R_c$.

TABLE 7.3.

Observed and calculated values of sidereal diurnal and semi-diurnal amplitudes relating to the north-pointing cubical telescope (detector $C$). The calculated values are derived from the observations with detectors $A$ and $B$, for the cases $\gamma = 0$ and $\gamma = -2$.

| | Observed | Calculated ($\gamma = 0$) | Calculated ($\gamma = -2$) |
|---|---|---|---|
| $v_{1\,C\,max}(\%)$ | 0·043 ± 0·008 | 0·042 | 0·050 |
| $v_{2\,C\,max}(\%)$ | 0·024 ± 0·008 | 0·021 | 0·027 |

Substituting the variable subscript $l$ for $B$ in equation (7.5) and re-arranging, we obtain

$$V_{1l\,max} = \frac{\cos\delta_l}{\cos\delta_A}\left\{V_{1A\,max} + 4\tan d\frac{\sin\delta_l - \sin\delta_A}{\cos\delta_A}V_{2A\,max}\right\} \qquad (7.15)$$

Consider the solution $d = -38{\cdot}5°$, for $\gamma = 0$ and $R_c = 85$ GV, presented in 7 (d) above. Having derived

$$V_{1A\,max}\left(=\frac{v_{1A\,max}}{K_{1A}}\right) \text{ and } V_{2A\,max}\left(=\frac{v_{2A\,max}}{K_{2A}}\right) \text{ from Table 7.1,}$$

we use equation (7.15) to obtain the curve $V_{1l\max}$ *versus* $\delta_l$. Naturally, $V_{1A\max}$ (Hobart vertical) and $V_{1B\max}$ (Budapest vertical) must give two points on the curve, at latitudes $\delta_A$ and $\delta_B$ respectively. Now, to test the validity of the model for the anisotropy, and of the estimate of $d$, we determine $V_{1C\max}$ from observations with the north-pointing cubical telescope at Hobart and note whether the result agrees with the prediction of the latitude-curve for the appropriate asymptotic latitude of viewing.

In Figure 7.3 (a) it can be seen that the point representing $V_{1C\max}$ does in fact fall on the latitude curve that derives from the observations with the semi-cubical telescopes at Hobart and Budapest. It is noted that the free-space amplitude deduced from observations with the south-pointing telescope also falls on the curve at $\delta_l = -55°$, although the statistical error is much too large for this result to be of significance.

The dashed line in Figure 7.3 (a) is the latitude-curve which conforms with $V_{1A\max}$ (Hobart vertical) for the declination $d = -66°30'$. Since the co-ordinates of the normal to the plane of the ecliptic, looking south, are $RA = 0600$ and $d = -66°30'$, this direction was thought to be worth examining because of its Right Ascension. However, it is clear from the figure that as an axis for the anisotropy the normal to the ecliptic would be quite inconsistent with the observations from Budapest and from the north-pointing cubical telescope at Hobart. In fact, the two curves serve to illustrate

(a) that the declination of fit can be rather precisely determined, for given values of $\gamma$ and $R_c$, if sufficiently accurate observations are available from several well-spaced latitudes of viewing; and (b) that compatibility between the free-space amplitudes would be unlikely if the amplitude deduced from any one of the detectors was based on a false observation.

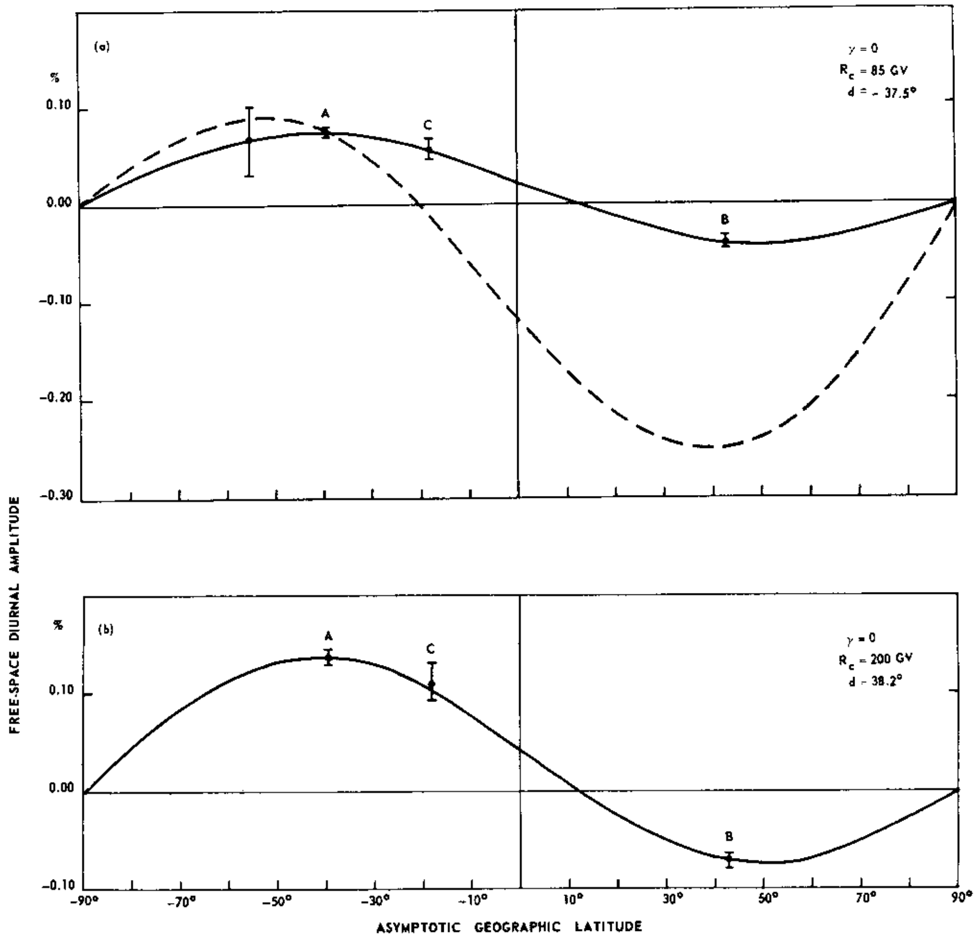



FIG. 7.3. Sidereal anisotropy. Dependence of free-space diurnal amplitude on asymptotic latitude of viewing, calculated for the value of the declination $d$ which conforms with the estimated amplitudes A and B, when

(a) $\gamma = 0$, $R_c = 85$ GV
(b) $\gamma = 0$, $R_c = 200$ GV.

The dashed line in (a) is calculated for $d = -66°30'$, and so as to conform with the estimated amplitude A.

A.—Hobart vertical, 1958 — 1962
B.—Budapest vertical, 1959 + 1961
C.—Hobart 30°N, 1961 + 1962
Unlettered—Hobart 45°S, 1961 + 1962.

Standard error tails are shown.

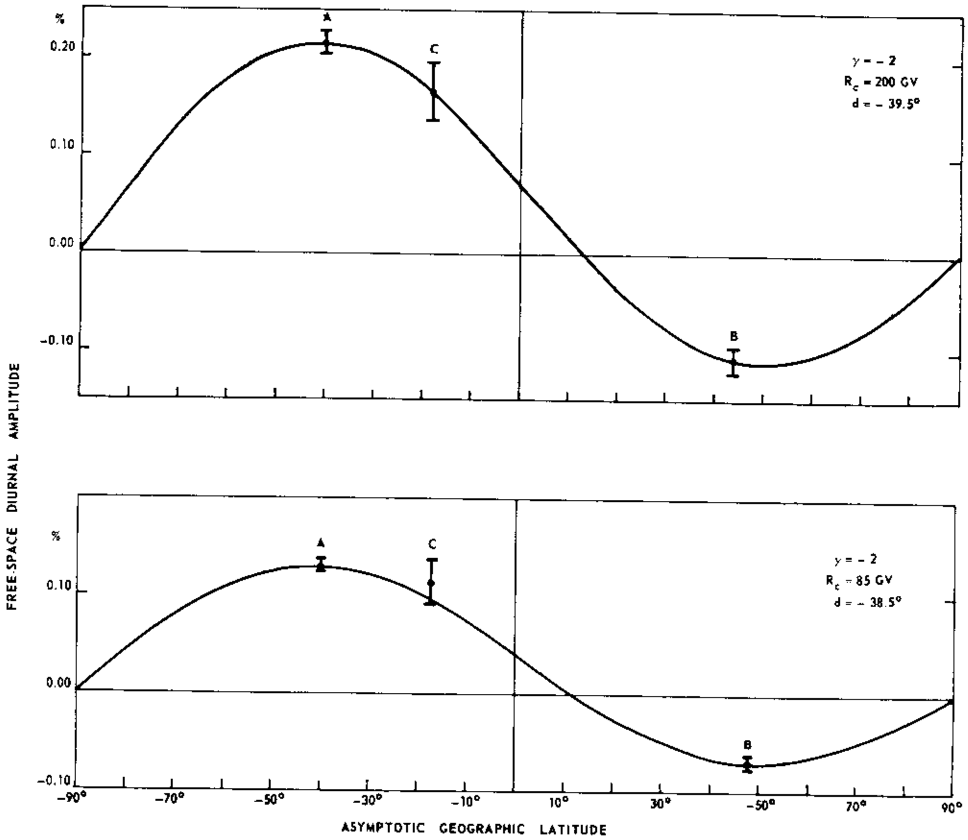



FIG. 7.4. Sidereal anisotropy. Dependence of free-space diurnal amplitude on asymptotic latitude of viewing, calculated for the value of the declination $d$ which conforms with the estimated amplitudes $A$ and $B$, when

(a) $\gamma = -2$, $R_c = 200$ GV
(b) $\gamma = -2$, $R_c = 85$ GV.

Standard error tails are shown.

Figures 7.3 and 7.4 show that the estimate of $d$ and the compatibility between amplitudes $V_{1A\max}$, $V_{1B\max}$ and $V_{1C\max}$ do not depend at all strongly on the values assigned to $\gamma$ and $R_c$, at least in the ranges 0 to $-2$ for $\gamma$ and 85 $-200$ GV for $R_c$.

*(g) The latitude-dependence of the free-space semi-diurnal amplitude*

On the basis of the model for the anisotropy it is evident from equation (7.14) that the ratio $\frac{V_{2l\max}}{\cos^2 \delta_l}$ is constant. Consequently, if $V_{2A\max}$ is estimated from observations with detector A, for a pair of values of $\gamma$ and $R_c$, the free-space amplitude at any other latitude of viewing may be determined without requiring knowledge of the declination $d$. Using the semi-diurnal amplitude observed at Hobart with the vertical semi-cubical telescope (detector A) as the basic observation, we have

$$V_{2l\max} = \frac{\cos^2 \delta_l}{\cos^2 \delta_A} \quad V_{2A\max} \tag{7.16}$$

The latitude curves in Figure 7.5 have been calculated so as to conform with $V_{2A\max}$ for the pairs of values of $\gamma$ and $R_c$ that are indicated. It is clear that the curves also conform with $V_{2B\max}$ and $V_{2C\max}$. Thus, within the errors of the measurements, it is evident that the three most accurate semi-diurnal amplitudes derived from the observations underground are compatible with the model for the anisotropy. The free-space amplitude obtained from observations with the south-pointing telescope is not shown. Although anomalously large, it is not significantly different from zero.

From analysis of amplitudes of the second harmonic, it cannot be said that any one value of $\gamma$ or $R_c$ is to be preferred, within the ranges considered.

## 7.2. THE ESTIMATED RIGHT ASCENSION (RA)

The RA of the anisotropy in the southward direction beyond the earth's field region is estimated from the time of maximum of the sidereal daily variation observed in the vertical direction underground at Hobart, averaged over the eight years 1958-1965 (Table 5.6) and from the phase displacements $\phi_1$ (diurnal component) and $\phi$ (semi-diurnal).

Calculated values of $\phi_1$ versus $R_c$ and $\gamma$ that are listed in Tables 6.2 and 6.3 for the vertical telescopes are seen to be practically the same as the corresponding values of $\phi_2$. Further, the observed first and second harmonics in the vertical direction are in phase, within the errors of observations, and therefore on the basis of the model are in accord with the calculations of $\phi_1$ and $\phi_2$. Consequently, the time of maximum, 0600, of the sum of harmonics is taken as the best estimate of the phase at Hobart and from this the RA of the anisotropy versus $R_c$ and $\gamma$ is determined by application of the phase displacement $\phi_1(\sim \phi_2)$. The result is shown in Figure 7.6. It is clear that the dependence of the estimate on the nature of the variation spectrum only becomes important for threshold rigidities below about 100 GV. In lieu of a better criterion, the choice of a value for $R_c$ shall for the present be centred on the upper limiting rigidity of the solar anisotropy, $R_u$. The RA would then be 0640 with an uncertainty of perhaps a quarter of an hour, depending on the range of values within which $R_c$ and $\gamma$ might reasonably be thought to lie.

Much of the error of estimate would seem to come from a year to year variability in the time of maximum of the observed sidereal daily variation. The phase of the smoothed annual average daily variation shown in Figure 5.10 exhibits a fluctuation of approximately $\pm$ 2 hours about the average, somewhat in excess of the variation to be expected from count-rate statistics. Therefore it would seem fair to assign to the eight-year average an uncertainty of approximately $\pm$ 1 hour, accruing from statistical and quasi-statistical fluctuations. Thus only a rough figure can be given for the error of estimate of RA, and it is suggested that that would be $\pm$ 1·25 hours, noting that some of this might arise from genuine fluctuations in the direction of the anisotropy as specified outside the region of the earth's field.

## 7.3. THE DIRECTION OF THE ANISOTROPY AND OF THE GALACTIC MAGNETIC FIELD

The co-ordinates of the axis of the anisotropy in the southward direction are

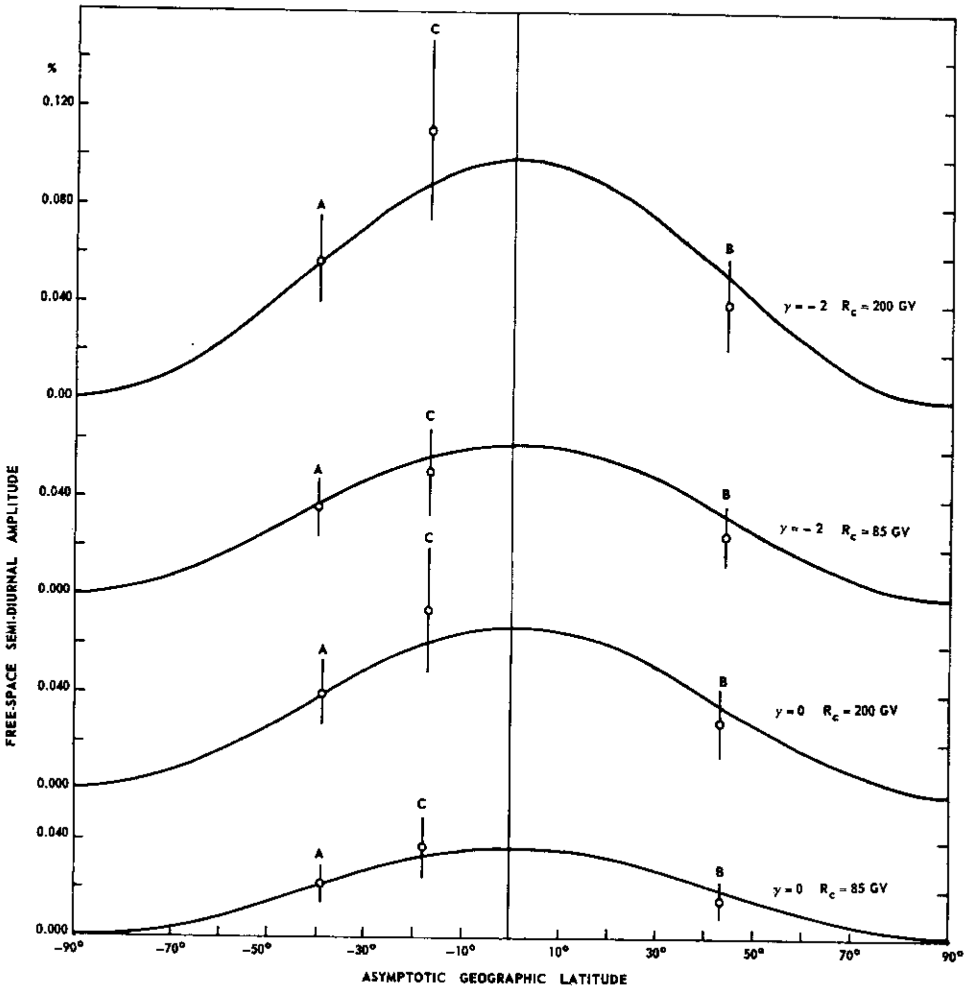



FIG. 7.5. Sidereal anisotropy. Dependence of free-space semi-diurnal amplitude on asymptotic latitude of viewing, calculated for the values of $\gamma$ and $R_c$ shown, so as to conform with the estimated amplitude A. Standard error tails are shown.

provisionally estimated to be (RA = 0640 ± 0115, Dec. = − 38 ± 7°). The corresponding galactic longitude and latitude co-ordinates in the new system (Blaauw *et al.* 1959) would be approximately ($l^{II} = 245°$, $b^{II} = -15°$). In the northward direction of the axis the co-ordinates would be (RA = 1840, Dec. = +38°; and $l^{II} = 65°$, $b^{II} = +15°$).

In Section I reference was made to the optical and radio measurements, which suggest that there is a relatively uniform galactic magnetic field in the vicinity of the solar system, lying approximately in the direction of the local spiral arm, towards $l^{II} = 70°$, $b^{II} = 0$ in the southward, or outward, direction. Table 7.4 lists some of the most recent determinations of the direction of the local galactic magnetic field by radio and optical methods. The optical polarization measure-

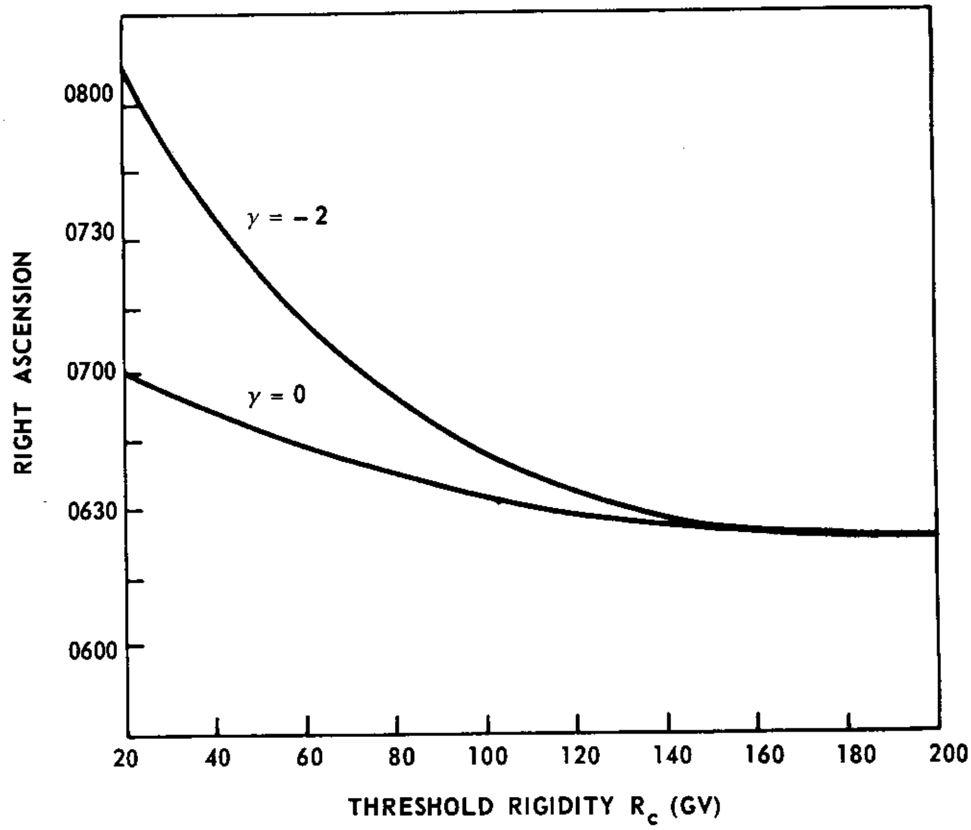



FIG. 7.6. The estimated right ascension of the two-way anisotropy, versus threshold rigidity, for two values of the index of the variation spectrum.

ments (except those of Behr) give the direction of the field within about 3 kilo parsecs (kpc) of the sun, while the radio determinations appear to refer to much closer regions. Notably, the regions of polarized synchrotron emission are thought to be located at distances of perhaps only 100 to 200 parsecs from the sun (Mathewson *and* Milne 1965).

TABLE 7.4

Recent optical and radio determinations of the galactic longitude (new system) of the local spiral arm field. Galactic latitude $b^{II} = 0°$ in all cases.

| *Optical polarization of starlight* | | *Longitude* ($l^{II}$) |
|---|---|---|
| Northern hemisphere | | |
| Stars > 600 parsecs from sun | Serkowski (1962) | 230° — 50° |
| Stars < 250 parsecs from sun | Behr (1964) | 242° — 62° |
| Southern hemisphere | Smith (1956) | 240° — 60° |
| *Faraday rotation of source polarization* | Gardner and Davies (1966) | 275° — 95° |
| *Linear polarization of synchroton emission* | Mathewson and Milne (1965) | 250° — 70° |

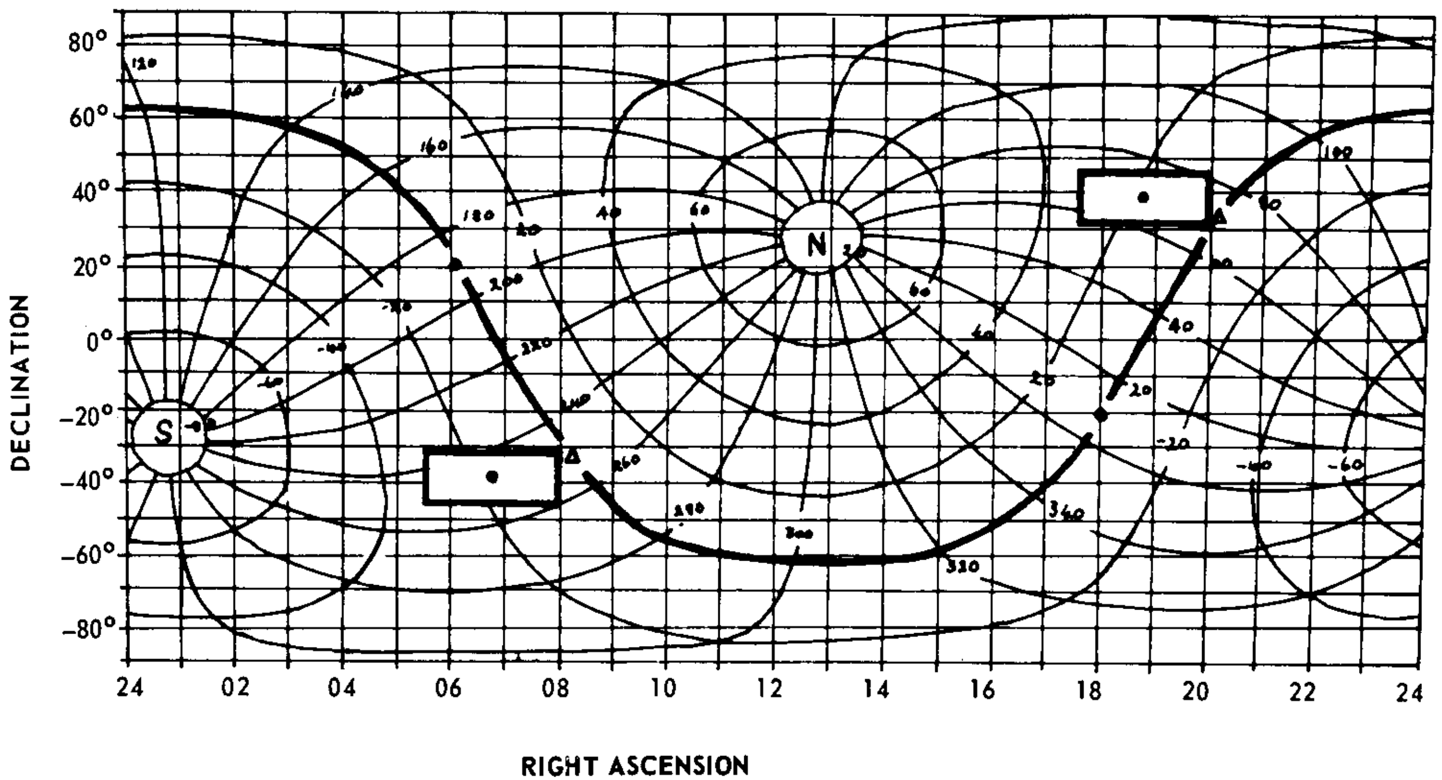



FIG. 7.7. The estimated direction [•] of the proposed cosmic ray anisotropy compared with a representative estimate, from optical and radio measurements, of the direction △ of the local galactic magnetic field. The directions are referred to the new system of galactic co-ordinates (latitude $b^{\pi}$, longitude $l^{\pi}$) super-imposed on the celestial grid (declination $d$, right ascension $\alpha$).

Arising largely out of these determinations, opinion now seems to favour a local galactic field which preserves a uniform direction out to large distances from the sun. The direction appears to be, within about 20 degrees, approximately parallel with the local spiral structure as outlined by the concentrations of H II regions, OB stars and galactic clusters (e.g., Becker 1964)) although it is not clear that this is coincident with the directions given by the distribution of neutral hydrogen (e.g., Beer 1961; Kerr 1966).

In Figure 7.7 the apparent difference between the direction of the galactic magnetic field and of the cosmic ray anisotropy is shown on a celestial-galactic co-ordinate grid. The estimated northward and southward directions of the anisotropy, positioned within approximate zones of uncertainty, may be compared with a representatative direction ($l^{II} = 70°$, $b^{II} = 0°$) of the field. Because of the uncertainties of measurement it cannot be said at present that there is a significant difference.

The estimated direction of the anisotropy is consistent with the propagation of excess fluxes of primaries with steep helices along the field, since the maxima of the diurnal and semi-diurnal components coincide (Table 1.1). If the anisotropy does originate in the galactic field, as its type and direction suggest, and if a real difference is established between the cosmic ray and the other estimates of direction, two possibilities suggest themselves:

(1) The direction of the galactic field immediately beyond the solar system may not be the same as that derived from measurements extending over relatively vast distances remote from the sun.

(2) The interplanetary magnetic field may produce a distortion of direction of the anisotropy, of a type that would be evident in annual averaged observations.

# 8. THE AMPLITUDE OF THE ANISOTROPY AND THE AMPLITUDE OF RESPONSE AT GROUND-LEVEL.

## 8.1. THE ESTIMATED AMPLITUDES OF THE COMPONENTS OF THE ANISOTROPY

### (a) *Derivation*

The proposed description of the anisotropy in the frame of reference in which it is fixed (Section 6.A1) gives the differential intensity maxima in opposite directions as

$$A_{1R}I_{OR} = \left\{1 + \frac{R^\gamma}{3}(2n + 3m)\right\} I_{OR}.$$

and

$$A_{2R}I_{OR} = \left\{1 + \frac{R^\gamma}{3}(2n - 3m)\right\} I_{OR}.$$

The amplitude of the integrated intensity maximum in the $A_1$ direction, expressed as a fraction of the total intensity, is therefore (for the non-positive values of $\gamma$ considered here)

$$\frac{\Delta I_1}{I_o} = \frac{2n + 3m}{3} \frac{\int_{Rc}^{\infty} R^\gamma I_{OR} dR}{\int_{Rc}^{\infty} I_{OR} dR}$$

$$= k_c \frac{2n + 3m}{3} \tag{8.1}$$

Similarly, in the opposite direction, where streaming effects a reduction of intensity,

$$\frac{\Delta I_2}{I_o} = k_c \frac{2n - 3m}{3} \tag{8.2}$$

It can be seen, then, that the contribution from streaming is $S = mk_c$ and and from the symmetrical two-way anisotropy is $T = \frac{2n}{3} k_c$.

Now, the free-space diurnal amplitude derived from observations with detector A is (equation (6.7))

$$V_{1A\max} = k_c\, m \cos d \cos \delta_A + k_c \frac{n}{2} \sin 2d \sin 2\delta_A \tag{8.3}$$

Equation (7.19) may be conveniently expressed as

$$V_{1A\max} = X \cos \delta_A + Y \sin 2\delta_A \tag{8.4}$$

where $X$ is the (equatorial) maximum of the latitude-dependent diurnal amplitude term due to streaming, and $Y$ is the corresponding (mid-latitude) maximum amplitude due to the two-way diurnal component. Given $X$ and $Y$, the anisotropic intensity maxima are

streaming: $$S = \frac{X}{\cos d} \tag{8.5}$$

two-way: $$T = \frac{4Y}{3 \sin 2d} \tag{8.6}$$

To determine the two component maxima of the anisotropy as functions of statistically independent variables, it is necessary to estimate the declination $d$ from two of the detectors (A and B) and employ that value of $d$ in the estimations of $S$ and T from observations with a third detector (C).

The amplitude terms $X$ and $Y$ appear quite simply by suitably re-arranging equation (7.15). In fact, it is the most convenient way of determining the total diurnal amplitude. From the equation we obtain

$$X = \frac{1}{\cos \delta_C} V_{1C\,\max} - \frac{4 \tan \delta_C}{\cos \delta_C} \tan d \,.\, V_{2C\,\max} \tag{8.7}$$

and

$$Y = \frac{2}{\cos^2 \delta_C} \tan d \,.\, V_{2C\,\max} \tag{8.8}$$

The two components of the latitude curve of the free-space *diurnal* amplitude are shown, with their maxima $X$ and $Y$, in Figure 8.1 for the case $\gamma = 0$ and $R_C = 85$ GV. The sum curve is the same as that shown in Figure 7.3 (a).

It follows from the equations above that the streaming and two-way components of the intensity maxima in the frame of reference of the anisotropy are

$$S = \frac{X}{\cos d} = \frac{1}{\cos \delta_C} \frac{V_{1C\,\max}}{\cos d} - \frac{4 \tan \delta_C}{\cos \delta_C} \frac{\tan d}{\cos d} V_{2C\,\max} \tag{8.9}$$

and

$$T = \frac{4Y}{3 \sin 2d} = \frac{S}{6 \cos^2 \delta_C} \frac{1}{\cos^2 d} V_{2C\,\max} \tag{8.10}$$

Treating $S$ as a function of the independent chance variables $V_{1C\,\max}$, $V_{2C\,\max}$ and $d$, we have

$$\sigma^2(S) = \left(\frac{\partial S}{\partial(V_{1C\,\max})}\right)^2 \sigma^2(V_{1C\,\max}) + \left[\frac{\partial S}{\partial(V_{2C\,\max})}\right]^2 \sigma^2(V_{2C\,\max}) + \left(\frac{\partial S}{\partial d}\right)^2 \sigma^2 d \tag{8.11}$$

Thus

$$\sigma^2(S) = \left(\frac{1}{\cos \delta_c \cos d}\right)^2 \sigma^2(V_{1C\,\max}) + \left(\frac{4 \tan \delta_c \tan d}{\cos \delta_c \cos d}\right)^2 \sigma^2(V_{2C\,\max})$$
$$+ \left[\frac{1}{\cos \delta_c \cos^2 d} V_{1C\,\max} + \frac{4(2 - \cos^2 d) \tan \delta_c}{\cos^3 d \cos \delta_c} V_{2C\,\max}\right]^2 \sigma^2 d \tag{8.12}$$

Similarly,

$$\sigma^2(T) = \left(\frac{8}{6 \cos^2 \delta_c \cos^2 d}\right)^2 \sigma^2(V_{2C\,\max}) + \left(\frac{4}{\cos^2 \delta_c \cos^3 d} V_{2C\,\max}\right)^2 \sigma^2 d \tag{8.13}$$

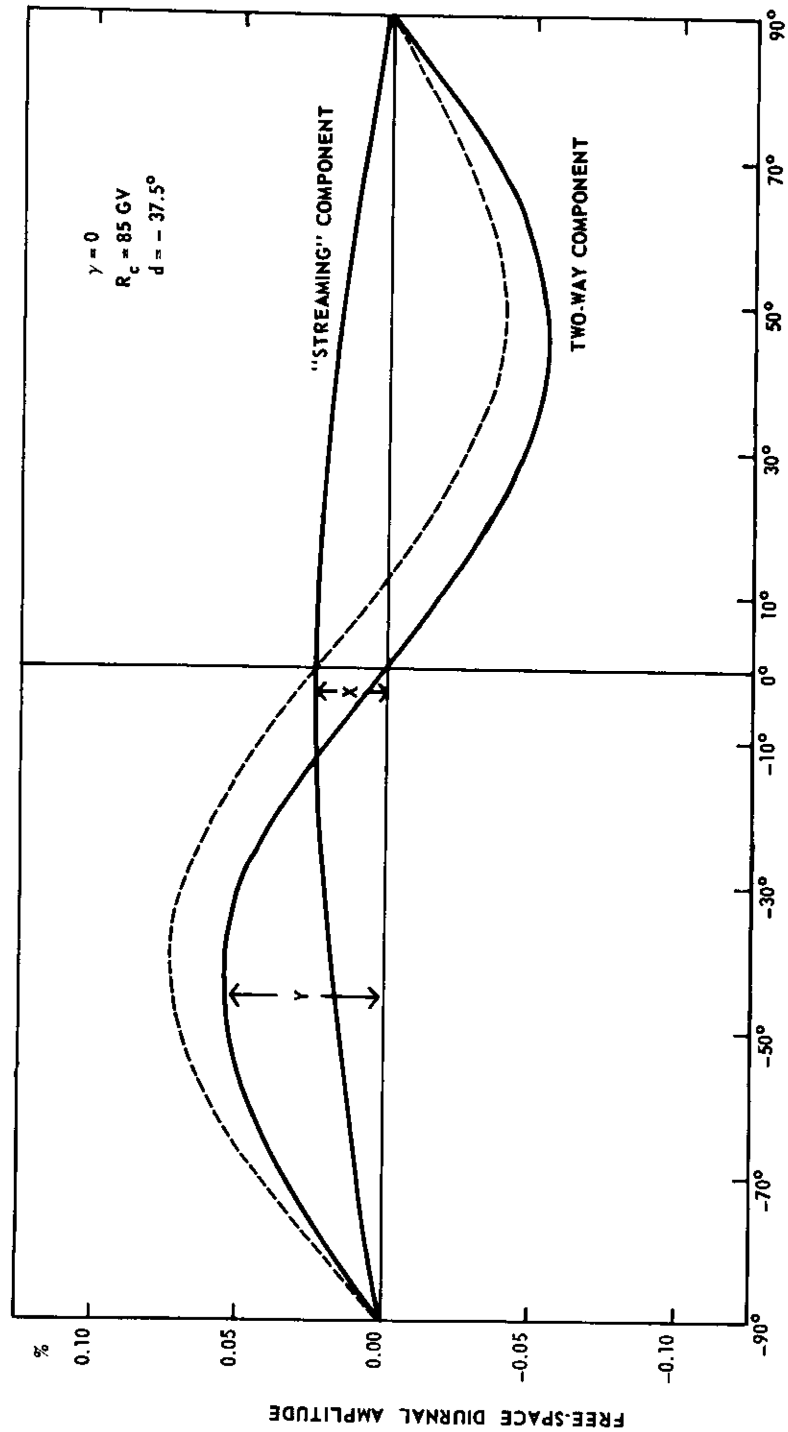



FIG. 8.1. Sidereal anisotropy. Dependence of free-space diurnal amplitude (dashed line) on asymptotic latitude of viewing, reproduced from Figure 7.3(a) and showing the two diurnal components predicted by the model, the dashed line representing their sum.

*(b) Estimation*

The declination was estimated from the sidereal diurnal and semi-diurnal amplitudes observed in the vertical direction at Hobart and Budapest (detectors A and B). The component intensity maxima $S$ and $T$ were then estimated from observations with the north-pointing cubical telescope (detector C) using the value of $d$ obtained from detectors A and B.

The results of the computations are listed in Table 8.1 for different values of $\gamma$ and $R_C$.

TABLE 8.1

Estimates of the component sidereal intensity maxima S and T in the frame of reference of the anisotropy outside the earth's field region, for different values of $\gamma$ and $R_C$. The estimates derive from observations in the vertical direction at Hobart (1958–1962) and Budapest (1959, 1961) and in the direction 30°N of zenith at Hobart (1961–1962). The errors shown are the S.E.'s of estimate.

| | $\gamma = 0$ | | $\gamma = -2$ | |
|---|---|---|---|---|
| $R_C(GV)$ | 85 | 200 | 85 | 200 |
| Streaming maximum % (S = $mk_c$) | 0·027 ±0·022 | 0·056 ±0·043 | 0·053 ±0·043 | 0·076 ±0·068 |
| Two-way maximum % (T = 2/3 $nk_c$) | 0·086 ±0·047 | 0·175 ±0·096 | 0·179 ±0·096 | 0·272 ±0·150 |
| Total maximum, northward direction (T + S) % | 0·059 | 0·119 | 0·126 | 0·196 |
| Total maximum, southward direction (T − S) % | 0·113 | 0·231 | 0·232 | 0·348 |

None of the estimates of anisotropic intensity maxima listed in Table 8.1 differ from zero by as much as two standard errors. It is clear that a very large increase in counting rate is needed if a reliable estimate of the contribution due to streaming is to be found. However, it is interesting to note that about 95% of $\sigma^2(S)$ is due to $\sigma^2(V_{1C\max})$ and $\sigma^2(V_{2C\max})$. On the other hand, about 65% of $\sigma^2(T)$ is due to $\sigma^2(d)$. Taking these factors into account, it would seem from Table 8.1 that an increase in counting rate with cubical telescopes in the north-pointing direction by a factor of ten, to ~ 170,000 pcles/hour, would reduce $\sigma(S)$ by a factor of about 2·6 and give a significant result for $S$ over a biennial period. At the same time, $\sigma(T)$ would be reduced by a factor of only 1·2 and this should give a barely significant result for the two-way component. If, together with the increase in count rate in the north-pointing direction at Hobart, the counting rates of each of the vertical semi-cubical arrays A and B were increased by a factor of two, the reduction factor for $\sigma(T)$ would increase to about 1·7, although $\sigma(S)$ would not be appreciably altered. In terms of a biennial period, the actual counting rates required from the semi-cubical vertical arrays at each of the two latitudes of viewing would be ~ 350,000 pcles/hour.

On the basis of the model and of the observations that have been used here, it seems unlikely that the intensity maximum due to streaming would have exceeded approximately 0·2% outside the earth's field region or that the two-way maximum would have exceeded 0·6%, if the actual values of $\gamma$ and $R_C$ lay somewhere in the ranges considered. The corresponding upper limits beyond the interplanetary field would be expected to be higher, because of the smoothing of amplitudes effected by the field.

### 8.2. THE RELATIVE RESPONSE AT GROUND-LEVEL

The procedures for determining the asymptotic cones of acceptance for underground meson telescopes were applied to the determination of the cones of acceptance of a vertical cubical telescope at ground level, Hobart, using the coupling coefficients of Webber *and* Quenby (Figure 6.6). Integrated constants of response analagous to those listed in Table 6.2 and 6.3 were computed. From these the relative amplitudes of response $\frac{\text{sea-level}}{\text{underground}}$ could be calculated in repect of a sidereal anisotropy, for different values of $\gamma$ and $R_C$. The calculations were made for three values of threshold rigidity (50, 70 and 100 GV) and for $\gamma = +1$, 0, $-2$ and $-4$. The resulting curves of fit (Figure 8.2) give an approximate indication of the relative diurnal and semi-diurnal amplitudes of response.

It appears from the calculations that, unless $\gamma$ is negative and the anisotropy is observable at a very low threshold rigidity, the amplitudes of the harmonics at sea level should be $< 50\%$ of the amplitudes underground. If $\gamma = +1$, amplitudes at sea level must be less than 20% of the amplitudes underground, i.e., the actual values would be typically less than 0·01%.

Averaged over the five years 1958-1962, the observed harmonic components from the telescope at ground level were:

diurnal amplitude 0·035 ± 0.002%, time of maximum 0900 ± 0014;
semi-diurnal amplitude 0·008 ± 0·002%, time of maximum 0540 ± 0030;
amplitude of sum of harmonics 0·037%, time of maximum 0730.

The corresponding results from observations of vertical intensity underground over the same period are listed in Table I, Paper 5, and lead to the following values for the amplitude ratio $\frac{\text{sea-level}}{\text{underground}}$:

diurnal: ~ (80 ± 7) %;
semi-diurnal: ~ (100 ± 45) %.

The intercomparison implies a very low value for $R_C$ and a negative value for $\gamma$. Alternatively, it could be interpreted as a further indication that a persistent and significant part of the observed effect at ground level is spurious, while being approximately in phase with the proposed anisotropic component. The great year-to-year and shorter term variability of the surface observations in sidereal time has been summarized in Figures 3.5 to 3.7 and emphasizes the problem of identifying a sidereal component in the data either from meson telescopes or neutron monitors. It is likely that much of the variability arises from the fact that it does not require great percentage modulation (e.g., ~ 20% of amplitude) to produce annual sideband amplitudes of ~ 0·05% in sidereal time from significant components of the large solar daily variation at ground level. Short-term irregular changes may also be important.

Perhaps observations with crossed telescopes would remove much of the variable and presumably spurious content in the annual apparent sidereal effect

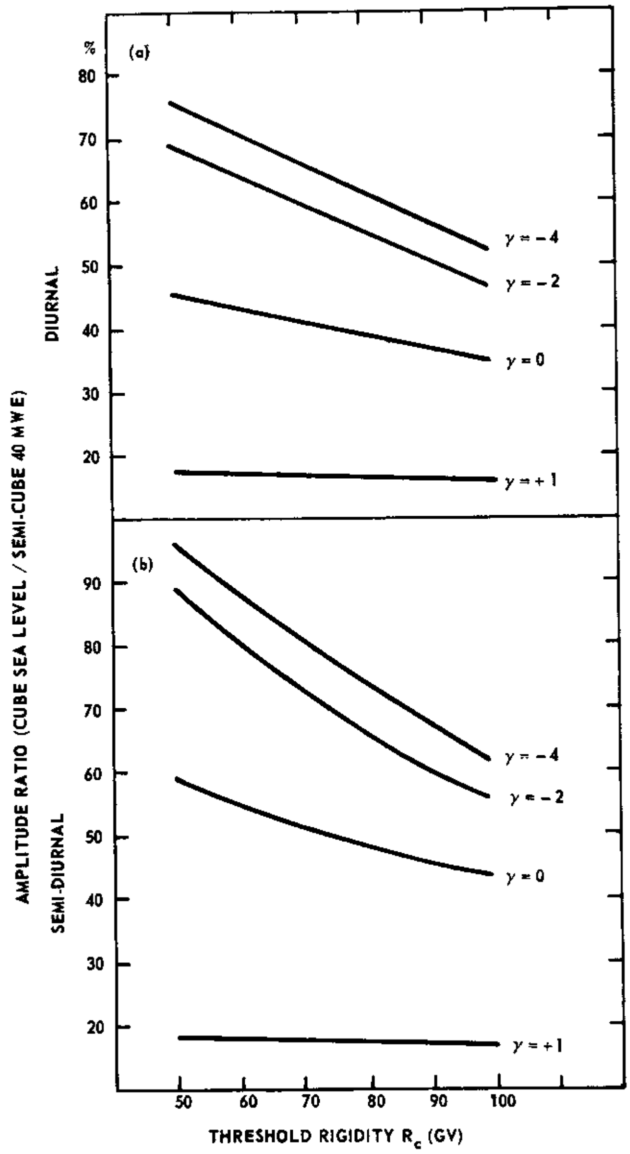



FIG. 8.2. Sidereal anisotropy. (a) Calculated ratios of amplitudes of first harmonics at Hobart for a vertical cubical telescope at sea level and a vertical semi-cubical telescope at a depth of 40 m.w.e. (b) Calculated ratios of amplitudes of the second harmonics.

above ground. It seems that it would be desirable to obtain long-term records from several crossed-pair arrays, each array being identified with its own well-defined asymptotic latitude strip. However, two points should be noted. At the important mid-latitude locations the differential asymptotic latitude of viewing $\delta_R$ varies considerably with rigidity over the range of rigidities where most of the response occurs. Typically, the range of variation of $\delta_R$ is about 40°, as Baker *et al.* (1966) indicate in Figure 1 of their paper on the crossed telescope experiment at Denver. Consequently, an initial assumption has to be made as to the likely value of $R_C$, since this enters rather critically into the design of experiments and the interpretation of the data. In the case of experiments underground, on the other hand, $\delta_R$ does not change greatly with $R$ unless telescopes are directed towards very high latitudes. If we consider the vertical semi-cubical telescopes at Hobart and Budapest, for example, Figure 6.2 shows that the total range of variation of $\delta_R$ with $R$ is not more than 10°. Because of factors of this kind, a rudimentary description of the anisotropy can be obtained from underground data without having to specify $R_C$ and $\gamma$. As to the second point, if $R_C$ is in fact not less than about 100 GV, very high counting rates would seem to be necessary at ground level, because of the small expected amplitudes (Figure 8.2). Putting this in another way, the anisotropy would affect less than 20% of the counting rate above ground as against 75% at 40 m.w.e., or 5% as against 25% if $R_C$ were greater than about 300 GV (see Figure 6.6). Nevertheless, because of the information it would give concerning $\gamma$ and $R_C$, unambiguous and significant evidence for the anisotropy from detectors with relatively low mean rigidities of response would be of great value.

# 9. CONCLUDING REMARKS

## 9.1. GENERAL COMMENTS ON THE EVIDENCE

The investigation began with observation of a persistent annual apparent sidereal effect in the underground data from Hobart. The purpose of subsequent experiments and analyses of data has been essentially to determine the origin of the underground effect.

In the first place, there can be no doubt as to the existence of a characteristic annual sidereal daily variation at Hobart. The basic features can be identified in each of the eight years of data. In the long-term average, the diurnal amplitude is found to differ from zero by approximately twenty standard errors and the semi-diurnal amplitude by four.

Evidence has accumulated that the underground effect is due to a two-way sidereal anisotropy and, on the basis of the empirical model used, an additional streaming component. Four major lines of evidence have been presented:

1. the evidence from the annual daily variations of vertical intensity in solar, sidereal and anti-sidereal time at Hobart;
2. the evidence from the phase anomalies;
3. the evidence for a 12-hour phase difference of sidereal origin between the two hemispheres, as verified by observations in the vertical and inclined (70°N) directions at Hobart;
4. the evidence from the diurnal and semi-diurnal components which have been shown to conform with the empirical model for the anisotropy in free space, notably in respect of their latitude dependences of amplitude.

The possibility of a significant spurious component in the observations has been a primary consideration. The evidence concerning this was of two kinds:

(1) The sideband analysis of the vertical intensity at Hobart showed that, if the persistent and uniform annual daily variation in sidereal time was spurious, it could not be reconciled with the behaviour of the daily variations in solar and anti-sidereal time. A major difficulty was the lack of evidence for a solar component large enough to generate the required sidereal sideband component. If there had been such a solar component, it would have been difficult to explain the absence, in most years, of a significant sideband component in anti-sidereal time. It was proposed that in 1958 and 1959, when the anti-sidereal effect was prominent, modulation of the sidereal component itself may have occurred.

(2) The elements of evidence for the two-way anisotropy showed significant compatibility. Thus, the occurrence of the phase anomalies were consistent with the secular trends of amplitude of the observed solar and sidereal components; the sidereal times of maximum obtained from annual averages and from the phase

anomalies agreed; the 12-hour difference in the sidereal diurnal maxima between the hemispheres agreed with the phase difference observed in the vertical and inclined (70°N) directions at Hobart and was consistent with the presence of appropriate second harmonics; and the compatibility of the estimated free-space amplitudes relating to three widely spaced latitudes of viewing were demonstrated for both diurnal and semi-diurnal components. It is submitted that, if the observed sidereal effect had been spurious, or had contained a large spurious component, there should instead have been an obvious lack of agreement amongst these varied contributions to the evidence. In fact, no significant incompatibilities were found.

The coherent nature of the evidence from the few detecting units was a main source of information that the data selected for analysis were reliable. A useful indication also came from the response to the solar anisotropy: subject to the estimated statistical errors, which were serious only in the data from the south-pointing telescope, there was good agreement amongst all detectors as to the response underground, particularly so between the Hobart and Budapest vertical telescopes. Again, analysis of variance indicated that, if systematic effects were removed, the bi-hourly values of vertical intensity at Hobart would follow essentially the expected Poisson distribution.

The data from ground level that were examined showed much variability, both year-to-year and shorter-term, and it was concluded that the usual methods of analysis would be unsuitable. Nevertheless, there were some indications that the neutron and charged particle detectors were responding to the sidereal anisotropy.

Finally, it will have been noted that, if we disregard the latitude dependence of free-space harmonics, the evidence that a sidereal anisotropy exists did not depend on assumptions as to the nature of the interplanetary magnetic field. In fact, it did not require that the primaries be charged, although it seemed unlikely from other considerations that the anisotropy would be due to neutral primaries. Even when the evidence relating to the free-space harmonics is included, it is apparent that a wide range of values of $R_C$ would be tolerated.

## 9.2. THE TWO-WAY ANISOTROPY

The underground observations are consistent with a simple two-way anisotropy (equal amplitudes in opposite directions) and a superimposed uni-directional component. The estimation of the southward direction of the bi-directional component takes account of the influence of the earth's magnetic field only, and thus gives an annual average direction in interplanetary space, the co-ordinates being ($RA = 0640 \mp 0118$, Dec. $= -38 \pm 7°$). The direction is close to that estimated for the galactic magnetic field in the local region. On the other hand, if it were assumed that the declination was that of the normal to ecliptic, i.e., $d = \pm\ 66°\ 30'$, there would be clear disagreement with the latitude dependence of free-space diurnal amplitude (Figure 7.3). Therefore, a galactic origin for the bi-directional component is indicated, although it does not follow that this would also hold for the uni-directional component.

The calculations, based on a simple empirical model, require assumptions to be made as to the values of $\gamma$, the index of the variation spectrum, and $R_C$, the primary threshold rigidity for observations. From the few estimations made, it is clear that $\gamma$ may take values from 0 to $-2$ at the least, and the choice for $R_C$ may range from about 30 GV to not less less than 200 GV, without affecting the estimate of the direction of the anisotropy.

Two determinations of $R_u$, the upper limiting rigidity of the solar anisotropy, have been made and indicate that $R_u$ may have decreased from approximately 100 GV in 1958 to perhaps 60 GV in 1962. It seems unlikely that $R_C$ would be less than $R_u$ and this has guided the choice of a range of values for $R_C$ in the calculations of response.

There has been some evidence of a response to the anisotropy from the data at ground level, notably from the phase anomaly in 1954. If there has been a response, typical amplitudes can hardly have been less than approximately 0·02%. Guided by the response characteristics of the sea-level telescope at Hobart, this suggests that, unless the amplitude of the anisotropy was much greater then than it is now, $\gamma$ could not have been positive.

The counting rates from the underground experiments were too low to allow significant values of the individual intensity maxima in the frame of reference of the anisotropy to be determined. However, preliminary estimates gave approximately 0·05%, not exceeding 0·2% for the uni-directional component, and approximately 0·2%, not exceeding 0·6% for the two-way component.

There are indications from the eight years' observations of vertical intensity underground that the sidereal daily variation at Hobart is subject to both seasonal and year-to-year changes. It is believed that pronounced semi-annual variations of amplitude and, less markedly, of phase, may have occurred during 1958 and 1959 and it was tentatively suggested in Section 5.7 (b) that these seasonal changes may have been connected with the scattering and reflection of primaries in the interplanetary field as they approached the sun. As to the year-to-year change, it is evident that the annual amplitude has decreased since 1958, although the trend is not a very obvious one (Figure 5.9) and that an amplitude peak occurred between 1961 and 1962. The latter, being associated with a minimum of amplitude of the solar daily variation and with the occurrence of a well-defined phase anomaly, has been interpreted as evidence of a temporary decline in the ability of the interplanetary field to trap the higher energy primaries and thus to restrict observation of the sidereal anisotropy. On the other hand, it is not yet clear that the longer-term decrease of sidereal amplitude is associated with solar activity.

### 9.3. THE UNI-DIRECTIONAL COMPONENT

The proposal that a streaming component might explain the larger diurnal amplitudes in the southern hemisphere is a very tentative one. There has been no indication from the analyses as to whether a single mechanism, producing a two-way anisotropy with unequal amplitudes in the two hemispheres, is operating or whether there are two mechanisms, one producing a simple two-way anisotropy symmetrical about the equatorial plane and the other producing a superimposed uni-directional anisotropy. The declination of the uni-directional term has been taken to be the same as that of the bi-directional term, but it is most important to note that only the declination of the latter has been determined. If the inclinations of the two components happened to be different, the only parameter whose esimate would be affected would be the uni-directional amplitude constant $m$. The observations suggest that the right ascensions of both components would be at least approximately in alignment.

On the hypothesis of a single mechanism, with the anisotropy being entirely of galactic origin, it might be worth considering that the opacity of the inter-planetary medium differs between the two hemispheres, in such a manner that $R_C$ has been appropriately smaller in the southern hemisphere over the period of observations.

On the hypothesis of two mechanisms, both galactic, it could be argued that the uni-directional anisotropy is due to the streaming of cosmic rays along the galactic magnetic field. On the other hand, if the uni-directional anisotropy were to be of solar origin, the possibility that it derives from a radial density gradient of cosmic ray intensity in the presence of the sectorised interplanetary field must be considered. This possibility will be referred to again in Sections 9.4 and 9.5 below.

## 9.4. RECENT DEVELOPMENTS, 1967-1969

Since the investigations described in the body of this report were concluded, some interesting new results have been reported, both from experiments in other countries and from measurements with directional telescopes at Mawson and Hobart. It would not be possible to do justice in a short space to the varied experimental arrangements and analytical methods that have been used. It must suffice here to list the experiments and associated papers known to the author, with one or two comments on outstanding feaures.

### *(a). Measurements at sea level*

(i) Large air Cerenkov narrow-angle telescopes have been employed by Sekido and his co-workers to detect high energy mesons (> 10 GEV at the instrument) at a zenith angle of 60° in the east and west directions (1964-65) and in the equatorial plane (1968). The results of sidereal analyses of the (E-W) difference have been reported (Sekido *et al.* 1968, 1969).

(ii) Using intensity difference methods, Nagashima, Ueno, Mori *and* Sagisaka have compared sidereal analyses of 20 years of ion chamber data with analyses of meson and neutron data over the period of the IGY (Nagashima *et al.* 1968).

(iii) Geiger counter telescopes set at a zenith angle of 45° to record the intensity of mesons from the west and east geomagnetic headings have been operating continuously at Hobart and at Mawson since 1956. The (W-E) difference has been used in a sidereal analysis of 10 years' data (1957-1966) from each place (Jacklyn *and* Vrana 1969).

(iv) A compound Geiger counter telescope has been operating continuously at Mawson since early 1968. With directions of viewing in the north and south geographic directions at a zenith angle setting of 76°, the atmospheric depth is approximately 40 m.w.e. A sidereal analysis of the observations in 1968 has been carried out for the individual directions of viewing (Jacklyn *and* Vrana 1969).

Significant observations of the diurnal and semi-diurnal components were obtained from investigations (i), (iii) and (iv), and of the diurnal component from investigation (ii). In all cases the results were in accord with the descriptive model for the anisotropy.

*(b). Observations underground*

(i) Some results of measurements with very large plastic scintillator telescopes at depths from 25 m.w.e. to 40 m.w.e. at Mt. Chacaltaya, Bolivia and at Embudo Cave near Alburquerque, New Mexico, have been reported by Swinson (1969). The observations from 8 telescopes in the two hemispheres include a result from a telescope pointing into the northern hemisphere from Mt. Chacaltaya. All the diurnal maxima conform with the bi-directional model. Swinson has proposed that the bi-directional diurnal effect can be explained as a solar phenomenon, in terms of a radial cosmic ray density gradient in the presence of the sectorised interplanetary magnetic field, producing unequal intensity maxima normal to the plane of the ecliptic. However, aside from other considerations, in its assumption that there is no important semi-diurnal contribution, his suggestion does not take account of the central observational feature, the latitude-dependent interplay between the diurnal and semi-diurnal components. In fact, the model which describes the bi-directional anisotropy in terms of two harmonics distinguishes clearly (viz., Figure 7.3) between the best estimate of the declination, $-38°$, and the declination of the normal to the ecliptic, $-66° 30'$, required by a density gradient hypothesis. If there is an annual residual density gradient effect, it is considered that it would be essentially uni-directional, inequalities of intensity maxima in the two directions transforming to small higher harmonics, the general form of the anisotropy being that proposed by L. Davis (1954). It can only be conjectured at present that such an effect might explain the "streaming" component (Figure 8.1).

(ii) Measurements with large plastic scintillator telescopes located at a depth of 70 m.w.e. at Torino, Italy, are being carried out by Cini-Castagnoli and co-workers. Within their limits of error, the sidereal diurnal and semi-diurnal components reported for the year 1966 conform with expectations for the northern hemisphere. (Cini-Castagnoli *et al.* 1969).

### 9.5. FINAL DISCUSSION OF THE ANISOTROPY AND OF SOME OF THE PROBLEMS OUTSTANDING

The principal finding from the underground experiments at Hobart and Budapest, the high zenith angle experiments at Nagoya, the East-West experiments at sea level at Hobart and Mawson and the high zenith angle North-South experiment at Mawson is that there exists a two-way sidereal anisotropy with diurnal and semi-diurnal components. The bi-directional nature of the diurnal component has been additionally confirmed by the recent American underground experiments. Evidence from somewhat greater depths underground, at London and at Bologna, and from sea level, where many years of data from the ion chambers of Forbush *and* Lange, and data from a number of directional telescopes in the U.S.S.R., have been analysed, is consistent with the other findings. Finally, to the author's knowledge there have been no statistically significant reliable observations to the contrary.

It has also been shown that the total anisotropy resolves into bi-directional and uni-directional components and it is conceivable that these have different origins. The estimate of direction from the underground latitude survey, and the preliminary estimate of the threshold rigidity from the North-South experiment, indicate that the bi-directional part of the anisotropy is of galactic origin and that it is apparently aligned with the local direction of the galactic magnetic field. It will be important to substantiate this. As for the uni-directional component, it will be necessary to determine whether it is essentially of galactic or of solar origin.

To confirm the galactic nature of the two-way anisotropy, in the first place the direction should be accurately re-determined from a latitude survey, preferably using large detectors underground. The present moderate depths seem to be the most practicable. Secondly, it will be necessary to obtain a firm estimate of the energy range. This is the major objective of the experimental programme being developed by the Antarctic Division for the Mawson station. In the first stage of the programme an array of high zenith angle North-South telescopes is to be installed at sea level. At the mean zenith angle of viewing, 76°, the atmospheric depth is approximately 40 m.w.e. A North-pointing telescope scans equatorially and observes the maximum of the sidereal semi-diurnal component. The asymtotic longitude of viewing is not very dependent on energy. On the other hand, a South-pointing telescope scans mid-latitudes and observes the maximum of the sidereal diurnal component. The asymptotic longitude of viewing is an extremely sensitive function of energy. In consequence, the sidereal diurnal and semi-diurnal phase relationships from the North-South experiment are expected to give basic information about the energy range of the bi-directional effect. They should also help us to determine the nature of the uni-directional component, particularly whether or not it is of solar origin. A description of the experimental arrangement and results of the first year of observation with a prototype telescope have been published (Jacklyn *and* Vrana 1969). The observations are in accord with the descriptive model and already suggest that the threshold rigidity for the total anisotropy is not less than about 100 GEV.

On the evidence, the bi-directional effect is galactic, while we have no information as to the origin of the uni-directional sidereal component. If the latter should happen to be of solar origin (through the radial density gradient), it should not be observable at a sufficiently great depth underground and we should see only a simple bi-directional anisotropy. To allow for the possibility that the upper limit of energy of the effects of solar activity in sidereal time may be greater than the upper limit for the solar co-rotation anisotropy, the effective primary threshold energy of response underground might have to be not less than 200 GEV. Then it seems that the depth of observation should lie somewhere between 200 m.w.e. and 300 m.w.e. Undoubtedly, observations with the desired accuracy of amplitude of $\pm$ 0·005% at these depths would be of great value. However, to achieve an annual result to the level of accuracy at any one location, approximately seventy semi-cubical detectors, each of one metre-square surface area, or an equivalent arrangement, would be required. If detecting installations on this large scale are not to be contemplated, it seems clear that the advantage at present lies with varied experiments at depths nearer 40 m.w.e., where not only is the intensity much greater but the earth's magnetic field can still be used effectively for determining primary energy response.

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The study of small intensity variations over a long period has called for very reliable Geiger-counter performance and painstaking and efficient processing of data. It is a pleasure, therefore, to acknowledge the high standard of the services of Mr. M. Mason, glassblower in the Physics Department, and his assistant, Mr. M. P. Cronley, and of Mrs. P. S. James, Mrs. F. R. Chappell and other members of the Cosmic Ray computing staff at Hobart.

## 11. REFERENCES


BAKER, P., CHASSON, R. L., *and* KISSELBACH, V. J. (1966). *Nature* **211**: 354.

BALIGA, S. P., *and* THAMBYAHPILLAI, T. (1959). *Philosophical Magazine* **4**: 973.

BECKER, W. (1964) *Zeitzschrift für Astrophysik* **58**: 202.

BEER, A. (1961). *Monthly Notices of the Royal Astronomical Society* **123**: 191.

BEHR, A. (1959). *Veröflentichungen der Universitätssternwarke zu Gottingen* No. 126.

BERCHOVITCH, M. (1963). *Proceedings of the International Conference on Cosmic Rays, Jaipur, India* **2**: 350.

BERDICHEVSKAYA, T. M., *and* ZHUKOVSKAYA, N. A. (1960) [*Transactions Ya Fan.* (Cited in Dorman (1963) below)].

BETHE, Z. (1932). *Physica* **76**: 293.

BLAAUW, A., GUM, S. C., PAWSEY, J. L., *and* WESTERHOURT, G. (1959). International Astronomical Union Information Bulletin No. 1: 4-6.

BLOCH, Z. (1935). *Physica* **81**: 363.

BRUNBERG, E. A., *and* DATTNER, A. (1953). *Tellus* **5**: 269.

CHANDRASEKHAR, S., *and* FERMI, E. (1953). *Astrophysical Journal* **118**: 113.

CINI-CASTAGNOLI, G., DODERO, M. A., *and* ANDREIS, L. (1968). *Canadian Journal of Physics* **46**: 5839.

CLAY, J. (1939). *Reviews of Modern Physics* **11**: 128.

COMPTON, A. H., *and* GETTING, I. A. (1935). *Physical Review* **47**: 817.

CONFORTO, A. M., *and* SIMPSON, J. A. (1957). *Nuovo Cimento* **6**: 1052.

COUSINS, J. E., *and* NASH, W. F. (1962). *Advances in Physics* **11**: 349.

COUSINS, J. E., NASH, W. F., *and* POINTON, A. J. (1957). *Nuovo Cimento* **6**: 1113.

COWAN, C., RYAN, D., *and* BUCKWALTER, G. (1965). *Proceedings of the International Conference on Cosmic Rays, London* 2: 1041.

DAVIES, R. D., SHUTER, W. L. H., SLATER, C. H., *and* WILD, P. A. T. (1964). *Monthly Notices of the Royal Astronomical Society* **126**: 353.

DAVIS, L. (1954). *Physical Review* **96**: 743.

DEMMELMAIR, A. (1937). *Sitzungberichte der Preussischen Akademie der Wissenschaften zu Berlin* **11**: 146.

DORMAN, L. I. (1957). Cosmic ray variations. Moscow State Publishing House. (Translation by Technical Document Liaison Office, U.S. Air Force).

DORMAN, L. I. (1963). *In* "Progress in Elementary Particle and Cosmic Ray Physics." (Vol. VII). (North-Holland: Amsterdam).

DORMAN, L. I., *and* INOZEMTSOVA, O. I. (1961). Results of IGY (Moscow) Cosmic Rays. No. 4: 209.

DUPERIER, A. (1949). *Proceedings of the Physical Society* (A) **62**: 684.

DUTT, J. C., *and* THAMBYAHPILLAI, T. (1965). *Journal of Atmospheric and Terrestrial Physics* **27**: 349.

ELLIOT, H., *and* DOLBEAR, D. W. N. (1951). *Journal of Atmospheric and Terrestrial Physics* **1**: 205.

FARLEY, F. J. M., *and* STOREY, J. R. (1954). *Proceedings of the Physical Society* (A) **67**: 996.

FENTON, A. G. (1963). *Proceedings of the International Conference on Cosmic Rays, Jaipur, India* **2**: 185.

FENTON, A. G., JACKLYN, R. M., *and* TAYLOR, R. B. (1961). *Nuovo Cimento* **22**: 285 [PAPER 3 OF REPORT].

FENTON, K. B. (1951). Ph.D. Thesis, University of Tasmania.

FERMI, E. (1954). *Astrophysical Journal* **119**: 1.

FERMI, E. (1949). *Physical Review* **75**: 1169.

GARDNER, F. F., *and* DAVIES, R. D. (1966). *Australian Journal of Physics* **19**: 129.

GINSBERG, V. L., *and* SYROVATSKII, S. I. (1964). "Origin of Cosmic Rays." (Pergamon Press: New York).

GLOKOVA, E. S. (1960). *Proceedings of Moscow Cosmic Ray Conference* **4**: 251.

HALL, J. S. (1958). *Publications of the United States Naval Observatory* (second series) **17**: 275.

HESS, V. F., *and* STEINMAUER, R. (1933). *Sitzungberichte der Preussischen Akademie der Wissenschaften zu Berlin* (Phys.-Math. KI.) **15**.

HILTNER, W. A. (1951). *Astrophysical Journal* **114**: 241.

HOGG, A. R. (1949). *Memoirs of the Commonwealth Observatory, Canberra.* No. 10.

ILLING, W. (1936). *Terrestrial Magnetism and Atmospheric Electricity* **41**: 188.

JACKLYN, R. M. (1962) *Nuovo Cimento* **24**: 1034. [PAPER 2 OF REPORT].

JACKLYN, R. M. (1963a). *Nature* **200**: 1306. [PAPER 1 OF REPORT].

JACKLYN, R. M. (1963b). *Novo Cimento* **30**: 40. [PAPER 4 OF REPORT].

JACKLYN, R. M. (1963c). *Proceedings of the International Conference on Cosmic Rays, Jaipur, India* **2**: 235. [PAPER 6 OF REPORT].

JACKLYN, R. M. (1965). *Nuovo Cimento* **37**: 1135. [PAPER 5 OF REPORT].

JACKLYN, R. M. (1966). *Nature* **211**: 690. [PAPER 8 OF REPORT].

JACKLYN, R. M. *and* HUMBLE, J. E. (1965). *Australian Journal of Physics* **18**: 451. [PAPER 7 OF REPORT].

JACKLYN, R. M., *and* VRANA, A. (1969). *Proceedings of the Astronomical Society of Australia* **1** (6): 278.

KERR, F. J. (1966). Symposium, Radio and Optical Studies of the Galaxy, Mount Stromlo Observatory **1**: 22.

KITAMURA, M., *and* KODAMA, M. (1960). *Proceedings of Moscow Cosmic Ray Conference* **4**: 202.

KOLHORSTER, D., *and* VON SALIS (1926). *Naturwissenschaften* **14**: 1963.

KUZMIN, A. I. (1960). *Transactions Ya Fan* (Physics Series). [Cited in Dorman (1963) above].

LANGE, L., *and* FORBUSH, S. E. (1948). Volume 14 of the Researches of the Department of Terrestrial Magnetism. (Carnegie Institute of Washington Publication 175).

LINDLEY, D. V., *and* MILLER, J. C. P. (1953). "Cambridge Elementary Statistical Tables." (Cambridge University Press).

MCCRACKEN, K. G., *and* RAO, U. R. (1965). *Proceedings of the International Conference on Cosmic Rays, London* **1**: 213.

MCCRACKEN, K. G., RAO, U. R., *and* SHEA, M. A. (1962). Massachussetts Institute of Technology Laboratory for Nuclear Science Technical Paper No. 77 (NYO-2870).

MATHEWS, T. (1963). *Philosophical Magazine* **8**: 387.

MATHEWSON, D. S., *and* MILNE, D. K. (1965). *Australian Journal of Physics* **18**: 635.

MATHEWSON, D. S., BROTON, N. W., *and* COLE, D. J. (1966). *Australian Journal of Physics* **19**: 93.

MORRIS, D., *and* BERGE, G. L. (1964). *Astrophysical Journal* **139**: 1388.

MURAYAMA, T. (1960). *Proceedings of Moscow Cosmic Ray Conference* **3**: 140.

MURDOCH, H. S., OGILVIE, K. W., *and* RATHGEBER, H. D. (1960). *Proceedings of Moscow Cosmic Ray Conference* **1**: 304.

NAGASHIMA, K., UENO, H., MORI, S., *and* SAGISAKI, S. (1968). *Canadian Journal of Physics* **46**: S611.

PARKER, E. N. (1964). *Planetary and Space Science* **12**: 735.

PARKER, E. N. (1965). "The Dynamical Properties of Cosmic Rays." (University of Chicago Press: Chicago).

PARSONS. N. R. (1959). Ph.D. Thesis, University of Tasmania.

PARSONS, N. R. (1957). The design and operation of ANARE cosmic ray recorder "C". *ANARE Interim Reports* No. 17.

QUENBY, J. J., *and* THAMBYAHPILLAI, T. (1960). *Philosophical Magazine* **5**: 585.

QUENBY, J. J., *and* WEBBER, W. R. (1959). *Philosophical Magazine* **4**: 90.

RAO, U. R., MCCRACKEN, K. G., *and* VENKATESAN, D. (1963). *Journal of Geophysical Research* **68**: 345.

SANDOR, T., SOMOGYI, S., *and* TELBISZ, P. (1960). *Nuovo Cimento* **17**: 1.

SARABHAI, V. A., *and* SUBRAMANIAN, G. (1963). *Proceedings of the International Conference on Cosmic Rays, Jaipur, India 2*: 307.

SCHONLAND, B. J. F., DELATITZKY, B., *and* GASKELL, J., (1937). *Terrestrial Magnetism and Atmospheric Electricity* **42**: 137.

SEKIDO, Y., KONDO, I., MURAYAMA, T., KAMIYA, K., UENO, H., MORI, S., OKUDA, H., MAKINIO, T., SAKAKIBARA, S., *and* FUJIMOTO, K. (1963). *Proceedings of the International Conference on Cosmic Rays, Jaipur, India* **4**: 194.

SEKIDO, Y., NAGASHIMA, K., KONDO, I., MURAYAMA, T., OKUDA, H., SAKAKIBARA, S., *and* FUJIMOTO, K. (1968). *Canadian Journal of Physics* **46**: S 607.

SEKIDO, Y., NAGASHIMA, K., KONDO, I., MURAYAMA, T., *and* SAKAKIBARA, S. (1969). *Proceedings of the International Conference on Cosmic Rays, Budapest, Hungary.*

SERKOWSKI, K. (1962). "Advances in Astronomy and Astrophysics." (Ed. Z. Kopal). (Academic Press: New York).

SMITH, E. VAN P. (1956). *Astrophysical Journal* **124**: 43.

SWINSON, D. (1969). *Journal of Geophysical Research* **74**: 5591.

THAMBYAHPILLAI, T., DUTT, J. C., MATHEWS, T., *and* ROMERO, F. (1965). *Proceedings of the International Conference on Cosmic Rays, London* **1**: 138.

THOMPSON, J. L. (1939). *Physical Review* **58**: 11.

THORNDIKE, A. M. (1952). "Mesons." (McGraw-Hill: New York).

WHITTAKER, E. T., *and* ROBINSON, G. (1944). "The Calculus of Observations." (Fourth edition). (Blackie: Glasgow).

WILCOX, J. M., *and* NESS, N. F. (1965). *Proceedings of the International Conference on Cosmic Rays, London* **1**: 302.

WILSON, B. G. (1959). *Canadian Journal of Physics* **37**: 19.

WOLLAN, E. O. (1939). *Review of Modern Physics* **11**: 160.

## APPENDIX

## RADIATION SENSITIVITY AS A FUNCTION OF ASYMPTOTIC DIRECTION OF VIEWING

### I.1. RADIATION SENSITIVITY SPECIFIED IN THE ALT-AZIMUTH FRAME OF REFERENCE

In Section 6A.5 (see also Parsons 1957) the differential radiation sensitivity associated with the elementary solid angle of arrival $\omega_r$ is given by

$$I(R,\omega_r) = Y_R\, F(\omega_r) \cos {}^n Z_r,$$

where $Y_R$, the differential coupling coefficient, gives the fraction of the counting rate due to primaries of rigidity $R$, $F(\omega_r)$ is the geometric sensitivity and $\cos {}^n Z_r$ expresses the zenith-angle dependence of intensity. $F(\omega_r)$ is initially specified in the frame of reference of the telescope, referred to the axis and the plane of a tray. (In this respect the treatment of $F(\omega_r)$ for inclined telescopes differs somewhat from that given by Parsons.) To obtain $I(R,\omega_r)$, it is necessary to transform to the frame of reference of altitude and geographic azimuth referred to the zenith and the plane of the horizon. Naturally, if the telescope axis is vertical, only a formal exchange of symbols is required (depending on the reference direction for azimuth in the plane of the tray), since the two frames of reference are then coincident.

### I.2. THE DEFLECTION DATA SPECIFIED IN THE ALT-AZIMUTH FRAME OF REFERENCE

Brunberg *and* Dattner (1953) give asymptotic bearings in geomagnetic latitude and longitude for particles which arrive in directions specified by altitude and geographic azimuth. When the asymptotic bearings are expressed in this form, interpolation from the diagrams become impracticable for some directions if the cone of viewing of the telescope, such as the south-pointing cube at Hobart, takes in the polar axis. It was therefore found more convenient to express the asymptotic latitude and longitude data in the form of deflections in zenith angle and geographic azimuth, thus transforming from the geomagnetic equatorial to the alt-azimuth system of co-ordinates.

### I.3. CONVERSION OF CO-ORDINATES USING A TERRESTRIAL GLOBE

Since the facilities of a high-speed computer were not available when the calculations were to be made, a terrestrial globe (Figure I.1) was employed to convert directly from, say, bearings in geomagnetic latitude and longitude to bearings in altitude and geomagnetic azimuth, at the given geomagnetic latitude of observation. By drilling a suitable pair of conjugate holes for the axis of rotation, transformation from one spherical co-ordinate system to another, involving simple rotations of axes, was effected by tilting the vertical axis of one frame of reference (represented by the the celluloid strips) with respect to that of the other (represented by the co-ordinate grid). With the exception of directions very close to the pole, co-ordinates of direction could be read off to an accuracy of a degree without difficulty.

### I.4. RADIATION SENSITIVITY AS A FUNCTION OF ASYMPTOTIC DIRECTION OF VIEWING

The steps are now listed by which sets of values of $I(R, \omega)$ and the associated asymptotic latitude and longitude co-ordinates, $(\delta_r)_R$ and $(\phi_r)_R$, may be obtained.

1. Initially, $F(\omega_r)$ is expressed as a function of inclination $(Z')$ to the telescope axis and azimuth $(A')$ in the plane of a tray.

2. Co-ordinates of directions are transformed from the telescope system to the alt-azimuth system (axis of rotation of globe set at $Z$ = inclination of telescope axis to zenith). $I(R, \omega_r)$ is obtained as a function of altitude $(a)$ and geographic azimuth $(A)$.

3. Asymptotic bearings in geomagnetic latitude and longitude, from the diagrams of Brunberg *and* Dattner, are converted to bearings in altitude and geomagnetic azimuth (axis of rotation of globe set at $\lambda$ = geomagnetic latitude of place of observation). Deflections in altitude and geographic azimuth are obtained by adding to the bearings in geomagnetic azimuth the constant angle $\alpha$ = difference between true north and geomagnetic north at the latitude of the place of observation. Figures I.2 to I.9 give the deflection data referred to these co-ordinates, for particles of rigidity 50 GV and 150 GV which arrive at the geomagnetic latitude $\lambda = -50^\circ$, approximating the latitude of Hobart.

4. From 2. and 3. above, $I(R, \omega_r)$ is obtained as a function of asymptotic direction, referred to the alt-azimuth co-ordinate system.

5. $I(R, \omega_r)$ is obtained as a function of asymptotic geographic latitude and longitude of viewing by converting from the alt-azimuth to the terrestrial equatorial co-ordinate system (axis of rotation of globe set at $\delta$ = geographic latitude of place of observation).

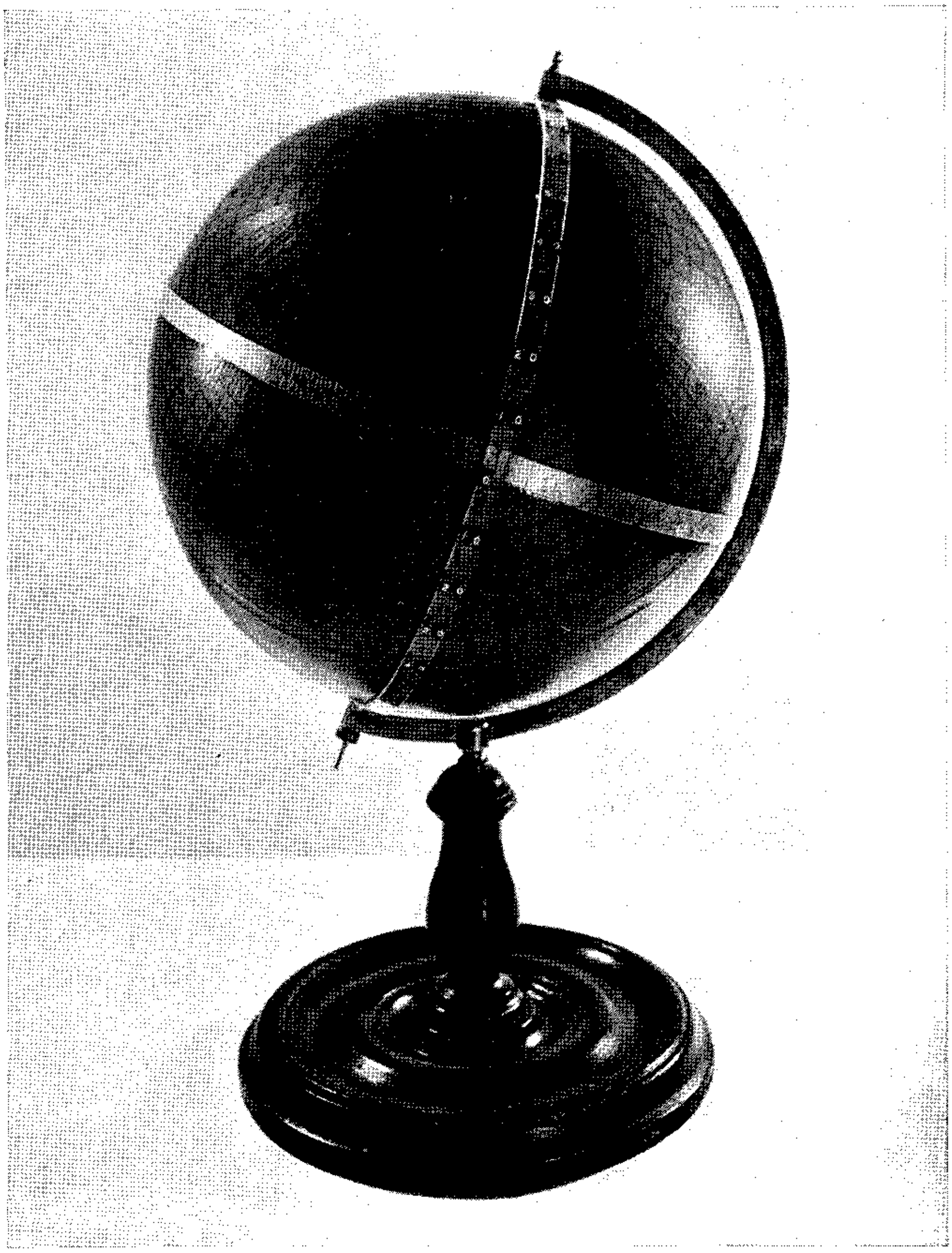

FIG. I.1. Terrestrial globe.

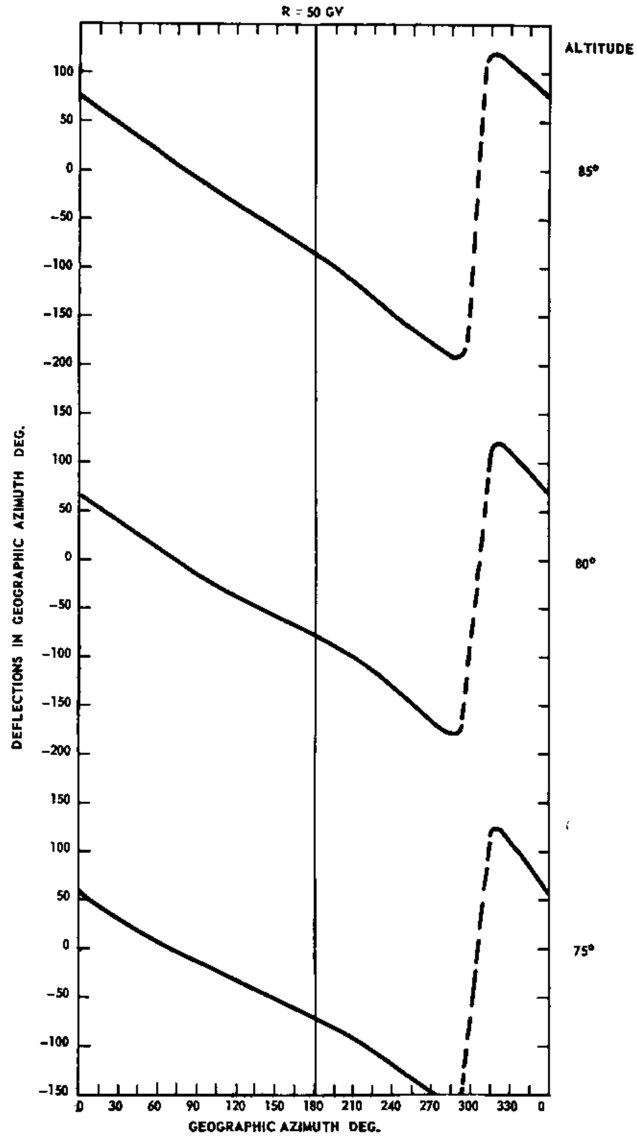


FIG. I.2. Deflections in azimuth of protons of rigidity 50 GV in centred dipole field for given directions of arrival at geomagnetic latitude —50°.

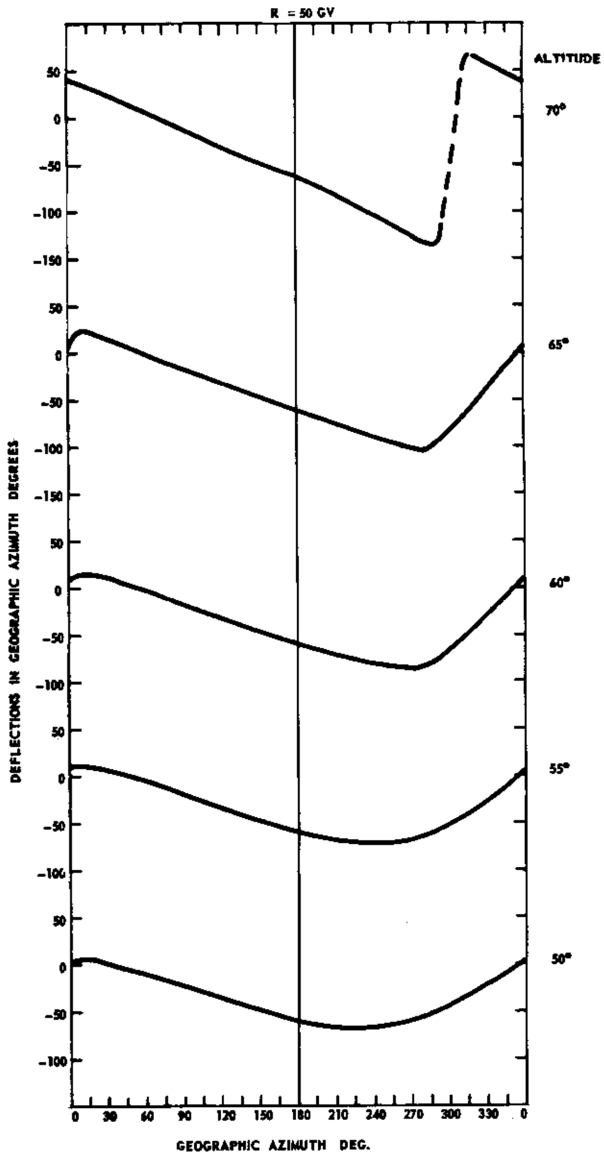


FIG. I.3. Deflections in azimuth of protons of rigidity 50 GV in centred dipole field for given directions of arrival at geomagnetic latitude —50°.

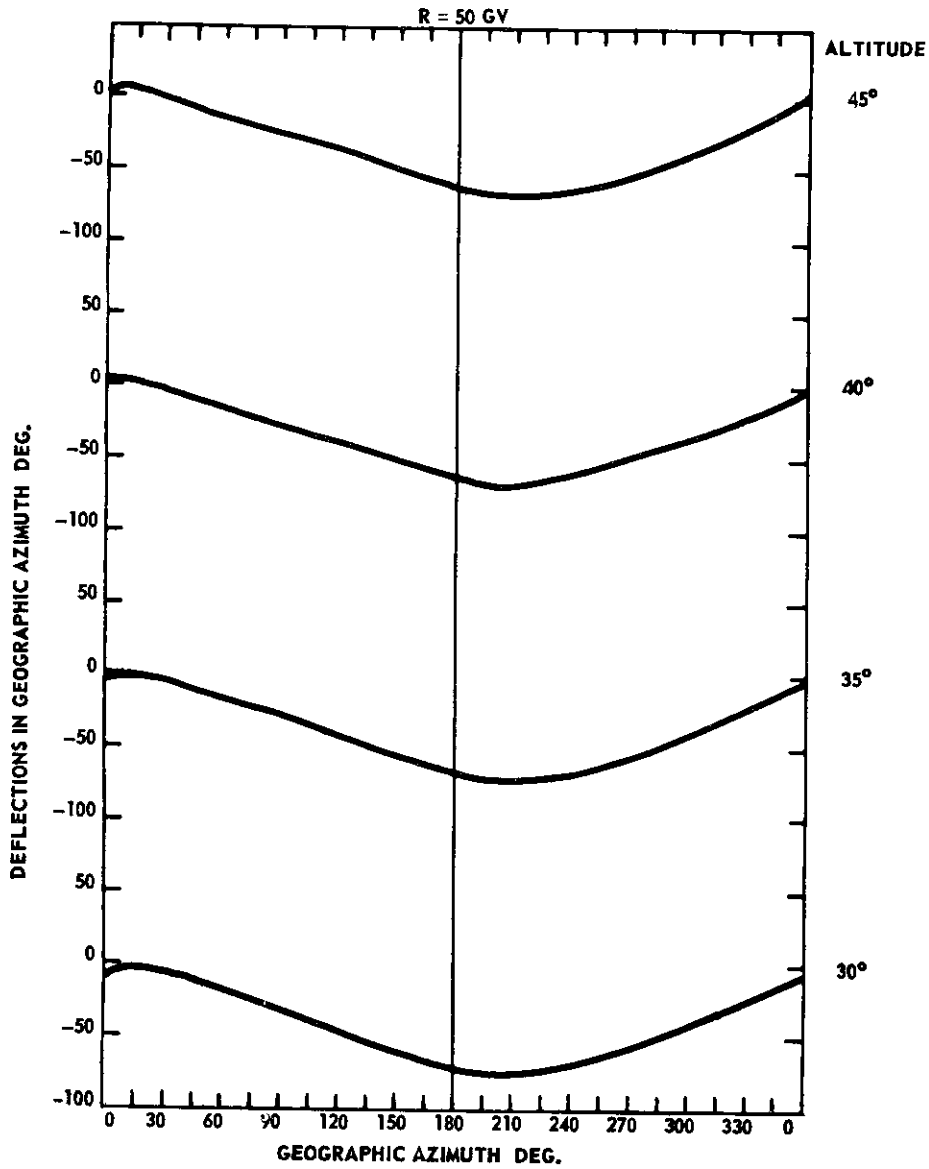


FIG. I.4. Deflections in azimuth of protons of rigidity 50 GV in centred dipole field for given directions of arrival at geomagnetic latitude —50°.

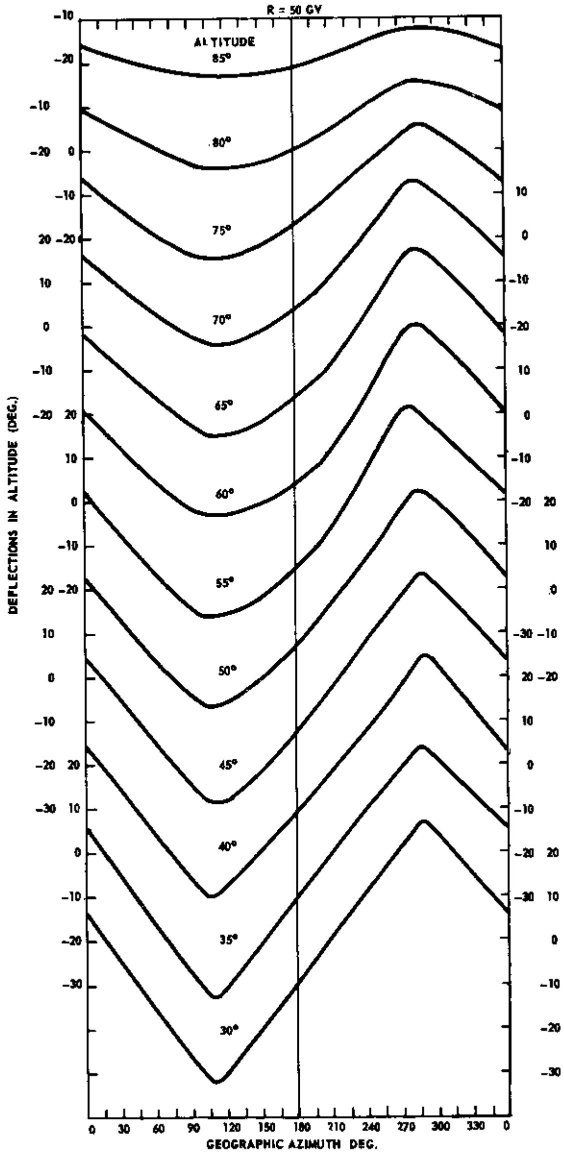


FIG. I.5. Deflections in altitude of protons of rigidity 50 GV in centred dipole field for given directions of arrival at geomagnetic latitude —50°.

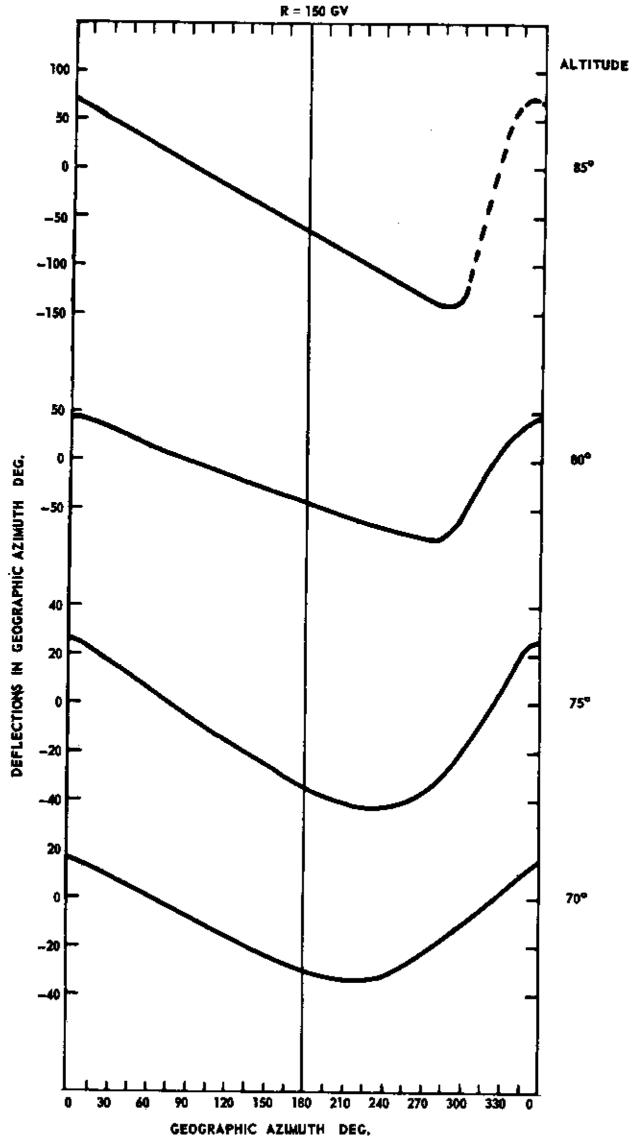


FIG. I.6. Deflections in azimuth of protons of rigidity 150 GV centred dipole field for given directions of arrival at geomagnetic latitude —50°.

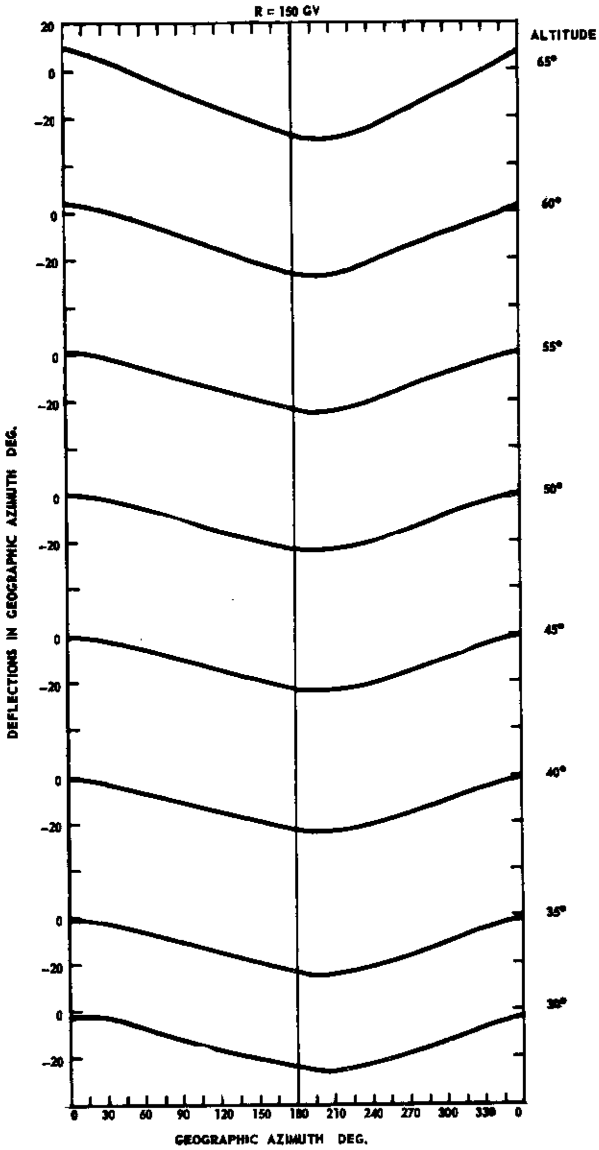


FIG. I.7. Deflections in azimuth of protons of rigidity 150 GV in centred dipole field for given directions of arrival at geomagnetic latitude —50°.

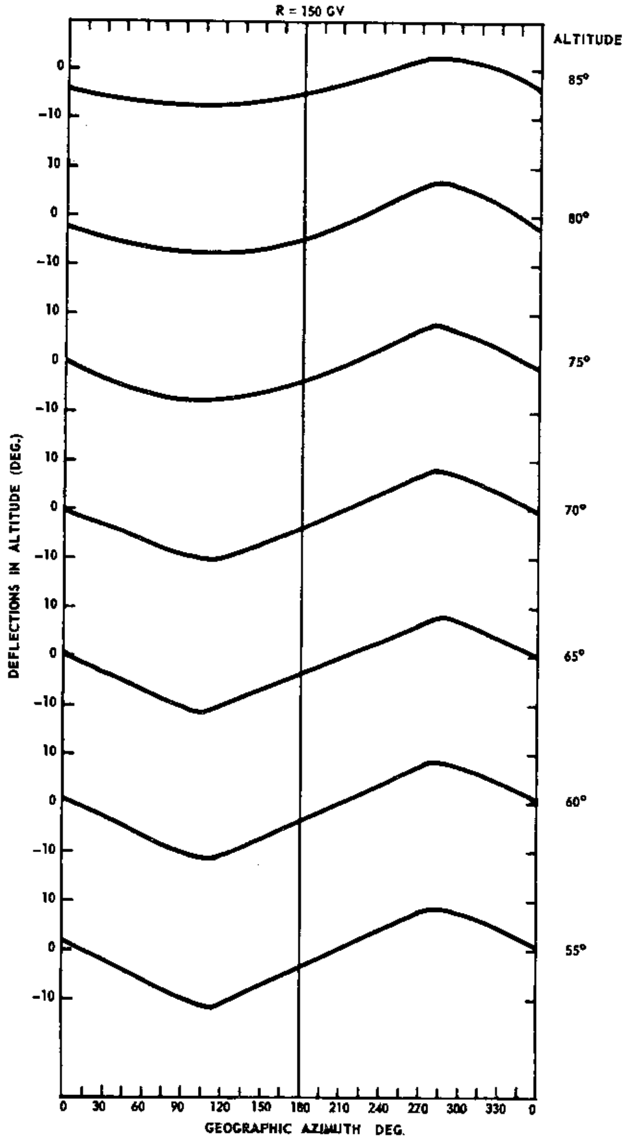


FIG. I.8. Deflections in altitude of protons of rigidity 150 in centred dipole field for given directions of arrival at geomagnetic latitude —50°.

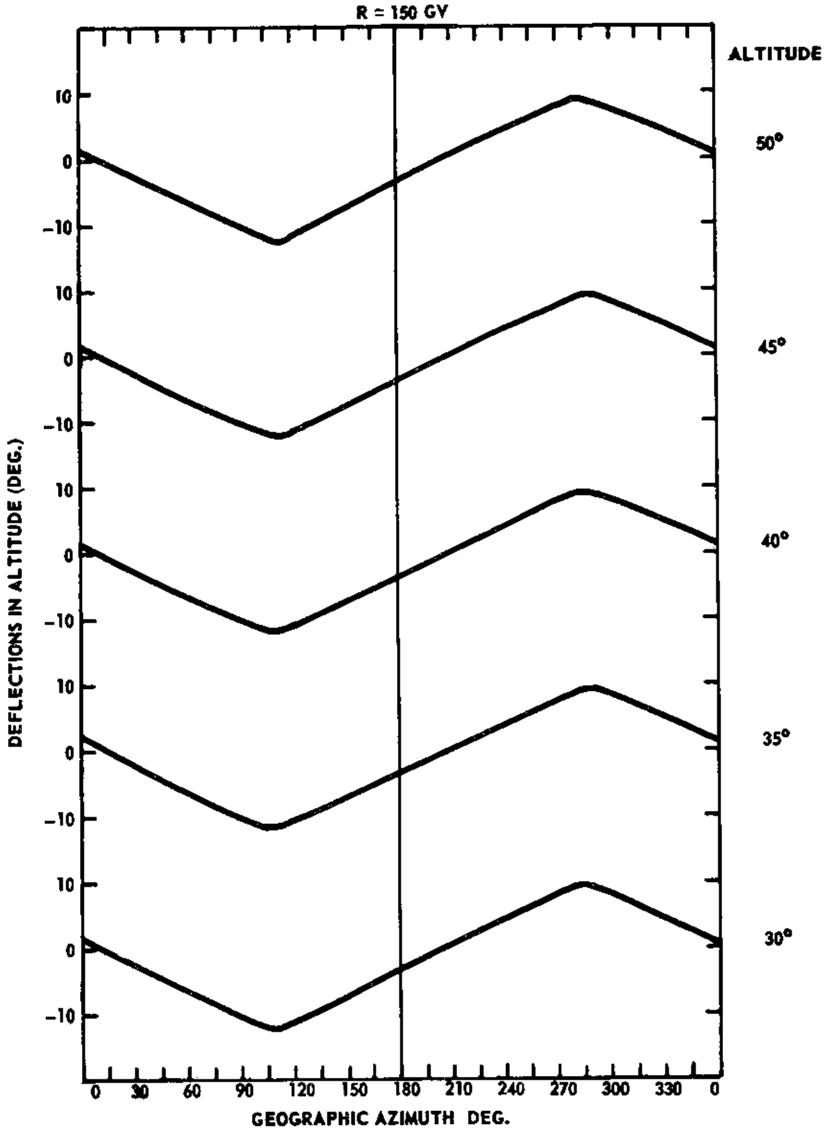


FIG. I.9. Deflections in altitude of protons of rigidity 150 GV in centred dipole field for given directions of arrival at geomagnetic latitude —50°.